Minimum Differentiation in Commercial Media Markets

Supplemental Research Note

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In this note we illustrate the robustness of the principle of minimum differentiation under several modifications to the original model. In particular, we consider the following modifications:

(i) an alternative pricing mechanism in which stations set, rather than negotiate, the unit price of advertising;

(ii) the case when either the media or product market is not fully served;

(iii) viewers incorporate the informational benefit of advertising;

(iv) stations receive additional source of revenue.

C.1 (Alternative Pricing Mechanism for Advertising)

In this section we examine the case when stations quote "take-it-or-leave-it" prices, rather than entering bilateral negotiations with producers. We argue that when stations compete in advertising price, competing stations continue to have the incentive to offer identical programming. Specifically, we demonstrate numerically that profits are maximized when they minimally differentiate programming.

To formally examine "take-it-or-leave-it" pricing by stations, we modify the original model by allowing stations to move first, in stage 0, by choosing their location $d_j \in [0, \frac{1}{2})$ $j = 1,2$. In stage 1, each station $j = 1,2$ offers a single unit price a^j for advertising on station j. The remainder of the game is as in the original model. Recall that, in stage 2, given these advertising prices each producer $i = 1,2$ chooses advertising levels φ_i^1, φ_i^2 and product price p_i, as in the original model. Finally, consumers make their media choice and subsequent product choice in stage 3.

Since producers' and consumers' objectives remain as in the original model, we begin with stations' pricing decisions in stage 1, using producers' first order conditions (7) and (8) to characterize all subsequent choices by producers and consumers in stages 2 and 3. Note that, since stations set a single price, we can assume that producers choose symmetrically with respect to a given station. Therefore, we can denote the level of advertising chosen by each producer on station j as $\varphi^j = \varphi_1^j = \varphi_2^j$. Using (7) and (8), we can then write a producer's inverse demand for advertising on station j as

$$a^j(\varphi^j) = \frac{X^1 D^1 + X^2 D^2}{-\left(X^1 \frac{\partial D_i^1}{\partial p_i} + X^2 \frac{\partial D_i^2}{\partial p_i} \right)} \left[X^j D^j \frac{G'(\varphi^j)}{G(\varphi^j)} - \frac{\gamma(D^j - D^r)}{2t_s(1-d_1-d_2)} \right], \qquad \text{(C.1.1)}$$

with the right-hand expression evaluated at $\varphi^j = \varphi_1^j = \varphi_2^j$, where X^j, D^j, are as defined in the main text in (10) and (11) and

$$\left. \frac{\partial D_i^j}{\partial p_i} \right|_{\varphi_1^j = \varphi_2^j = \varphi^j} = \frac{-G^2(\varphi^j)}{2t_p}$$

for $j = 1,2$. By specifying advertising price, a^j, station j implicitly determines the amount of advertising that producers purchase with station j in stage 2. Therefore, station j's objective in stage 1 is equivalent to choosing advertising level φ^j in order to maximize $C^j = 2a^j(\varphi^j)\varphi^j$ subject to (C.1.1). The first order necessary condition for station j in stage 1 is

$$\frac{da(\varphi^j)}{d\varphi^j} \left[a^j(\varphi^j) \right]^{-1} = \frac{-1}{\varphi^j}. \qquad \text{(C.1.2)}$$

Given symmetric location decisions $d = d_1 = d_2$ in stage 0, we denote the level of advertising in a symmetric equilibrium of the subgame starting in stage 1 by $\varphi_d^* = \varphi^1 = \varphi^2$, which is a function of d and implicitly characterized by the equation

$$\frac{G''(\varphi_d^*)}{G'(\varphi_d^*)} - \frac{G'(\varphi_d^*)}{2-G(\varphi_d^*)} \left(1 - \frac{1}{G(\varphi_d^*)} \right) - \frac{2\gamma(3-2G(\varphi_d^*))}{t_s(1-2d)(2-G(\varphi_d^*))} = \frac{-1}{\varphi_d^*},$$

and derived from (C.1.2).

