Technical Appendix

Strategic Assortment Reduction by a Dominant Retailer

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This Technical Appendix contains the technical details that establish all propositions stated in the text. In addition, we provide here supporting arguments to two incidental claims made in the main text of the manuscript. In particular, we show the following.

1. Proofs to Propositions 1-4.
2. a) The notion of marginal cost increasing in assortment size is not at odds with decreasing average costs (economies of scope).

   b) The notion of marginal cost increasing in assortment size is not at odds with a system that makes it easier to monitor additional SKU's (marginal cost of monitoring decreasing in assortment size)
3. The channel incentives identified in the "Cournot" model of the main text can be demonstrated in a "Bertrand" model, with the low cost retailer setting lower prices.

TA.1 Proofs of Propositions 1-4

The following lemma is used to prove Proposition 1.

LEMMA TA.1 (Equilibrium of the Subgame Starting at Stage 1)

(i) When product i is carried by both retailers, the manufacturer's and retailers' product-specific profits are, respectively,

$$\Pi_i^M = \frac{1}{6b_i}\left(a - \frac{c^A + c^B}{2}\right)^2; \qquad \Pi_i^j = \frac{1}{36b_i}\left(a - \frac{7c^j - 5c^{-j}}{2}\right)^2,$$

with the distribution given by

$$q_i^j = \frac{1}{6b_i}\left(a - \frac{7c^j - 5c^{-j}}{2}\right) \qquad q_i^{-j} = \frac{1}{6b_i}\left(a - \frac{7c^{-j} - 5c^j}{2}\right)$$

for $j = 1,2$; $-j \neq j$.

(ii) When product i is carried by only retailer j then product-specific profits and quantity are, respectively,

$$\Pi_i^M = \frac{(a - c^j)^2}{8b_i}; \qquad \Pi_i^j = \frac{(a - c^j)^2}{16b_i}, \qquad q_i = \frac{a - c^j}{4b_i}.$$

Proof of Lemma TA.1

Retailers' optimal reactions to wholesale prices w_i, *i* = 1,2 are given in equation (2). For *i* = 1,2, the manufacturer maximizes $\Pi_i^M = w_i(q_i^A + q_i^B)$, subject to the reactions in (2) over $w_i \geq 0$. Substitute these values back in to (2) to obtain equilibrium quantities. Equilibrium profits for product *i* are computed by substituting the optimal w_i's and q_i^j into (3) and (4). Q.E.D.

Proof of Proposition 1

The expressions given Table 1 follow from Lemma TA.1 as follows. For outcome *F*, substitute $c^A = c^B = c$ into the expressions in part (i) for products *i* = 1,2 and compute profits $\Pi^{j} = \sum_i \Pi_i^j$ for $j = M, A$, and B. For *Sp*, substitute $c^A = 0$ and $c^B = c$, with product 1 carried by both retailers using expressions of part (i) of Lemma TA.1 and product 2 carried by retailer *B* only using expressions of part (ii) of the lemma. Profits are $\Pi^{j} = \sum_i \Pi_i^j$ for $j = M$ and B and $\Pi^A = \Pi_1^A$. For *Ex*, substitute $c^A = c^B = 0$, with product 1 carried by *A* and product 2 carried by *B* using the expressions in part (ii) of the above lemma. Profits are $\Pi^M = \Pi_1^M + \Pi_2^M$, $\Pi^A = \Pi_1^A$, and $\Pi^B = \Pi_2^B$. For *S*, substitute $c^A = c^B = 0$, with product 1 carried by *A* and *B* using expressions of part (ii) of the lemma. Profits are $\Pi^j = \Pi_1^j$, for $j = M, A$ and B. Q.E.D.

Next, we present two intermediate results that are used to prove Proposition 2.

PROPOSITION TA.1

Let $(c/a, b_1, b_2) \in [0,1]^2$ *and denote* $\zeta \equiv \frac{4\sqrt{3}-5}{3} \approx 0.64$. *Then there exist functions* $f(b_1/b_2)$, $g(b_1/b_2)$, *and* $h(b_1/b_2)$ *with* $0 < g(b_1/b_2) < f(b_1/b_2) < 1$ *for all* $b_1/b_2 \in (0,1)$, $g(b_1/b_2) < h(b_1/b_2) < f(b_1/b_2)$ *for all* $b_1/b_2 \in (\frac{1}{3}, \zeta)$ *and* $h(b_1/b_2) < g(b_1/b_2)$ *for all* $b_1/b_2 \in (\zeta, 1)$ *which characterize the equilibria of the M-Dominant game as follows.*

(i) If $0 < b_1/b_2 < \frac{1}{3}$ *then the unique equilibrium outcome is*

F if and only if $0 < c/a < g(b_1/b_2)$;

Sp if and only if $g(b_1/b_2) < c/a < f(b_1/b_2)$;

S if and only if $f(b_1/b_2) < c/a < 1$.

(ii) If $\frac{1}{3} < b_1/b_2 < \zeta$ *then the unique equilibrium outcome is*

F if and only if $0 < c/a < g(b_1/b_2)$;

Sp if and only if $g(b_1/b_2) < c/a < h(b_1/b_2)$;

Ex if and only if $h(b_1/b_2) < c/a < 1$.

(iii) If $\zeta < b_1/b_2 < 1$ *then the unique equilibrium outcome is*

F if and only if $0 < c/a < g(\zeta)$;

Ex if and only if $g(\zeta) < c/a < 1$.

Proof of Proposition TA.1

In the *M-Dominant* game, the manufacturer implements its preferred distributional strategy based on outcome leading to the most profits. Comparing profit levels across these four outcomes requires pair-wise comparisons using the profit expressions in Proposition 1. Specifically, five (5) such comparisons are sufficient to determine the equilibrium in all regions of the parameter space $[0,1]^2$. Direct comparisons of profits leads to the following:

$$\Pi_F^M \gtrless \Pi_{Ex}^M \quad \Leftrightarrow \quad c/a \lessgtr 1-\sqrt{3}/2. \tag{TA.1}$$

$$\Pi_S^M \gtrless \Pi_{Ex}^M \quad \Leftrightarrow \quad b_1/b_2 \lessgtr 1/3. \tag{TA.2}$$

$$\Pi_F^M \gtrless \Pi_{Sp}^M \quad \Leftrightarrow \quad c/a \lessgtr (\delta-1)/(\delta-\tfrac{1}{2}) \equiv g(b_1/b_2). \tag{TA.3}$$

where $\delta \equiv \sqrt{1+b_1/(4b_2)}$. The function *g* is strictly increasing in b_1/b_2 on [0,1] and represents *M*'s indifference curve for *Sp* and *F*. The last two comparisons that are need are the following.

