Supplemental Appendix to

“Selective Reporting of Factual Content by Commercial Media”

This appendix provides additional analysis and clarifying details to support the claims made in the paper.

1. Model with Preference Structure without Facts

The purpose of our research regards the incentives of commercial media for factual content production. Therefore, the main model embeds consumers’ desire for additional facts. It is reasonable to posit that the underlying motivation of any curious reader is to ultimately know the underlying state of the world – the truth. This generates two questions about our main model. First, *how does a preference for additional facts relate to the desire to know the truth?* We show here that the preference structure of our main model is consistent and, moreover, implies an inherent desire to know the truth. The second question we address here is: *What happens in a stripped down version of our model in which all that consumers’ desire is simply to know the truth, independent of reading facts?* The analysis below shows that such a model leads to same qualitative results of the main model.

The simplest way to capture the direct preference for truth is the absolute distance between the truth (the state) and the media stance. Since no agent in our model observes the exact truth, we can substitute the observed state *t* (an unbiased estimator about the truth) to approximate the distance. Consumers hope to minimize the expected distance between the stance and *t*, or

$$T = -E(|s-t|)\,. \tag{1.1}$$

In specifying preferences in this way, the expected benefit for a consumer (with opinions *b*) obtains utility from a medium with a stance is given by

$$E\left[u_b(s,p)\right] = E[V + mT - d(s-b)^2 - p]\,. \tag{1.2}$$

In this formulation, note that the parameter *m* reflects the relative preference for knowing the truth. This corresponds to the parameter *M* in the main model, which captures the consumer’s preference for additional facts.

1.1 Preference for Facts Implies Desire for Truth

Consider the following two objectives: $\text{argmax}_s(N)$ and $\text{argmax}_s(T)$, where

$$N = \begin{cases} \frac{\Omega_1}{s} = \frac{\Omega \times t}{s}; \text{if } t < s \\ \frac{\Omega_0}{1-s} = \frac{\Omega \times (1-t)}{1-s}; \text{otherwise,} \end{cases}$$

as defined in the main model. We show here that these two objectives (i) imply the same optimum at *t* and (ii) respond similarly to changes in the stance. (i) We can clearly see that when $s = t$, $N = \Omega$ which is the maximum facts a medium can report. Similarly, $\max(T) = 0$ when $s = t$. Hence $\text{argmax}_s(N) = \text{argmax}_s(T) = t$. To show (ii), we show that when *s* moves away from *t* in both directions, both objectives decrease. In the section 2.2 we established that the further a medium's stance is from the state, the fewer facts it can use in its report to support that stance. It is straightforward to see that when *s* moves away from *t* in both directions, T decreases as well.

1.2 A Model with No Facts – Consumers Directly Value Knowing the State

In this section we show that a model in which consumer preferences are characterized by equations (1.1) and (1.2) above imply the same basic results of our main model in which consumers desire more facts. Specifically, we show

1. In a monopoly:
    a. There does not exist a fully informative equilibrium in monopoly.
    b. If *m* is sufficiently large, there exists a partially informative equilibrium. Otherwise there is only the uninformative equilibrium.
2. Competition (weakly) reduces media informativeness.

We start with (1-a). Assume there exists a fully informative equilibrium in which the monopoly medium's stance strategy is a one-to-one mapping $s^*: [0,1] \to [0,1]$ between *s* and *t*. Under a fully informative equilibrium, for any $t_2 \neq t_1$, we must have $\pi(t_1 \mid s_1) \neq \pi(t_2 \mid s_1)$ otherwise the medium could report either $s_1 = s^*(t_1)$ or $s^*(t_2) \neq s_1$ to earn equilibrium payoffs. Consumers could not, therefore, update their beliefs fully informatively. Without loss of generality, we assume $t_1 \le s_1$. By setting the equilibrium price, the profits are expressed:

$$\pi(t_1 \mid s_1) = V - m \mid t_1 - s_1 \mid - d \max\{(s_1 - 1 + z)^2, (s_1 - z)^2\}; \text{ and}$$
$$\pi(t_2 \mid s_1) = V - m \mid t_2 - s_1 \mid - d \max\{(s_1 - 1 + z)^2, (s_1 - z)^2\}.$$