By numerically solving this equation for φ_d^* at values of $d \in (0,.5)$, we demonstrate that, consistent with the original model, lower levels of advertising correspond to higher levels of d. The top section of Figure C.1.1 illustrates some typical numerical results using the specification $G(\varphi) = \varphi^\eta$. These results additionally suggest that the comparative statics of φ_d^* with respect to parameters γ and t_s, are also consistent with the original model. Higher values of the ratio γ / t_s

imply that listeners are more sensitive to advertising, thereby inducing stations to reduce advertising levels.

In order to illustrate stations' incentive to minimally differentiate, we compute the station profits $C_d^* = 2a(\varphi_d^*)\varphi_d^*$ as a function of d. Using the intuition from the main model, we expect that higher values of d increase stations' profits since reduced advertising levels, implied by similar programming, reduces product competition, thereby permitting stations to quote higher advertising prices. Numerical results confirm this intuition. (See lower section of Figure C.1.1.)

The above analysis suggests that the minimum differentiation result (Corollary 1) is robust to the way in which one models the determination of advertising price. The unprofitable merger result of section 5, however, does not hold up to this modification. If stations merge before setting advertising prices, the merged entity can ensure that each station individually earns as much profit as without merging since the merged firm can always set advertising prices as if stations were competing.

Overturning this result is not surprising since the merged firm now has complete market power in the exchange of advertising space vis-à-vis producers. Such an extreme distribution of market power implied by a merger is exogenously imposed by the price setting assumption. The bargaining framework of the original model, on the other hand, endogenizes the distribution of market power in accordance with the relative bargaining positions.

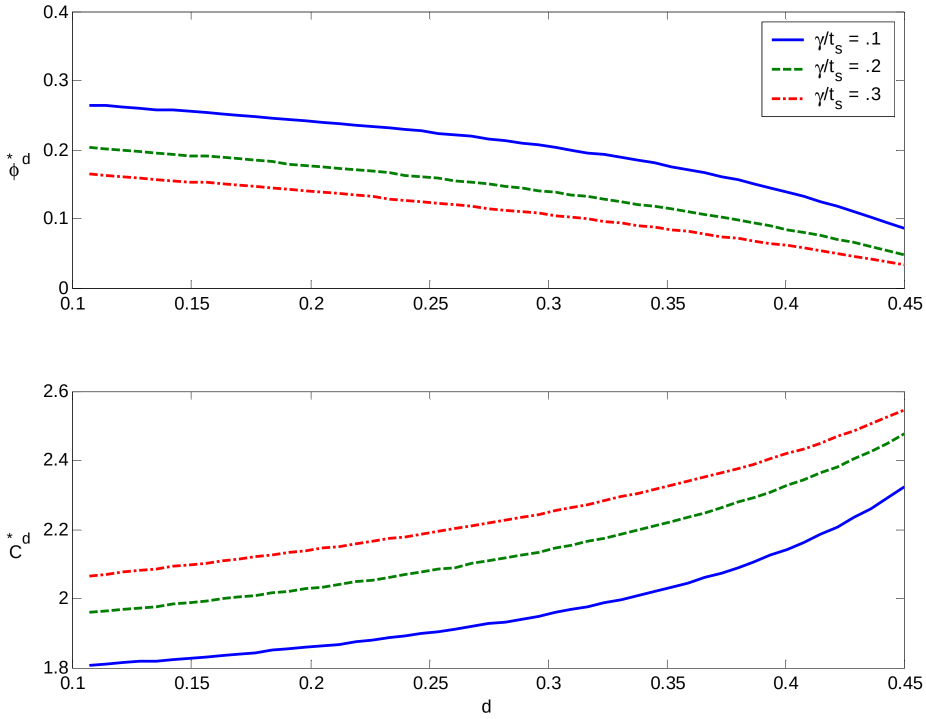

Figure C.1.1: Station Advertising & Profits as a Function of Location ($\eta = 0.70$)

C.2 (Partial Market Coverage)

In this section we investigate the robustness of the principle of minimum differentiation with regard to the assumptions that both the product and the media markets are fully served. We show, in particular, that the incentive for stations to minimally differentiate may not hold when the product market is not fully covered. However, the principle of minimum differentiation can still hold if the media market is not fully covered.