$$\Pi_S^M \gtrless \Pi_{Sp}^M \quad \Leftrightarrow \quad \frac{b_1}{b_2} \gtrless \frac{(4-c/a)(c/a)}{3(1-c/a)} \equiv \hat{f}\left(c/a\right). \tag{TA.4}$$

$$\Pi_{Sp}^M \gtrless \Pi_{Ex}^M \quad \Leftrightarrow \quad \frac{b_1}{b_2} \lessgtr \frac{\frac{1}{3}(2-c/a)^2-1}{1-(1-c/a)^2} \equiv \hat{h}\left(c/a\right). \tag{TA.5}$$

Because $\hat{f}$ is strictly increasing and $\hat{h}$ is strictly decreasing in c/a on [0,1], we define for $b_1/b_2 \in [0,1]$, $f(b_1/b_2) \equiv \hat{f}^{-1}(b_1/b_2)$ and $h(b_1/b_2) \equiv \hat{h}^{-1}(b_1/b_2)$, which represent *M*'s indifference curve for outcomes *S* versus *Sp* and *Ex* versus *Sp*, respectively. Conditions (A.4) and (A.5) can be rewritten in the canonical form

$$\Pi_{Sp}^M \gtrless \Pi_S^M \quad \Leftrightarrow \quad c/a \lessgtr f(b_1/b_2). \tag{TA.4´}$$

$$\Pi_{Sp}^M \gtrless \Pi_{Ex}^M \quad \Leftrightarrow \quad c/a \lessgtr h(b_1/b_2). \tag{TA.5´}$$

The function *g* is strictly increasing in b_1/b_2 on [0,1] and represents *M*'s indifference curve for *Sp* and *F*. It is verified (numerically) that $f(b_1/b_2) > g(b_1/b_2)$ for all $b_1/b_2 >$

0, as claimed in the condition of the proposition. Furthermore, since *f* and *g* are increasing and intersect *h* at exactly one point (specifically, at $1/3$ and at ζ, respectively) we can write the following:

$$f(b_1/b_2) \lessgtr h(b_1/b_2) \quad \Leftrightarrow \quad b_1/b_2 \lessgtr 1/3; \tag{TA.6}$$

$$g(b_1/b_2) \lessgtr h(b_1/b_2) \quad \Leftrightarrow \quad b_1/b_2 \lessgtr \zeta \equiv \hat{h}\left(1-\sqrt{3}/2\right). \tag{TA.7}$$

Note that $1/3 < \zeta$. To show (i), let $b_1/b_2 \in [0,1/3)$ then (TA.2) implies *Ex* is dominated by S and thus can never be an equilibrium for any c/a. If $0 < c/a < g(b_1/b_2)$ then (TA.3) and (A.4´) imply $\Pi_F^M > \Pi_{Sp}^M > \Pi_S^M$, yielding *F* as the equilibrium. If $g(b_1/b_2) < c/a < f(b_1/b_2)$, then (TA.3) and (TA.4´) imply $\Pi_{Sp}^M > \Pi_F^M, \Pi_S^M$, yielding *Sp* as the equilibrium. Finally if $f(b_1/b_2) < c/a < 1$, then (TA.3) and (TA.4´) imply $\Pi_S^M > \Pi_{Sp}^M >, \Pi_F^M$. To show (ii), let $b_1/b_2 \in (1/3, \zeta)$. (TA.2) implies *S* is dominated by *Ex*. Conditions (TA.3) (TA.5´) and (TA.6) imply the ordering required for the equilibrium description in the proposition. To show (iii), let $b_1/b_2 \in (\zeta, 1]$. (TA.2) implies *S* is dominated by *Ex* and conditions (TA.3), (TA.6) and (TA.7) imply that *Ex* dominates *Sp*. Therefore, *F* and *Ex* are the only outcomes possible in equilibrium. Finally, (TA.1) implies the ordering required for the equilibrium description in the proposition. Q.E.D.

PROPOSITION TA.2 *Let g and h be the functions determined in Proposition 1,* $\zeta \equiv \frac{4\sqrt{3}-5}{3} \approx 0.64$, *and* $(b_1/b_2, c/a)$ *be in* Θ. *Then the equilibria of the A-Dominant game is described as follows.*

(i) If $0 < b_1/b_2 < \zeta$ *then the unique equilibrium outcome is*

F *if and only if* $0 < c/a < k(b_1/b_2)$;

Sp *if and only if* $k(b_1/b_2) < c/a < g(b_1/b_2)$.

(ii) If $\zeta < b_1/b_2 < 1$ *then the unique equilibrium outcome is*

F *if and only if* $0 < c/a < k(b_1/b_2)$;

Sp *if and only if* $k(b_1/b_2) < c/a < h(b_1/b_2)$;

Ex *if and only if* $\max\{h(b_1/b_2), k(b_1/b_2\} < c/a < g(\zeta)$.

Proof of Proposition TA.2

As argued in the text, *A* can only implement *M*'s second-best outcome (*F* being the first), which is either *Sp* or *Ex*. From (TA.5') in the proof of Proposition A.1, we already know that *M*'s second-best is *Sp* for $(b_1/b_2, c/a)$ below *h* and *Ex* above. (i.e., *Sp* and *Ex* are the corresponding equilibrium of the subgame starting in period 1.) *A*, in period 0, will not abandon product 2 if (5) holds. Further, note that *k*, as defined in (5) satisfies the following. For any $b_1/b_2 \in (0,1)$,

$$k(b_1/b_2) < g(b_1/b_2)\text{, and} \tag{TA.8}$$

$$k(b_1/b_2) \lessgtr h(b_1/b_2), \qquad \text{for } b_1/b_2 \lessgtr \eta, \tag{TA.9}$$

where $\eta \approx 0.99$. To verify (TA.8), observe that both *k* and *g* are both continuous and increasing and then it can be shown that $k(1) < g(1)$ and $k(x) = g(x)$ has no solution in (0,1). The condition (TA.9) can be verified by first noting that both *k* and *h* are continuous with *k* strictly increasing and *h* strictly decreasing on (0,1). This implies that they cross at most once, which occurs at η. The inequalities in (TA.9) then follow from the fact that $k(0) = 0 < h(0) = 2 - \sqrt{3}$. It follows from these two conditions that for any $b_1/b_2 \in (0,1)$, *F* is the equilibrium if $c/a < k(b_1/b_2)$.