Considering a case when $t_1 \le s_1 < t_2$. The condition $\pi(t_1 \mid s_1) \neq \pi(t_2 \mid s_1) \;\; \forall t_2 \in (s_1, 1]$, requires $s_1 - t_1 \neq t_2 - s_1$. This implies $s_1 > \frac{1}{2}$. We now show that when $t_1 = 0$, the medium has a profitable deviation. With $t_1 = 0$, $T = -\mid s_1 - t_1 \mid < -\frac{1}{2}$. Therefore, the medium can be better off by deviating with a report $s = \frac{1}{2}$, since no matter what consumers' belief about *t* when $s = \frac{1}{2}$, $\pi(t \mid s = \frac{1}{2}) = V + mT - d \max\{(\frac{1}{2} - 1 + z)^2, (\frac{1}{2} - z)^2\} > \pi(t_1 = 0 \mid s_1)$ since $-\mid \frac{1}{2} - t \mid \ge -\frac{1}{2}$, $\forall t$ and $\max\{(\frac{1}{2} - 1 + z)^2, (\frac{1}{2} - z)^2\} \le \max\{(s_1 - 1 + z)^2, (s_1 - z)^2\} \;\; \forall s_1$. Therefore, the medium will at least prefer $s = \frac{1}{2}$ than $s^*(t_1 = 0) \neq \frac{1}{2}$, which implies that $s^*(t)$ cannot be an equilibrium strategy.

Next we consider the possibility of an equilibrium in a monopoly that is not fully informative. We use the PBE equilibrium concept similar to that defined in the main text.

A less-than-fully informative equilibrium is characterized by a set of dividing points with $a_0 = 0 < a_1 < \cdots < a_{x-1} < a_x = 1$ *with* $x \ge 1$, *and a set of media stances* $(s_i)_{i=1,\cdots,x}$, *where* $s_i \neq s_{i'}, \forall i, i' \in \{1, \cdots, x\}$ *such that:*

1. $(a_i)_{i=0,\cdots,x}$ *and* $(s_i)_{i=1,\cdots,x}$ *satisfy* $\pi(s_i, a_{i-1}, a_i) = \pi(s_{i+1}, a_i, a_{i+1})$, *for* $i = 1, \ldots, x-1$ *where:*

$$\pi(s_i, a_{i-1}, a_i) = V + \frac{1}{a_i - a_{i-1}} m[-s_i^2 + s_i(a_i + a_{i-1}) - \tfrac{a_i^2}{2} - \tfrac{a_{i-1}^2}{2}] - \begin{cases} d(s_i - 1 + z)^2, \text{if } a_i \le \frac{1}{2} \\ d(s_i - z)^2, \text{if } a_{i-1} \ge \frac{1}{2} \end{cases}$$

2. $s_i = \arg\max_s \pi(s, a_{i-1}, a_i)$ *with* $s \in [a_{i-1}, a_i]$ *for any* $t \in [a_{i-1}, a_i]$;

3. *There is symmetry around the middle point:* $a_i = 1 - a_{x-i}$, $s_i = 1 - s_{x+1-i}$ *for all* $i = 1, \ldots, x$; and

4. $\mu(t \mid s_i)$ *is uniformly supported on* $[a_{i-1}, a_i]$ *if* $s_i \in [a_{i-1}, a_i]$.

Using this definition, we now establish (1-b). We can see if the medium provides the same report regardless of the state, then the optimal stance is $s = \frac{1}{2}$, which is uninformative. When the medium cannot update consumers belief about the truth, the medium caters its stance to consumers' opinions and earns profit $\Pi = V - \frac{m}{4} - d(\frac{1}{2} - z)^2$. We can see when *m* is really small, the medium is better off by reporting uninformatively. Intuitively, when consumers don't value the truth, the medium has no incentive to report close to the truth. We can show there exists a cutoff point m_1 such that when $0 < m < m_1$, it is impossible to have a partially informative equilibrium. If we take a first order condition on the medium's profit function with respect to s_i by assuming $a_i \le \frac{1}{2}$, we solve for the optimum:

$$s_i = \frac{m(a_i + a_{i-1}) + 2d(1-z)(a_i - a_{i-1})}{2m + 2d(a_i - a_{i-1})}. \qquad (1.3)$$