Partially Covered Product Market

When the prices of the products are sufficiently high, some consumers may withdraw from the market as illustrated in Figure C.2.1. The threshold consumer x^* is indifferent between buying product 1 and withdrawing from the market and consumer $(1-y^*)$ is indifferent between buying product 2 or withdrawing, where the values of x^* and y^* are determined as follows:

$$x^* = \left(\frac{v_p - p_1}{t_p}\right)^{1/2} \quad \text{and} \quad y^* = \left(\frac{v_p - p_2}{t_p}\right)^{1/2}.$$

The share of station j's listeners who end up buying product i is given as:

$$D_i^j = \left(\frac{v_p - p_i}{t_p}\right)^{1/2} G(\varphi_i^j).$$

Note that this expression is independent of the decision made by the competitor of i since each producer has a local monopoly in this case. Other than the two different expressions for D_i^j, equations (4)-(8) still govern the negotiations among the parties and the optimization problems of the producers. Substituting the different expressions for D_i^j at the symmetric equilibrium when $d_1 = d_2 = d$ yields:

$$p^* - c = \frac{v_p - c}{2} \quad \text{and} \quad \varphi^* = T^{-1}\left(\frac{\gamma}{2t_s(1-2d)}\right).$$

Substituting the above into the agreement payoff of each station in (4) yields

$$C^* = \frac{(v_p - c)^{3/2} G(\varphi^*)}{4(2t_p)^{1/2}}\left[1 + \frac{\gamma\varphi^*}{t_s(1-2d)}\right].$$

When the elasticity of the outreach probability is non-increasing with φ, the expression $\gamma\varphi^* / t_s(1-2d)$ is decreasing when the ratio $\gamma / t_s(1-2d)$ declines. However, since the extent of advertising φ^* increases in this case, the expression multiplying the brackets in C^* is larger. The profits of each station may increase, therefore, when t_s is bigger or d is smaller, thus contradicting the minimum differentiation result we obtain in the main text. This result depends crucially on the existence of price competition in the product market, which is absent when the market is less than fully covered.

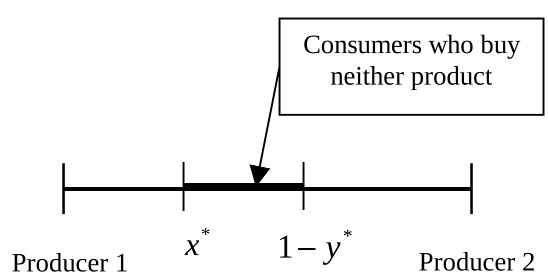

Figure C.2.1 Product Market Less than Fully Covered

Partially Covered Media Market

Using the derivations included in the proof of Lemma 1, it follows that consumers who patronize a single station are most likely to drop out of the media market. Assuming that all the three regions reported in Lemma 1 still exist, in Figure C.2.2, we depict the net utility of the consumer as a function of her location in the distribution. In the figure, we assume that $\varphi_1^2 + \varphi_2^2 > \varphi_1^1 + \varphi_2^1$ and we also assume that the consumers who are located close to the two edges of the distribution find it optimal to withdraw from the market since their net utility is negative.