Let $b_1/b_2 \in (0,\eta)$ and $c/a \in (k(b_1/b_2), \min\{g(b_1/b_2), h(b_1/b_2)\})$, retailer *A* implements *Sp* when abandoning product 2. Because *Sp* is more profitable than *F* for retailer *A*, it is the equilibrium outcome. Finally, let $b_1/b_2 \in (\zeta,\eta)$ and $c/a \in (\max\{h(b_1/b_2), k(b_1/b_2)\}, g(\zeta))$. Then retailer *A* implements *Ex* when abandoning product 2. *A* finds this more profitable than *F* if and only if

$$c/a > l(b_1/b_2) \equiv 1 - \frac{3}{2}\sqrt{\frac{b_1/b_2}{1 + b_1/b_2}}.$$

Observe that $l'(b_1/b_2) < 0$ for all b_1/b_2 and that $l(\zeta) \approx 0.04174 < \frac{5-\sqrt{21}}{4} = h(1)$. Thus,

$$l(b_1/b_2) < l(\zeta) < h(1) < h(b_1/b_2)$$

for all $b_1/b_2 \in (\zeta, 1]$, where the last inequality follows from the fact that *h* is decreasing. Hence, *Ex* is the equilibrium outcome in the region $b_1/b_2 \in (\zeta, \eta)$ and $c/a \in (\max\{h(b_1/b_2), k(b_1/b_2)\}, g(\zeta))$. Q.E.D.

Proof of Proposition 2

This follows directly from Propositions TA.1 and TA.2.

Proof of Proposition 3

First consider the *M-Dominant* game. Clearly, $\Pi^M_{SpA} > \Pi^M_{SpB}$ for $c^A < c^B$, where profit expressions can be directly deduced from Lemma TA.1. Also, the fact that $c^A < c$ implies that *SpA* dominates *F*, *Ex,* and *S* for parameter constellations $(b_1/b_2, c/a) \in \Theta_{Sp}$. Hence, *SpA* is optimal for *M* and therefore is the equilibrium outcome for the *M-Dominant* game.

In the *A-Dominant* game, if retailer *A* abandons product 2 in Stage 0, then $(b_1/b_2, c^B/a) \in \Theta_{Sp}$ and Proposition 2 imply that *M*'s optimal strategy is to distribute both products through retailer *B*. That is, *SpB* is the equilibrium of the subgame starting in Stage 1 given that retailer *A* does not carry product 2. Finally, *SpB* is the equilibrium of the overall game if and only if $\Pi^A_{SpA} < \Pi^A_{SpB}$. A direct comparisons of profit expressions in (6) implies that the following condition is sufficient:

$$\frac{b_1}{b_2} < \frac{8}{3}\frac{c^A}{a}\frac{(2 - c^A/a)}{(1 - c^A/a)^2} \equiv p(c^A/a). \qquad \text{(TA.10)}$$

First note that it can be directly verified that *p* is strictly increasing in c^A/a. Therefore, $p(c^A/a) > p[g(b_1/b_2)]$ since $c^A/a > g(b_1/b_2)$ by the assumption $(b_1/b_2, c^A/a) \in \Theta_{Sp}$. Finally, the condition (TA.10) follows by noting that

$$p[g(b_1/b_2)] = 8/3g(b_1/b_2)(4\delta)(\delta - \frac{1}{2})$$

$$= \frac{32}{3}\left[(1+\frac{b_1}{4b_2}) - \sqrt{1+\frac{b_1}{4b_2}}\right]$$

$$> \frac{b_1}{b_2}.$$

Q.E.D.

Proof of Proposition 4

Using the expressions for output quantities from Table 1 in equation (7), we arrive at the following:

$$CS_F = \frac{(a-c)^2}{9}\left(\frac{1}{b_1}+\frac{1}{b_2}\right)$$

$$CS_{Sp} = \frac{(a-c/2)^2}{18b_1} + \frac{(a-c)^2}{32b_2}$$

$$CS_{Ex} = \frac{a^2}{32}\left(\frac{1}{b_1}+\frac{1}{b_2}\right).$$

We first show that $CS_F > CS_{Sp}$ everywhere in Θ. Generally,

$$CS_F > CS_{Sp} \quad \Leftrightarrow \quad c/a < \frac{\sigma - 1}{\sigma - \frac{1}{2}} \tag{TA.11}$$

where $\sigma \equiv \sqrt{2 + \frac{23b_1}{16b_2}}$. The right-hand side of (TA.11) is increasing and greater than $\frac{2(\sqrt{2}-1)}{2\sqrt{2}-1}$ for all b_1/b_2. Since $\frac{2(\sqrt{2}-1)}{2\sqrt{2}-1} > 1 - \frac{\sqrt{3}}{2} = g(\zeta)$, it follows that $CS_F > CS_{Sp}$ everywhere in Θ.

Comparing consumer surplus in *Sp* and in *Ex*, we have the general condition

$$CS_{Sp} > CS_{Ex} \quad \Leftrightarrow \quad \frac{b_1}{b_2} < \frac{\frac{4}{9}(2-c/a)^2 - 1}{1-(1-c/a)^2} \equiv \hat{m}(c/a). \tag{TA.12}$$

Observe that $\hat{m}$ is decreasing and $\hat{m}(c/a) = 1 \Rightarrow c/a = \frac{17-3\sqrt{22}}{13} > g(\zeta)$. Therefore, $(b_1/b_2, c/a) \in \Theta \Rightarrow c/a < g(\zeta) \Rightarrow \hat{m}(c/a) \geq 1 > b_1/b_2$. Hence, (TA.12) holds everywhere in Θ.

Q.E.D.

TA.2 A Model of Assortment Costs:

In this section, we derive the notion of assortment costs used in the main text from a general model of assortment costs. Denote the retailer's assortment cost $C(\{q_i\}_{i=1}^{n}, n)$, where *n* is the assortment size (number of products in the assortment) and q_i is quantity of product *i* stocked and sold. As we show, there exists a plausible cost function that simultaneously possesses the following properties:

(1) Marginal (assortment) cost (on each unit q_i) is increasing in assortment size (*n*).

(2) Average assortment cost is decreasing in assortment size (*n*).

(3) Marginal monitoring cost is decreasing in assortment size (*n*).

To illustrate, consider the following generic cost function:

$$C(\{q_i\}_{i=1}^{n}, n) = \sum_{i=1}^{n} c(n)q_i + m(n) \qquad \text{(TA.13)}$$

where $c(n)$ is the cost that is incurred on each unit sold, which we refer to as the marginal (assortment) cost, and $m(n)$ is other costs. The marginal assortment cost $c(n)$ is increasing in assortment size and reflects the additional labor when moving and sorting on shelves. The second term in (TA.13), $m(n)$, captures the additional costs associated with, for example, monitoring and tracking SKU's. It is assumed to be increasing in assortment size, *n*, but at a decreasing rate. We consider the following specifications of *c* and *m*, which satisfy these properties: $c(n) = cn$, $c > 0$ and $m(n) = \sqrt{n}$.