If $m \to 0$, we see $s_i \to (1-z)$ which is outside the interval of $[a_{i-1}, a_i]$. Therefore, $s_i \in [a_{i-1}, a_i]$ if and only if $m \ge 2d(1 - z - a_i)$. Define $m_1 \equiv \min_{a_{i-1}, a_i \le \frac{1}{2}} [2d(1 - z - a_i)]$. We can see that when $0 < m < m_1$, it is impossible to have a partially informative equilibrium. From the definition we know $m_1 > 0$. This establishes the second part of (1-b).

To show the first part of (1-b), we see from the optimal stance of s_i given in (1.3), that if $m > 2d(\frac{1}{2} - z)$, there exists a partially informative equilibrium with *x=2*. Define $m_2 \equiv 2d(\frac{1}{2} - z)$, from the definition we know $m_2 \ge m_1$. We now show that it is more profitable for the medium to report partially informatively with *x=2* than to report uninformatively when $m > m_2$. When $m > m_2$, since we know the $s_1 = \arg\max\{\Pi_2(s) = V + m[-2s^2 + s - \frac{1}{4}] - d(s \ -1 + z)^2\}$ belongs to $[0, \frac{1}{2})$; therefore $\Pi_2(s_1) > \Pi_2(s = \frac{1}{2}) = V - \frac{m}{4} - d(\frac{1}{2} - z)^2$, where $\Pi_2(s = \frac{1}{2})$ is the profit when the medium reports uninformatively. Hence when $m > m_2$, it is more profitable for the medium to report partially informative with two intervals.

Next, we investigate the competitive media case in order to establish main result: *Competition reduces media informativeness*. We prove this by first showing there does not exist a fully informative equilibrium with two media. Then we show the partially uninformative equilibrium is never possible with competitive media. The only equilibrium is either both media are uninformative or exactly one medium is partially informative.

We first prove there does not exist a fully informative equilibrium with two media. If both media report truthfully, then it is clear both media report the same stance and engage into price competition. As a result, both media earn zero profit. Therefore, both media have an incentive to deviate from the true state. Similar to Proposition 3, we followed Lemma 1 in Battaglini (2002) such that no truthful informative equilibrium implies no fully informative equilibrium.

Next we show that one can never find a partially informative equilibrium with two media. A *partially informative equilibrium with two competitive media* is a pure-strategy equilibrium if there exists a stance profile vector for each medium with $S^l : [0,1] \to \{s_1^l, s_2^l, \ldots,\}$, $l \in \{A, B\}$ for each medium and a set of dividing points $(a_i)_{i=0,\cdots,x}$ with $x \geq 2$, so that $s^l(t) = s_i^l$ for any $t \in [a_{i-1}, a_i]$, that the stance is optimal for both media and that $\mu(t \mid s^A, s^B)$ is uniformly supported on $[a_{i-1}, a_i]$ if $s^A, s^B \in [a_{i-1}, a_i]$.

If such equilibrium exists and both media report with a single interval, then we can prove at least one medium has incentive to deviate to other interval to "jam" the other's signal. The intuition is similar to our Proposition 4 when $z \leq \frac{5}{12}$: assume both media are partially informative and report stances $s^A < s^B < \frac{1}{2}$ within $[0, a_1]$ with $a_1 \leq 1/2$. First, if $T^A \leq T^B$ (s^B is closer to $\frac{a_1}{2}$), then $\Lambda \equiv T^A - T^B \leq 0$ .In this case medium *A* is further away from the middle point $\frac{1}{2}$, and can be strictly better off by deviating to a stance of $s^A = a_1 + \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. Under this deviation, medium *A* not only jams the signal so that $\Lambda \geq 0$, but also positions itself closer to the center of the line, which improves its payoff. Second, if $T^A > T^B$ such that s^B is further away from $\frac{a_1}{2}$, it can be shown that *B* can also profitably deviate to a non-overlapping interval in order to jam *A* and be strictly better off. Like in the proof of Lemma 3, when $\frac{5}{12} < z \leq \frac{1}{2}$, we establish the case of non-existence of partially informative reporting numerically This part proves that there exists at most one partially informative medium. Similar to the main conclusion in the paper, we can also show that when *m* is large enough, competitive media are uninformative. Intuitively, the disadvantage of being the only uninformative medium is so large when consumers really value facts that it is more profitable to eradicate the rival's ability to be partially informative by jamming its stance. For brevity, the more detailed proof of this part is omitted but available upon request.