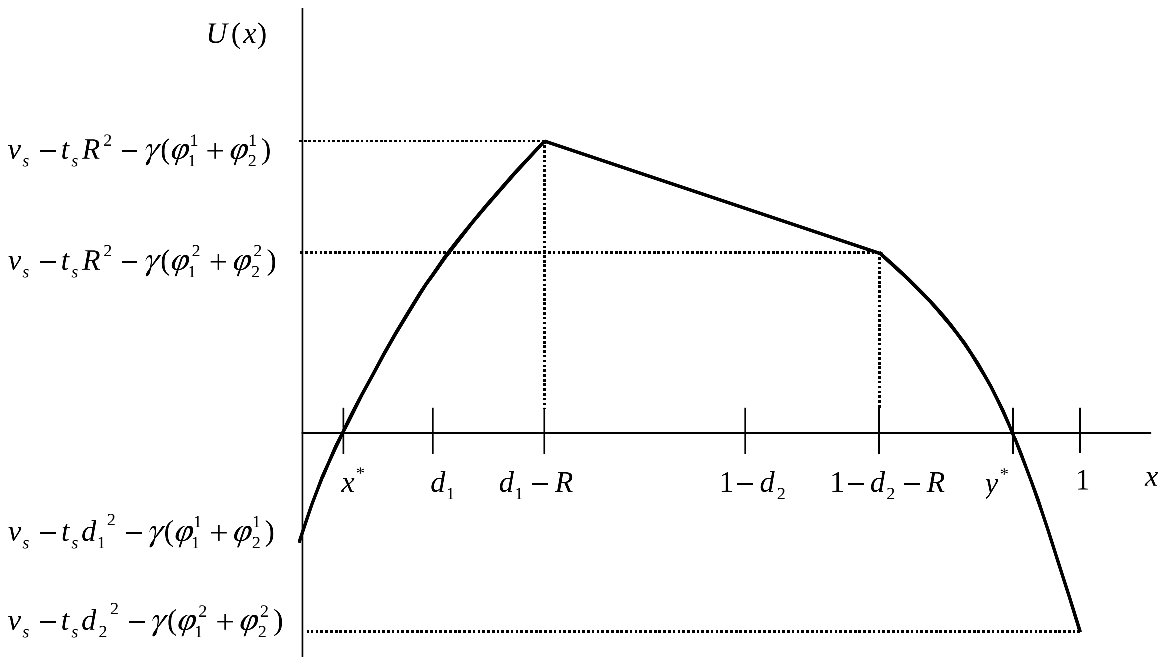

Figure C.2.2 Utility of Consumers in Partially Covered Media Market

The threshold consumers x^* and y^* who are indifferent between patronizing their preferred station or withdrawing from the media market satisfy the following equations:

$$x^* = d_1 - \sqrt{\frac{v_s - \gamma(\varphi_1^1 + \varphi_2^1)}{t_s}} \quad \text{and} \quad y^* = 1 - d_2 + \sqrt{\frac{v_s - \gamma(\varphi_1^2 + \varphi_2^2)}{t_s}}. \qquad \text{(C.2.1)}$$

For selected locations d_1 and d_2 of the two stations, we now describe the equilibrium of the second stage game assuming that the market is less than fully covered. We restrict attention to equilibria where the two producers behave symmetrically (i.e. $p_1 = p_2$ and $\varphi_1^i = \varphi_2^i = \varphi^i$, $i = 1,2$). The market shares of the stations can be obtained from (C.2.1) as follows:

$$X^1 = \frac{1-d_1-d_2}{2} + \sqrt{\frac{v_s - 2\gamma\varphi^1}{t_s}} - R \quad \text{and} \quad X^2 = \frac{1-d_1-d_2}{2} + \sqrt{\frac{v_s - 2\gamma\varphi^2}{t_s}} + R .$$

In case of disagreement between station 1 and one of the producers those market shares change as follows

$$\tilde{X}^1 = \frac{1-d_1-d_2}{2} + \sqrt{\frac{v_s - 2\gamma\varphi^1}{t_s}} - \frac{\gamma\left(\frac{\varphi^1}{2} - \varphi^2\right)}{t_s(1-d_1-d_2)}, \quad \tilde{X}^2 = \frac{1-d_1-d_2}{2} + \sqrt{\frac{v_s - 2\gamma\varphi^2}{t_s}} + \frac{\gamma\left(\frac{\varphi^1}{2} - \varphi^2\right)}{t_s(1-d_1-d_2)}$$