As we show, the cost function in (TA.13) simultaneously exhibits points (1) and (2) above. Specifically, the cost on an additional sale of product *i* changes with *n* according to

$$\frac{\partial^2 C}{\partial n \partial q_i} = \frac{dc(n)}{dn} = c; \qquad i = 1, \ldots, n$$

which is positive by *c* being positive. Therefore, marginal cost increases in *n*. However, consider the total assortment cost, averaged over total assortment size,

$$AC = \frac{1}{n} C((q_i)_{i=1}^{n}, n) = \frac{1}{n} \sum_{i=1}^{n} cnq_i + \frac{1}{\sqrt{n}}.$$

As the number of SKU's grows, average cost is decreasing since

$$\frac{\partial AC}{\partial n} = -\frac{1}{2n^{3/2}}$$

is negative for all *n*. Hence, a retailer can enjoy decreasing average costs (economies of scope), and decreasing marginal monitoring costs ($m''(n) = -n^{-3/2}/4 < 0$) despite the fact its marginal (assortment) costs are increasing in assortment size.

It is instructive consider the marginal impact on total assortment costs in (TA.13) when a product is added to a retailer's assortment. Consider the partial derivative

$$\frac{\partial C}{\partial n} = \sum_{i=1}^{n} cq_i + \frac{1}{2\sqrt{n}}.$$

For a large retailer (large *n*), the second term diminishes but the first term does not.[1] This reflects the notion that the impact of a growing assortment is dominated by the marginal (assortment) costs on each q_i sold. In other words, for retailers with large assortments, under this cost specification, the decision to widen assortment does not depend much on the adding another SKU to the monitoring or tracking system. Rather, it is the added cost to each unit (q_i) that is relatively important for the assortment decision.

TA.3 A Bertrand game for Assortment Reduction

In this section, we develop a generic model of a manufacturer and two differentiated retailers engaged in Bertrand competition. The purpose of this alternative model formulation is to illustrate the basic channel incentives associated with strategic assortment reduction, identified in the main text, are invariant to the choice of the strategic variable (price versus quantity).

Let *D* be a "generic" demand function such that $D:[0,\infty)^2 \to R$ is continuous, differentiable with $\partial D(p,p')/\partial p < 0 < \partial D(p,p')/\partial p'$. We assume the existence of some $\overline{p}$ such that $D(\overline{p},p) = 0$ for all $p \in [0,\overline{p}]$. Consider the following game. Retailers *A* and *B* face fixed wholesale prices $w \in [0,\overline{p}]$ for products $i = 1,2$ and simultaneously choose a pair of prices (p^{1j}, p^{2j}), for $j = A,B$. Specify demand for a popular good, labeled 1, at retailer *j* simply by $D^{1j} = D(p^{1j}, p^{1j'})$ for $j = A,B$; $j' \neq j$. For the specialty product,

[1] Note that *n* is number of products from *all* suppliers, even though each supplier may supply only a few products. In the model of the main text, for example, we focus on the assortment decision vis-à-vis a single manufacturer with two products.

labeled 2, scale demand by a factor $\alpha \in [0,1]$. That is, $D^{2j} = \alpha D(p^{2j}, p^{2j'})$ for $j = A, B$; $j' \neq j$. As assumed in the main text, demand is independent across products 1 and 2. Finally, recall that the dominant retailer, *A* in the main text, may choose to carry only product 1. In this case, retailer *B* is the exclusive seller of product 2. To save on notation, denote consumer demand by $D^{2B}(p^{2B}, -)$ which reflects the fact that *B*'s price is the only endogenous variable affecting the quantity sold.

By using this generic form demand formulation, we are able to consider a wide class of specifications. In what follows, we are interested in characterizing the demand conditions under which strategic assortment reduction is feasible in equilibrium.

We first consider a pricing game played by the two retailers, both of whom carry the same product *i*. Let $\Pi^{ij}(p^{ij}, p^{ij'})$ be the payoff to retailer $j = A, B$ from product $i = 1, 2$, when retailer $j' \neq j$ also carries product *i*. Then

$$\Pi^{ij}(p^{ij}, p^{ij'}) = (p^{ij} - c^j - w)D^{ij}(p^{ij}, p^{ij'}), \qquad \text{(TA.14)}$$

where c^j denotes the marginal cost for retailer $j = A, B$. These costs correspond to the assortment cost in the main text. Define a class of pricing games, in strategic form, $G_i(c^A, c^B) = (\Pi^{ij}, [0, \overline{p}])_{j=A,B}$, which are parameterized by costs. Our first objective is to establish the existence of a unique equilibrium, which is achieved in the following lemma.

LEMMA TA.2

If $\frac{\partial^2 D^{ij}}{\partial (p^{ij})^2} < -2 \frac{\partial D^{ij}}{\partial p^{ij}} / \overline{p}$ for all $p^{ij} \in [0, \overline{p}]$ then G_i has a unique equilibrium $(\hat{p}^{ij}, \hat{p}^{ij'}) \in [0, \overline{p}]^2$ such that

(a) $\hat{p}^{ij}$ is increasing in c^A and c^B for $i = 1, 2$ and $j = A, B$; and

(b) $c^j < c^{j'}$ implies that $\hat{p}^{ij} < \hat{p}^{ij'}$ for $i = 1, 2$ and $j, j' = A, B$; $j' \neq j$.

Proof: G_i is supermodular: (i) Strategy spaces are compact lattices; (ii) Payoffs are (trivially) supermodular in own strategies; and (iii) Payoffs exhibit increasing differences in rival's strategies (strategic complements). Thus G_i has an equilibrium (Vives 1990).

The condition on the second derivative of demand ensures uniqueness (dominant diagonal condition). To show (a), note that

$$\frac{\partial^2 \Pi^{ij}}{\partial p^{ij} \partial c^j} = \frac{\partial}{\partial p^{ij}} \left[-D^{ij}(p^{ij}, p^{ij'}) \right] > 0, \qquad \text{(TA.15)}$$

for $i = 1, 2$; $j, j' = 1, 2$ and $j' \neq j$. Hence, Π^{ij} has strictly increasing differences in (p^{ij}, c^j). By the monotone comparative statics property of supermodular games, we have the result in (a). (See Milgrom and Shannon 1995 or Vives 1999, Chapter 2.) To show (b), first consider the case of equal costs: $c^A = c^B = c$. Then, the unique equilibrium is symmetric: $\hat{p}^{iA} = \hat{p}^{iB}$. Depart from the symmetric case by unilaterally decreasing $c^A = c' < c$. This decreases *A*'s best response function, $B^{iA}(p^{iB}, c^A)$. That is, $B^{iA}(p^{iB}, c') < B^{iA}(p^{iB}, c)$ for all p^{iB}. On the other hand, *B*'s best response function $B^{iB}(p^{iA}, c^B)$ is unchanged. Since $B^{iB}(p^{iA}, c) > p^{iA}$ for all $p^{iA} \in [0, \hat{p}]$, we conclude that the equilibrium of $G_i(c', c)$ is characterized by a lower price for A than for *B*. In other words, the fixed point of the best reply function: $(B^{iA}, B^{iB}) : [0, \overline{p}]^2 \to [0, \overline{p}]^2$ lies "above" the 45^0 line. See Figure TA.1 Q.E.D.