2. A Rationale for the Favorable Criterion

In this section we provide an equilibrium selection criterion based on two desirable and general properties an equilibrium ought to possess. These properties, roughly speaking, insist that equilibria are (i) are mutually optimal (in a Nash sense) given consumers' beliefs about the state-of-the-world and (ii) "stable" to small mistakes in beliefs about stances. We then argue that the favorable criterion, as defined in the main text, is a sufficient and necessary condition for these two properties. In other words, an equilibrium satisfies the favorable criterion if and only if it satisfies these more generally desirable properties. To see why this criterion is desirable and filters out all but the intuitive equilibria, assume we are in an equilibrium that doesn't satisfy the criterion. Then there must exist a set of stances for the partially informative medium *A* such that for stance s_i^A, consumers believe the state $t \in [a_{i-1}^A, a_i^A]$ and formalize the expected number of facts to be $N_i^A(s_i^A)$. Because this equilibrium fails the favorable criterion, it does not have the mutually optimal property, (i) above. Therefore the equilibrium stance $s_i^A \neq \arg\max_s \pi^A[s, N_i^A(s_i^A), s^B, N^B]$. Essentially this means that conditional on the expected number of facts $N_i^A(s_i^A)$, and the rival medium's stance , the equilibrium stance s_i^A does not maximize medium *A*'s profit.[1] We can clearly see this equilibrium belief is not intuitive.

In what follows, it is sufficient to focus on equilibria in which exactly one firm is partially informative. Recall that in the main text, all fully informative and symmetrically partially informative equilibria are ruled out directly without appealing to the favorable criterion. Consequently, our arguments below start by assuming there is an equilibrium in

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[1] Note that s_i^A still constitutes an equilibrium stance because $s_i^A = \arg\max_s \pi^A[s, N_i^A(s), s^B, N^B]$.

which one firm is partially informative that does not satisfy the favorable criterion and then showing that neither of these properties can hold.

2.1 Mutual-Max Standard

Consider an equilibrium in which one medium is informative and the other is uninformative. In general, we should expect that both stances should maximize the medias' profits conditional on consumers' beliefs on the interval containing the state. Therefore, we specify the mutual-max standard as the follows. *In equilibrium, the choices of media stances by both media are optimal conditional on consumers' beliefs about the interval containing t.*

More specifically, in our framework, assume we have an equilibrium where medium A is partially informative and B is uninformative. Then there exists a stance profile vector for medium A with $S\ :[0,1]\to\{s_1^A,s_2^A,\ldots,\}$ and a set of dividing points $(a_i)_{i=0,\cdots,x}$ with $x\geq 2$, so that $s^A(t)=s_i^A$ for any $t\in[a_{i-1},a_i]$. Also that $\mu(t\,|\,s^A,s^B)=\mu(t\,|\,s^A)$ is uniformly supported on $[a_{i-1},a_i]$ if $s^A\in[a_{i-1},a_i]$. Then the mutual-max standard means that in this equilibrium, the stance choice vector s^A and s^B satisfies $s_i^A=\arg\max_s \pi^A(s,a_{i-1},a_i,s^B)$, and $s^B=\arg\max_s \pi^B(s,a_{i-1},a_i,s^A)$.

Claim 1: Any equilibrium that survives the favorable criterion satisfies the mutual-max standard and vice versa.