,

and in case of disagreement between station 2 and one of the producers, market shares change as follows:

$$\tilde{\tilde{X}}^1 = \frac{1-d_1-d_2}{2} + \sqrt{\frac{v_s - 2\gamma\varphi^1}{t_s}} - \frac{\gamma\left(\varphi^1 - \frac{\varphi^2}{2}\right)}{t_s(1-d_1-d_2)}$$

$$\tilde{\tilde{X}}^2 = \frac{1-d_1-d_2}{2} + \sqrt{\frac{v_s - 2\gamma\varphi^2}{t_s}} + \frac{\gamma\left(\varphi^1 - \frac{\varphi^2}{2}\right)}{t_s(1-d_1-d_2)} .$$

Agreement and disagreement payoffs remain as in (4) and (5) with the only change being the expressions for the market shares of stations. Solving for the Nash bargaining solution, yields expression (6), as when the market is fully covered, and, similarly optimizing with respect to the price still yields equation (8). The optimization with respect to advertising levels yields a different expression as follows:

$$\frac{\partial F_i}{\partial \varphi_i^j} = (p-c)\left\{ X^j D^j \left[\frac{G'(\varphi^j)}{G(\varphi^j)} - \frac{1}{2\varphi^j} \right] - \frac{\gamma(2D^j - D^r)}{4t_s(1-d_1-d_2)} - \frac{\gamma D^j}{2\sqrt{t_s(v_s - 2\gamma\varphi^j)}} \right\} = 0 .$$

At the symmetric equilibrium when $d_1 = d_2 = d$, $\varphi^1 = \varphi^2 = \varphi^*$ and $X^1 = X^2 = X^*$. The expression for φ^* and X^* are given as follows:

$$T(\varphi^*) = \frac{\gamma}{2X^*}\left[\frac{1}{2t_s(1-2d)} + \frac{1}{\sqrt{t_s(v_s - 2\gamma\varphi^*)}} \right]$$
$$X^* = \frac{1-2d}{2} + \sqrt{\frac{(v_s - 2\gamma\varphi^*)}{t_s}}, \tag{C.2.2}$$

where $X^* < \frac{1}{2}$. When the market is fully covered, the level of advertising per station φ^F satisfies the equation

$$T(\varphi^F) = \frac{\gamma}{2t_s(1-2d)}.$$

Hence, with less than full coverage of the media market, producers cut back on their advertising level (i.e. $\varphi^* < \varphi^F$).

Conducting a comparative statics analysis to evaluate how changes in the parameters affect equilibrium levels of advertising yields from (C.2.2) the following characterization. Advertising levels are unambiguously higher when γ or d are smaller, and when v_s is bigger. Changes in the parameter t_s have ambiguous implications on the equilibrium level of advertising. The latter result is different from the one obtained when the market is fully covered. When the transportation parameter t_s is higher, advertising levels are unambiguously higher when the market is fully covered. In contrast, when the market is less than fully covered, higher values of t_s have two counteracting effects on the incentives to advertise. On the positive side, consumers are more loyal to their preferred station implying that the station is less likely to lose audiences even when the producer increases his advertising with it. On the negative side, however, a higher value of t_s implies from (C.2.2) that each station has a smaller market share, thus reducing the marginal return to advertising.

Substituting the equilibrium levels of advertising and prices into the agreement payoff of each station, we obtain the following expression for the profits of each station at the symmetric equilibrium, where $d_1 = d_2 = d$.

$$C^* = \frac{t_p(2-G^*)^2}{2}\left[X^* + \frac{\gamma\varphi^*}{2t_s(1-2d)}\right], \tag{C.2.3}$$

where $G^* = G(\varphi^*)$.

Because of the complexity in conducting the derivation, we are unable to obtain a similar result to that reported in Proposition 3. Specifically, we cannot show the general result that $d_1 = d_2 = \frac{1}{2}$ is a local equilibrium when the market is partially covered. However, a numerical analysis with the example that $G(\varphi) = \varphi^\eta$ illustrates that the profit of each station in (C.2.3) is unambiguously increasing in d in the neighborhood of ½. Specifically, if both stations decide to reduce their degree of differentiation symmetrically when they are already located close to each other their profitability is enhanced. Figure C.2.3 illustrates some of our numerical calculations.