This lemma establishes the existence and uniqueness of the pricing game that ensues after the agents have determined the distributional strategy. Furthermore, part (b) of the lemma implies that lower assortment costs yield lower retail prices. Hence, under the conditions that yield a strategic assortment reduction, it also follows that the retailer with lower assortment costs also has lower prices.

Before identifying the incentives for choosing assortments, it is also necessary to consider the case when retailer *j* is the exclusive dealer of product *i*. Denote by $\Pi^{ij}(p^{ij}, -)$ as the payoff to retailer *j* from the sale of product *i* when retailer $j' \neq j$ is not selling product *i*. Then

$$\Pi^{ij}(p^{ij}, -) = (p^{ij} - c^j - w) D^{ij}(p^{ij}, -). \qquad \text{(TA.16)}$$

The maximization of (TA.16) is well defined under the condition in the lemma above.

Before examining the manufacturer's and the dominant retailer's preferences over distributional outcomes, we make two regularity assumptions on demand. These assumptions are made to give the manufacturer non-trivial trade-offs when deciding between distribution options.

ASSUMPTION TA.1 For $i, i' = 1, 2$, $j, j' = A, B$, $j' \neq j$.

(1) $D^{ij}(\hat{p}^{ij}, \hat{p}^{ij'}) < D^{ij}(\hat{p}^{ij}, -) < D^{iA}(\hat{p}^{iA}, \hat{p}^{iA}) + D^{iB}(\hat{p}^{iB}, \hat{p}^{iA})$

(2) $\left|\dfrac{\partial D^{ij}(\hat{p}^{ij}, -)}{\partial c^j}\right| \leq \left|\dfrac{\partial D^{i'A}(\hat{p}^{i'A}, \hat{p}^{i'B}) + \partial D^{i'B}(\hat{p}^{i'B}, \hat{p}^{i'A})}{\partial c^j}\right|$

In part (1), the first inequality states that a monopoly in product *i* has more sales than a competitor in duopoly. Without this condition, the manufacturer would never prefer *Sp* over *F*. The second inequality ensures that equilibrium sales are always higher with two retailers relative to one. Part (2) of this inequality simply states that a small change in assortment costs of one retailer has no smaller effect on sales when there are two sellers than when there is only one.

As in the main text, we suppose that each retailer faces the same marginal cost function, $c^j = c(n^j)$, where n^j is the number of products retailer *j* carries and that $c \equiv c(2) > c(1) = 0$. We now consider the four possible distributional arrangements *F*, *Sp*, *Ex*, and *S* when retailers set prices according to the corresponding equilibrium strategies derived above. Abbreviate notations with D_x^{ij} as the demand for product *i* at retailer *j* under the distributional arrangement $x \in \{F, Sp, Ex, S\}$. Similarly abbreviate payoffs Π_x^j. Finally, we make the following assumption.

ASSUMPTION TA.2

(1) $\left|\dfrac{\partial(D_{Sp}^{1A} + D_{Sp}^{1B})}{\partial c}\right| \leq \left|\dfrac{\partial(D_F^{1A} + D_F^{1B})}{\partial c}\right|$;

(2) $\left(D_S^{1A} + D_S^{1B}\right) - \left(D_{Sp}^{1A} + D_{Sp}^{1B}\right) \leq \left(D_{Sp}^{1A} + D_{Sp}^{1B}\right) - \left(D_F^{1A} + D_F^{1B}\right)$.

Generally, this assumption states that effects from assortment costs are no stronger at *Sp* than at *F*. Specifically, part (1) says that for a change in assortment costs, the impact on equilibrium sales levels is no stronger at *Sp* than at *F*. Similarly, part (2) ensures that the increase in overall sales of product 1 associated with retailer *A*'s abandonment of product 2 is not dramatically different than the sales increase in product 1 when retailer *B* is the second retailer to abandon product 2.

We start with the *M*'s incentives with respect to outcomes *S* and *Sp*. *M* prefers *S* over *Sp* if and only if $\Pi_{Sp}^{M} = w\left[D_{Sp}^{1A} + D_{Sp}^{1B} + \alpha D_{Sp}^{2B}\right] < \Pi_{S}^{M} = w\left[D_{S}^{1A} + D_{S}^{1B}\right]$. This is equivalent to the condition:

$$\Pi_{S}^{M} > \Pi_{Sp}^{M} \qquad \Leftrightarrow \qquad f(c) \equiv \frac{(D_{S}^{1A} + D_{S}^{1B}) - (D_{Sp}^{1A} + D_{Sp}^{1B})}{D_{Sp}^{2B}} > \alpha \,. \qquad \text{(TA.17)}$$

As in the main text, we evaluate the preferences for *M* (and later *A*) by examining indifference curves with respect to the parameters *c* and α. We preserve the notation with that of the Figures 2 and 3 of the main text by denoting *f* as the indifference curve for *M* with respect to outcomes *Sp* and *S*. In Figure TA.2, *M* prefers outcome *S*, for parameter values α and *c* lying above the curve *f*. The shape of this indifference curve can be seen by noting that (TA.17) holds with equality at $(\alpha, c) = (0,0)$. Furthermore, from Lemma TA.2 (part (a)), $f(c) > 0$ and increasing for $c > 0$.

Similarly, we can characterize *M*'s preference for *Sp* over *F* by the condition: $\Pi_{F}^{M} = w\left[\left(D_{F}^{1A} + D_{F}^{1B}\right) + \alpha\left(D_{F}^{2A} + D_{F}^{2B}\right)\right] < \Pi_{Sp}^{M}$. That is

$$\Pi_{Sp}^{M} > \Pi_{F}^{M} \qquad \Leftrightarrow \qquad g(c) \equiv \frac{D_{Sp}^{1A} + D_{Sp}^{1B} - D_{F}^{1A} - D_{F}^{1B}}{D_{F}^{2A} + D_{F}^{2B} - D_{Sp}^{2B}} > \alpha \,. \qquad \text{(TA.18)}$$