Proof: We prove this by first showing that any equilibrium that does NOT satisfy the mutual-max standard does NOT pass the favorable criterion. Without loss of generality, we assume there exists an asymmetric equilibrium in which medium *A* is partially informative and medium *B* is uninformative. For medium *A*, there must exist a stance profile vector for *A* with $S^A:[0,1]\to\{s_1^A,s_2^A,\ldots,\}$, and a set of dividing points $(a_i)_{i=0,\cdots,x}$ with $x\geq 2$, so that $s^A(t)=s_i^A$ for any $t\in[a_{i-1},a_i]$. Notice first that *B* can always respond to s_i^A without changing consumers' beliefs about medium *B*'s informativeness. Now if this equilibrium does not satisfy the mutual-max standard, it must be the case that there exists another $\dot{s}_i^A\in[a_{i-1},a_i]$ such that $\dot{s}_i^A=\arg\max_s \pi^A(s,a_{i-1},a_i,s^B)$, given the equilibrium beliefs and s^B. Then it is immediate that this equilibrium violates the favorable criterion from the fact that the deviation from s_i^A to $\dot{s}_i^A$ would be a profitable deviation under the favorable belief when consumers believe the deviation signals that the medium is at least as informative as before. Next we show that any equilibrium that fails the favorable criterion does not satisfy the mutual-max standard. Any favorable belief will reward the deviation of medium *A* under the equilibrium that fails the favorable criterion. Hence medium *A* has incentive to deviate conditional on the same belief about the informativeness ($t \in [a_{i-1}, a_i]$). Therefore the choice of s_i^A violates the mutual-max standard.

2.2 Trembling-Hand-Free

Our framework brings some additional difficulties in the equilibrium refinement: 1) different from existing research, in our framework the information senders (media) don't have any presumed types. Therefore any sender-type based refinement (as typically employed in classic signaling games) does not directly apply to our setting since they require receivers to update their out-of-equilibrium beliefs based on the sender's type. This is why the standard intuitive equilibrium or M1 refinements used in signaling games cannot be applied to our setting. 2) Even within the cheap talk literature, as mentioned before, there is no commonly accepted way to refine out-of-equilibrium beliefs in games with a continuum of types for the purpose of selecting reasonable equilibria.

We now follow a reviewer's suggestion to examine our equilibrium selection based on the trembling-hand equilibrium refinement. We show any equilibrium that does NOT satisfy the favorable criterion is not trembling hand free.

The trembling hand equilibrium refinements require the equilibrium can survive small perturbation of consumer beliefs. If equilibrium beliefs are slightly disturbed, then it should be possible to find an equilibrium of the game with perturbed beliefs, which is the same as the equilibrium of the unperturbed game. In other words, equilibria that are robust to the trembling hand refinement, are "stable" to small mistakes in consumer beliefs. Another way to interpret this concept is to say that consumers' beliefs are not restricted to a single stance, but rather can be a range of possible stances.

Similar to previous research in cheap talk equilibrium refinement (Battaglini 2002, Chen et al 2008, Lu 2011), we introduce the perturbed noise into the information transmission process by assuming that in equilibrium a consumers' equilibrium belief is slightly disturbed with small noise. More specifically, when a medium reports a stance s^j to signal its informativeness, consumers believe any media stance between $(s^j - \varepsilon, s^j + \varepsilon)$ signals the same interval, where ε is an arbitrary small number (when $\varepsilon = 0$ there is no perturbation).[2] In other words, the perturbation reflects that consumers have imprecise beliefs about the equilibrium media stances. One possible interpretation for this change is that consumers believe the media might make mistakes and therefore believe it is as informative despite seeing a stance that is very close to the equilibrium media stance. Under this newly defined perturbed game, we establish the following.

Claim 2: Any equilibrium that fails the favorable criterion will never be robust to an ε-perturbation (i.e. not satisfy the trembling-hand refinement) and vice versa.

Proof: For any asymmetric equilibrium (without loss of generality we assume that medium *A* is partially informative and *B* is uninformative), there must exist a media stance profile vector for media *A* with $S^A : [0,1] \rightarrow \{s_1^A, s_2^A, \ldots, \}$, and a set of dividing points $(a_i)_{i=0,\cdots,x}$ with $x \geq 2$, so that $s^A(t) = s_i^A$ for any $t \in [a_{i-1}, a_i]$. Under an ε-perturbed game, however, consumers have a slightly imprecise belief, which could potentially affect the media's equilibrium choice. Therefore, if consumers need to update their beliefs, they do so following Bayes' rule based on the common knowledge in the perturbed game. We now show that there does not exist an equilibrium to this perturbation which fails the favorable criterion.

First notice, if there is no perturbation, the beliefs of consumers hold that the media stance s_i^A signals $t \in [a_{i-1}, a_i]$ and the medium *A* will choose s_i^A. Under the perturbation, however, consumers believe a stance that is lightly different from the equilibrium choice is still as informative as before. The prior belief consumers have is that for any stance $\dot{s}_i^A \in (s_i^A - \varepsilon, s_i^A + \varepsilon)$, $t \in [a_{i-1}, a_i]$. Based on the definition of the favorable

[2] We consider perturbed beliefs defined on non-empty open sets of the (usual) subset topology with respect to the set $[a_{i-1}, a_i]$.

criterion, for the equilibrium that fails the criterion, any favorable belief rewards deviation, implying that at least one of the following must be true:

$$\lim_{\mu \to 0^+} \frac{\partial \pi^A (s_i^A + \mu)}{\partial s^A} > 0 \text{ and } \lim_{\mu \to 0^-} \frac{\partial \pi^A (s_i^A + \mu)}{\partial s^A} > 0$$ for any favorable out-of-equilibrium belief, where $\pi^A = \frac{[d(s^B - s^A)(s^A + s^B + 2 - 6z) + \Delta M]^2}{18d(s^B - s^A)(1-2z)}$. Therefore it must hold that under the prior belief, the medium will profitably deviate from the stance s_i^A. Specifically, s_i^A cannot be an equilibrium stance of the ε-perturbed game. Hence we proved that any equilibrium that fails the favorable criterion is not trembling hand free. Next we show that any equilibrium that satisfies the favorable criterion will be robust to an ε-perturbation. For the equilibrium that satisfies the favorable criterion, we know medium *A* will not deviate if consumers maintain the belief that the expected number of fact of any other $\dot{s}_i^A$ will be the same as s_i^A (otherwise this equilibrium will never satisfy the favorable criterion). Hence we can see for any small number of ε-perturbation this equilibrium is stable – i.e. satisfy the trembling-hand refinement.

3. Dynamics

Do the issues of dynamics and reputation undermine the central conclusions about competition and media informativeness found in the static model? For instance, it might seem that if consumers have repeated interactions with the media, then a medium would be induced to provide more facts to preserve its reputation. However, such an outcome depends on the additional conditions one assumes about how reputation is threatened. To illustrate, consider a dynamic game, with repeated interaction, reputation concerns, and the threat of retribution through, for example, unsubscribing. A necessary condition for consumers to "catch" media not reporting the truth is that they have means to verify all the facts, even those unreported. Hence, in a dynamic model, media would be inclined to report more facts and take stances closer to the truth if consumers have low verification costs.

It is also possible that dynamics and reputation reinforce the results of our static model. If "reputation" means that a medium "stands consistently with its position," it will want to maintain a constant stance. This does not seem to be a rare occurrence. For example, MSNBC and Fox maintain a reputation for their opposing political positions. Under this interpretation, we can see that a dynamic model would strengthen our model's prediction since the media's position is less flexible.

As these two scenarios suggest, the implication of dynamics could go either way depending on which conditions hold. Which scenario is more plausible is subject to debate. If verification costs were always sufficiently low for consumers to check unreported facts, then there would be no role for media in the first place. Thus, we feel that our setting is realistic in many plausible scenarios which are complex with a deep nature, such as "global warming." In those cases, the state of the world is not observable and depended from repeated interaction. However, our argument may not fit to some other simpler events, like "will the local sports team win a championship."