As with *f*, $g(0) = 0$ since the numerator is zero with no assortment costs and the denominator is positive under Assumption TA.1. This implies (TA.18) holds with equality at $(\alpha, c) = (0,0)$. To see that *g* is increasing in *c*, we evaluate the sign of its derivative. Bearing mind symmetry in outcome *F* (i.e. $D_{F}^{ij} = D_{F}^{i'j'}$ for $i, i' = 1, 2$ and $j, j' = A, B$), we have that $g'(c)$ is positive since

$$g'(c) = \frac{1}{x^2}\left\{ \frac{\partial\left(D_{Sp}^{1A} + D_{Sp}^{1B}\right)}{\partial c}\left(D_F^{1A} + D_F^{1B} - D_{Sp}^{2B}\right) - \frac{\partial\left(D_F^{1A} + D_F^{1B}\right)}{\partial c}\left(D_{Sp}^{1A} + D_{Sp}^{1B} - D_{Sp}^{2B}\right) \right.$$
$$\left. + \frac{\partial D_{Sp}^{2B}}{\partial c}\left(D_{Sp}^{1A} + D_{Sp}^{1B} - D_F^{1A} - D_F^{1A}\right) \right\}$$
$$\geq \frac{1}{x^2}\left\{ -\frac{\partial\left(D_{Sp}^{1A} + D_{Sp}^{1B}\right)}{\partial c} + \frac{\partial D_{Sp}^{2B}}{\partial c} \right\}\left(D_{Sp}^{1A} + D_{Sp}^{1B} - D_F^{1A} - D_F^{1B}\right)$$
$$\geq 0$$

where $x^2 = \left(D_F^{2A} + D_F^{2B} - D_{Sp}^{2B}\right)^2 > 0$. The first inequality follows from Assumption TA.2 part (1). The second inequality is a result of the facts that $D_{Sp}^{1j} > D_F^{1j}$ (by Lemma TA.2 part (a)) and Assumption TA.1 part (2). Therefore, *g* and *f* have similar curves in the parameter space (α, c). Lemma TA.3 gives an order relation for these two indifference curves.

LEMMA TA.3 Under Assumptions TA.1 and TA.2, $f(c) < g(c)$ for all $c > 0$. That is, for any fixed $c > 0$ and outcomes *F*, *Sp*, and *S*, *M* prefers *S* for $0 < \alpha < f(c)$, *Sp* for $f(c) < \alpha < g(c)$ and *F* for $\alpha > g(c)$.

Proof: The order $f(c) < g(c)$ can be shown by using Assumption TA.1 and part (2) of Assumption TA.2. This means that for $0 < \alpha < f(c)$, the conditions (TA.17) and (TA.18) hold (*S* is preferred to *Sp* is preferred to *F*). Increasing α beyond $f(c)$ but short of $g(c)$ implies (TA.17) is violated (*Sp* is preferred to *S*) but (TA.18) holds (*Sp* is preferred to *F*). Finally, for large $\alpha > g(c)$, both (TA.17) and (TA.18) do not hold (*F* is preferred to *Sp* is preferred to *S*). Q.E.D.

Figure TA.2 illustrates[2] the indifference curves *f* and *g* as characterized by Lemma TA.3. For the remainder of this technical appendix, we focus attention to regions of the parameter space where *F* is optimal for *M*. This corresponds to the region:

[2] The notation corresponds to that of Figure 2 in the main text.

$$\Theta_{Bertrand} = \{(\alpha, c) \in [0,1] \times [0, \overline{p}] \mid \alpha > g(c)\},$$

which is region in Figure TA.3 below and to the right of the indifference curve *g*. As defined in the main text: outcome *x* is a strategic assortment reduction equilibrium outcome if and only the following two conditions hold:

(i) $\Pi_x^A > \Pi_F^A$; and (ii) $\Pi_x^M > \Pi_y^M$ for all $y \neq x; y \in \Lambda$,

where $\Lambda = \{Sp, Ex, S\}$. As Figure TA.3 indicates, *S* is not a possible strategic assortment reduction equilibrium outcome (*Sp* is always preferred to *S* in $\Theta_{Bertrand}$). As for *Ex*, note that the *Sp* distribution is preferred over *Ex* by *M* if and only if $\Pi_{Sp}^M > \Pi_{Ex}^M = w\left[D_{Ex}^{1A} + \alpha D_{Ex}^{2B}\right]$. That is,

$$\Pi_{Sp}^M > \Pi_{Ex}^M \quad \Leftrightarrow \quad h(c) \equiv \frac{D_{Sp}^{1A} + D_{Sp}^{1B} - D_{Ex}^{1A}}{D_{Ex}^{2B} - D_{Sp}^{2B}} > \alpha. \tag{TA.19}$$

This indifference curve, *h*, is downward sloping since the numerator in (TA.19) is decreasing in *c* and the denominator increasing. Furthermore, $\lim_{c \to \infty} h(c) = 0$ (by Lemma TA.2 and Assumption TA.1) and $\lim_{c \to 0} h(c) = \infty$ which implies the shape depicted in Figure TA.2.[3]

Figure TA.2 displays the outcome of the *M* dominant game in some regions of the parameter space. For the purposes of this Technical Appendix, we omit discussion of the equilibrium outcomes in the regions above *g*.

We now turn to the preferences of retailer *A* in order to determine the conditions in which she would strategically refuse to carry product 2. Since her default payoff is her profit under *F*, it suffices to compare this with her payoffs in the outcomes *Sp* and *Ex*. If *A* prefers *Sp* over *F* then $\Pi_{Sp}^A = (\hat{p}_{Sp}^{1A} - w)D_{Sp}^{1A} > \Pi_F^A = (\hat{p}_F^{1A} - c - w)(1+\alpha)\left[D_F^{1A} + D_F^{2A}\right]$, which implies the condition:

$$\Pi_{Sp}^A > \Pi_F^A \quad \Leftrightarrow \quad k(c) \equiv \frac{(\hat{p}_{Sp}^{1A} - w)D_{Sp}^{1A}}{2(\hat{p}_F^{1A} - c - w)D_F^{1A}} - 1 > \alpha.$$

As for *A*'s preference for *Ex* over *F*, note that it always holds that $\Pi_{Sp}^A > \Pi_{Ex}^A$ since the only difference for *A* is that her rival's costs are higher in *Sp* than in *Ex*. Recall that *M*

[3] In the context of Figure TA.3, it is not possible to establish whether *h* intersects *g* within the relevant parameter space.

prefers *Ex* to *Sp* in the region between *g* and *h* so that if *A* abandons product 2 (in stage 0) *M* will distribute only product 2 through retailer *B*. Therefore, *Ex* may be optimally implemented by *A* if $\Pi_{Ex}^{A} = (\hat{p}_{Ex}^{1A} - w)D_{Ex}^{1A} > \Pi_{F}^{A}$, which implies the condition:

$$\Pi_{Ex}^{A} > \Pi_{F}^{A} \qquad \Leftrightarrow \qquad l(c) \equiv \frac{(\hat{p}_{Ex}^{1A} - w)D_{Ex}^{1A}}{2(\hat{p}_{F}^{1A} - c - w)D_{F}^{1A}} - 1 > \alpha \ .$$