4. Asymmetric Disutility from Opinion

It is reasonable to suppose that the disutility to a consumer depends on which "side" the difference occurs. For example, a consumer who is moderately left-leaning may have less disutility from a far-left stance than an equally distant stance to the right. The simplest way to capture this aspect is to have a demand model such that:

$$E[u_b(n, s, p \mid s)] = V + E[Mn \mid s] - d(s,b) \times (s-b)^2 - p$$

where $d_{Opp} \equiv d(s_L, b_R) = d(s_R, b_L) > d(s_L, b_L) = d(s_R, b_R) \equiv d_{Same}$ for $s_L, b_L < \frac{1}{2} < s_R, b_R$. With such a case, we expect, monopoly medium's incentive to slant could increase or decrease depending on the relative difference between d_{Opp} and d_{Same}. For instance, if $d_{Opp} >> d_{Same} \approx 0$ then media would suffer when drifting too far from the middle point. In contrast, if $d_{Same} >> d_{Opp} \approx 0$ would encourage media to appeal to a particular side. It is therefore not clear without more conditions how it would alter our results on polarization and bias. However the result of duopoly wouldn't change since each media covers half of the market. Therefore it would not affect the basic results about the relative informativeness of monopoly and duopoly since it would not affect consumers' abilities to update their beliefs.

5. Multiple Issues

Our model focuses on reports about a single issue. In reality, however, 1) there can be multiple issues regarding to the same topic; and 2) the media can discuss different issues in a single report. We discuss each possibility.

5.1 Multiple issues on the same topic

If consumers hold a general opinion about a given topic and their opinions on specific issues are consistent with their general opinion, then our model should have no problem. In this case, media in our model take stances on the general topic and provide a collection of reports consistent with that stance. For instance, in the case of global warming, the *New York Times* has a special section entitled "Global warming & Climate change"[3] that is solely dedicated to this topic. The articles in this section repeatedly report the global warming with similar stances, but with very different facts in each single issue.

5.2 Multiple issues in a single report

Because our model allows for only a single stance per topic, it may be difficult to interpret our model in this setting. For example, the controversial issue of global warming involves issues related not only to climate change but also to the development of alternative energy sources. Battaglini's (2002) analysis of standard "cheap-talk" in a multidimensional state space may provide some guidance. He shows, in fact, that multidimensionality can improve communication efficiency with multiple senders. Intuitively, multidimensionality can soften the conflicts among information senders and, therefore, leave room for senders to coordinate. This remains a limitation of our research, which future work can hopefully extend to address the case in which the state is conceived as having more than one dimension.

6. Non-Existence of Partially Informative Equilibria in Competitive Model with Dissimilar Partitions

In the main text we argued that for duopoly, a partially informative equilibrium does not exist under the favorable criterion. Our argument considered only partially informative equilibria in which both media share identical stance partitions. Here we show there does

[3] http://topics.nytimes.com/top/news/science/topics/globalwarming/index.html

NOT exist a partially informative with *dissimilar* partitions when $0 \leq z \leq 5/12$. Here the word "dissimilar" specifically means the set of dividing point can be different across media, which means that reporting intervals of different media must overlap in at least one instance. See Figure S2.1 for an illustration of this type of asymmetric equilibrium with 4 intervals for both media. However, it is not necessary that the partially informative equilibrium for both media have to have the same number of intervals.

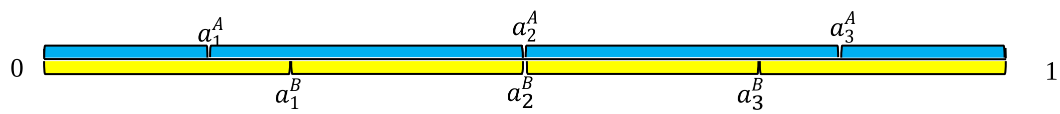

Figure S2.1: A Dissimilar Equilibrium with Two Competitive Media

Next, we define this type of equilibrium and show it doesn't exist.

Definition 8: A *partially informative equilibrium with two competitive media and dissimilar partitions* is a pure-strategy PBE if there exists a stance profile vector for each medium with $S^l : [0,1] \rightarrow \{s_1^l, s_2^l, \ldots,\}$, $l \in \{A, B\}$ for each medium and a set of dividing points $a_0 = 0 < a_1 < \cdots < a_{x^l - 1} < a_{x^l} = 1$ with $x^l \geq 2$, so that $s^l(t) = s_i^l$ for any $t \in [a_{i-1}^l, a_i^l]$, that the media choice is optimal for both media and that a consumer's updated belief $\mu(t \mid s^A, s^B)$ is uniformly supported on $[a_{i-1}^A, a_i^A] \cap [a_{i'-1}^B, a_{i'}^B]$ if $s^A \in [a_{i-1}^A, a_i^A]$ and $s^B \in [a_{i'-1}^l, a_{i'}^l]$. Consumer purchasing rules are given by:

$$\Psi_b^A(\cdot) = \begin{cases} 1, \text{if } E\left[u_b\left(N^A, s_i^A, p^A \middle| s_i^A\right)\right] \geq 0, \text{and } E\left[u_b\left(N^A, s_i^A, p^A \middle| s_i^A\right)\right] > E\left[u_b\left(N^B, s_i^B, p^B \middle| s_i^B\right)\right] \\ 0, \text{otherwise} \end{cases}.$$

Now we analytically show such equilibrium is never possible when $0 \leq z \leq 5/12$. Assume such equilibrium exists. Without loss of generality, we focus on the case when $a_1^A \leq a_1^B < 1/2$. We investigate the situation when $0 \leq t \leq a_1^A$ such that media chooses $s_1^A \in [0, a_1^A]$ and $s_1^B \in [0, a_1^B]$. In this case, consumers believe $t \in [0, a_1^A]$. First if $s_1^A < s_1^B$, we know that consumers expect that A's report has more facts, $\Delta = N_1^A - N_1^B > 0$, since s_1^A is further from ½. We show that medium B can profitably deviate by choosing the stance $\tilde{s}^B = \frac{1}{3}(s_1^A + 4 - 6z)$. Note that for $0 \leq z \leq 5/12$, we have $\tilde{s}^B =$

$\frac{1}{3}(s_1^A + 4 - 6z) > \frac{1}{2} > a_1^B$, which implies that medium B's stance jams s_1^A and $\Delta = 0$. Furthermore, $\tilde{s}^B = \text{argmax}_s\, \pi^B(s_1^A, s, \Delta = 0)$. Thus, $\pi^B(s_1^A, \tilde{s}^B, \Delta = 0) > \pi^B(s_1^A, s_1^B, \Delta = 0) \geq \pi^B(s_1^A, s_1^B, \Delta > 0)$. QED

7. Distribution of $t|\tau$

Here we show that the distribution of the portion of "1" facts to total facts t given the true (and unobserved) state of the world, τ follows a uniform distribution. Let $\tau \sim U[0,1]$. Assume we random draw N times. Define $t = \frac{1}{N}\sum Y_i \in [0,1]$, where $Y_i \sim Bernouli(\tau)$.

We know that for $0 \leq k \leq N$, $t \in [0,1]$. Furthermore,

$$P\left(t = \frac{k}{N}\right) = \int_0^1 P(\sum Y_i|\tau) f(\tau) d\tau = \int_0^1 \binom{N}{k} \tau^k (1-\tau)^{N-k} d\tau$$
$$= \binom{N}{k} \int_0^1 \tau^k (1-\tau)^{N-k} d\tau = \binom{N}{k} \frac{N-k}{k+1} \int_0^1 \tau^{k+1} (1-\tau)^{N-k-1} d\tau \ldots$$
$$= \binom{N}{k} \frac{1}{\binom{N}{k}} \int_0^1 \tau^N dt = \frac{1}{(N+1)}$$

This means that t has equal probability to be a value in $\left\{0, \frac{1}{N}, \frac{2}{N}, \cdots, 1\right\}$. Since we assume N to be a continuous variable we can see $f(t) = 1$ for any value of $t \in [0,1]$. Therefore the random variable t is uniformly distributed on [0,1].

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