By inspection, one can see that $k(0) = l(0) = 0$ and that *k* and *l* are both increasing in *c*. This implies that the indifference curves *k* and *l* pass through the origin, as depicted in the graph of Figure TA.3. Note that $l(c) < k(c)$ for all $c > 0$. Thus, for any parameter constellation located below *k*, retailer *A* prefers *F* and would never abandon product 2 in favour of *Sp*. In contrast, for points above the indifference curve *k*, *Sp* and *Ex* are a possible strategic assortment reduction. The condition under which this occurs depends crucially on the following notion. Because assortment costs lower margins for retailers, they imply a loss of sales. The precise amount of lost sales (of product 1) can be measured by the percentage $E \equiv \frac{D_{Sp}^{1A} + D_{Sp}^{1B}}{D_{F}^{1A} + D_{F}^{1B}} - 1 \geq 0$. In other words, *E* measures the extent to which lower assortment costs generate more category sales (as opposed to raising retailer margins).[4] If *E* is large, for example, then the manufacturer has a stronger incentive to release costs from the retailer by permitting assortment reduction. A larger *E* tends *M*'s indifference curve *g* downward to the horizontal axis in Figure TA.3. On the other hand, if *E* is small then by lowering assortment costs through the abandonment of product 2, retailer *A*'s advantage is strategic vis-à-vis its rival retailer. That is, as *E* decreases the indifference curve *g* rotates counter-clockwise. For small *E*, the curve *g* lies above the curve *k*. In this case, channel incentives are divergent in the wedge between these curves and strategic assortment reduction is the equilibrium outcome.

PROPOSITION TA.3 Suppose Assumptions TA.1 and TA.2 hold and define $\upsilon = g^{-1}(1)$.

[4] If *E*=0 then we have a fixed-demand model (e.g. Hotelling).

(a) If $E < k(c)\left[1 - \frac{D_{Sp}^{2B}}{D_F^{1A} + D_F^{1B}}\right]$ for all $c \in (0, \upsilon]$, then there exists a strategic assortment reduction equilibrium outcome *x* (*Sp* or *Ex*) for $g(c) < \alpha < \min\{k(c), 1\}$.

(b) If $E > k(c)\left[1 - \frac{D_{Sp}^{2B}}{D_F^{1A} + D_F^{1B}}\right]$ for all $c \in (0, \upsilon]$, then no strategic assortment reduction equilibrium outcome exists for any parameters.

Proof: (a) The condition on *E* implies directly that $g(c) < k(c)$ for $c \in (0, \upsilon]$. (Recall that $D_F^{ij} = D_F^{i'j'}$ for $i, i' = 1, 2$ and $j, j' = A, B$.) This implies that there is a region between these two curves for which $\Pi_{Sp}^A > \Pi_F^A$ and $\Pi_{Sp}^M, \Pi_{Ex}^M < \Pi_F^M$. In this case, *Sp* satisfies (i) and (ii) of the definition of strategic assortment reduction. Depending on the exact degree of the curve *h*, it is possible to have $\Pi_{Ex}^A > \Pi_F^A$ and $\Pi_{Sp}^M < \Pi_{Ex}^M < \Pi_F^M$ so that *Ex* is the strategic assortment reduction outcome. (b) The condition on *E* implies directly that $g(c) > k(c)$ for $c \in (0, \upsilon]$. In this case: $\Pi_F^M > \Pi_{Sp}^M \Rightarrow \Pi_F^A > \Pi_{Sp}^A, \Pi_{Ex}^A$. Q.E.D.

Proposition TA.3 gives a sufficient condition for the existence of a set of parameters for which strategic assortment reduction is possible.[5] This case is depicted in Figure TA.3. Recall, however, it is not guaranteed that *Ex* is a possible strategic assortment reduction equilibrium outcome without more conditions. Nevertheless, from this graphical analysis, we conclude that as long as $E > 0$ is not too large, the channel incentives for carrying the specialty product 2 differ for the manufacturer and for retailer *A* and the wedge between curves *g* and *k* (marked by ***Sp*** or possibly ***Ex***) represents these differences. Outside of this wedge, however, channel incentives are aligned. For large values of α and low values of *c*, both the manufacturer and retailer *A* want the additional market for product 2 served. This corresponds to the lower right region of the Figure

5 It is important to keep in mind that Proposition TA.3 is not exhaustive. Without more conditions on demand, it is not possible to rule out the possibility that the curves k and g intersect for $c > 0$. In this case, strategic assortment reduction is possible. We omit a full analysis of this case as it does not lend to additional insights.

TA.3. For large c and low α, the manufacturer does not find the channel losses from high costs worth the dual distribution of product 2.

It is worthwhile to relate this analysis to that of the model of the main text. In particular, compare Figure 3 with Figure TA.2. Loosely speaking, the horizontal axes in both graphs correspond to the "popularity" of the specialty variety. In particular, large values of α map to large value of the ratio b_1 / b_2. Similarly, the assortment costs are represented on the vertical axes of both graphs. The region of these parameters that supports *Sp* as an equilibrium correspond, as well.

Proposition TA.3 part (b) gives a sufficient condition for when strategic assortment reduction will never occur. Under this condition, the incentives of *M* and *A* are always aligned. Whenever it is optimal for *A* to reduce assortment, it will also be optimal for *M*.

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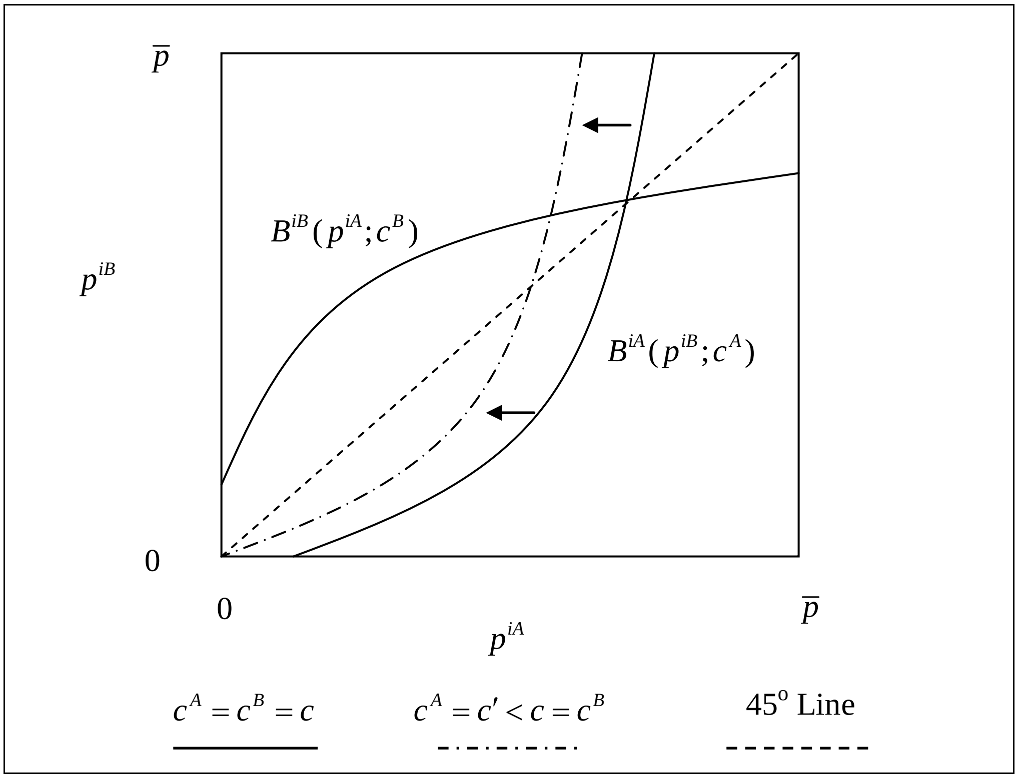

Figure TA.1 Best Response Curves of the Pricing Game

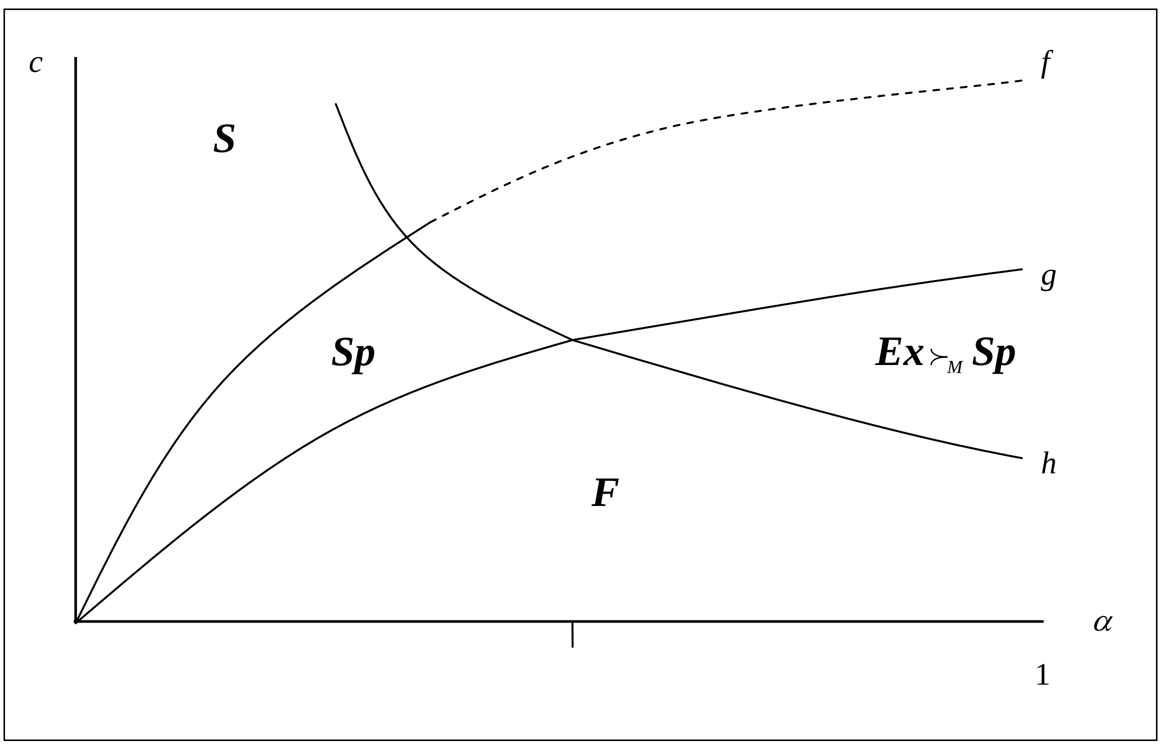

Figure TA.2 *M*'s Preferences in Some Regions of the Parameter Space

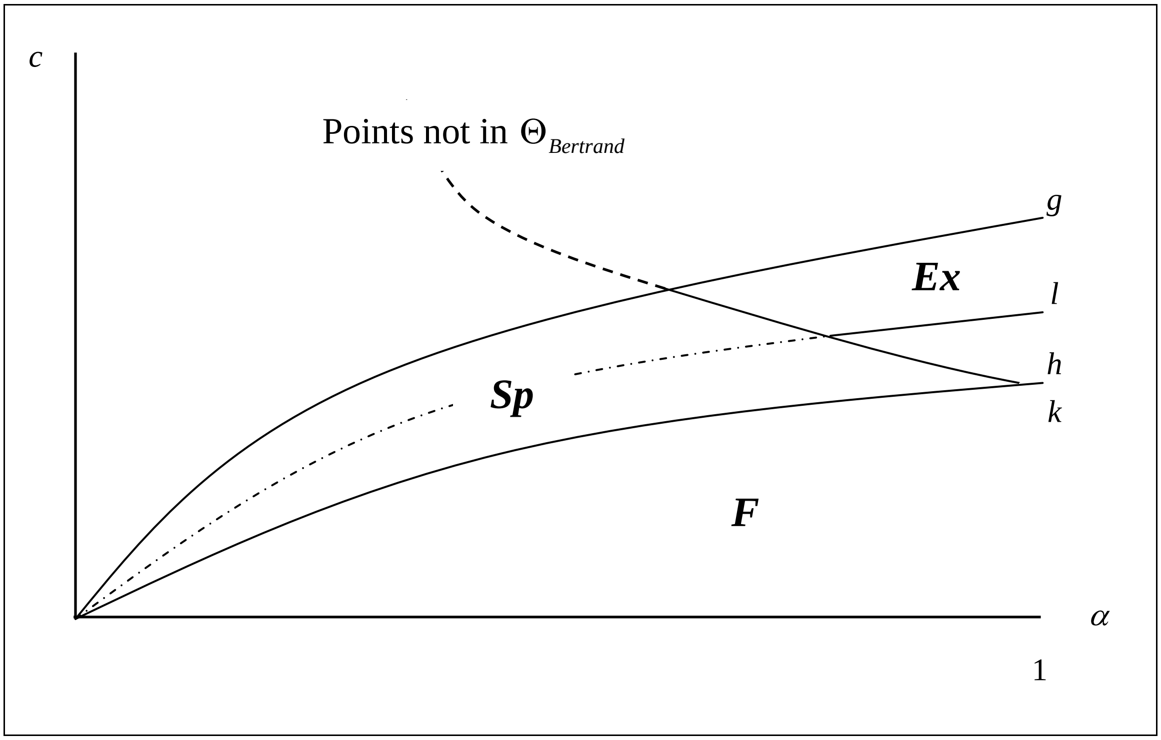

Figure TA.3 Strategic Assortment Reduction Equilibrium Outcomes