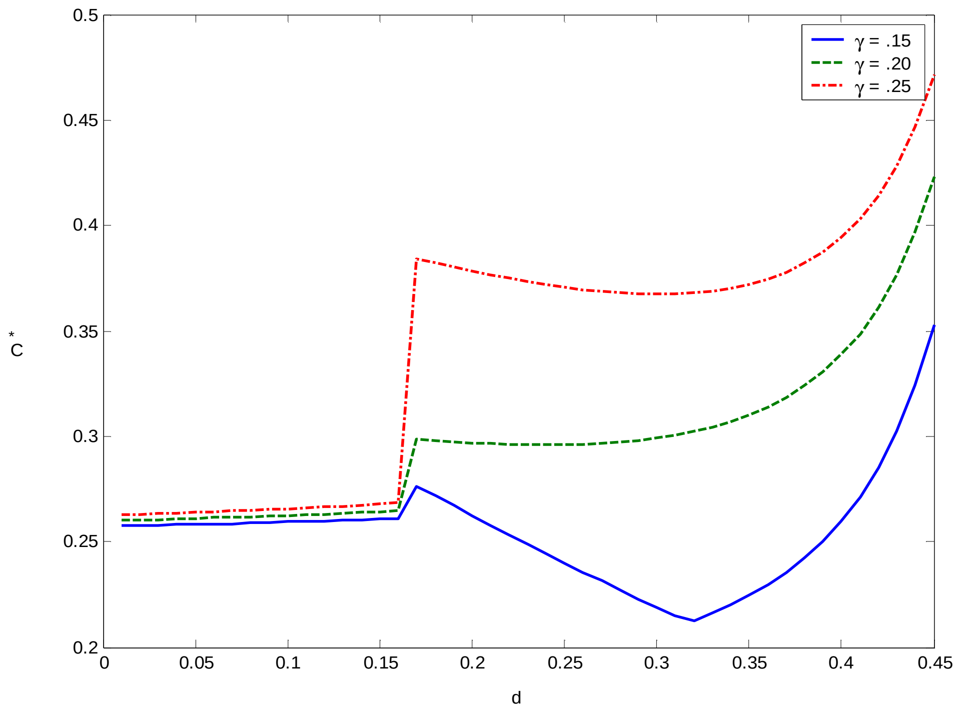

Figure C.2.3 Station Profit as a function d ($\eta = 0.75, t_s = 5, v_s = 0.5$).

Notice that the payoff of each station jumps discontinuously at the point where some consumers withdraw from the media market. For small d values (for instance for $d < 0.17$ when $\gamma = 0.25$) the market is fully covered, but as the stations move closer to each other, some consumers located close to the edges of the distribution withdraw. Levels of advertising decline discontinuously, which alleviates price competition and enhances the profitability of each station in a discontinuous manner.

In Figure C.2.3, there is a range of d values over which the profit of each station is a declining function of d. However, for large d values close to ½, each station's profits are definitely increasing with d. This result implies that the principle of minimum differentiation may still be valid even in an environment where the media market is less than fully covered.

C.3 (Incorporating the Informational Benefits of Advertising in the Specification of the Media Market Utility of Consumers)

In this section, we derive the aggregate level of advertising per station, Φ^*, that consumers consider to be optimal in view of the informational benefits they derive from advertising in the product market. To simplify the derivation, we assume that consumers cannot diversify their viewing experience, and have to choose, instead, one of the two stations. Given the advertising levels φ_i^j, $i, j = 1,2$ on the stations, the consumer who chooses to listen to station j anticipates the following benefit in the product market:

$$\begin{aligned} B_p^j &= \left[v_p - \min\left\{t_p y^2 + p_1, t_p (1-y)^2 + p_2\right\}\right] G_1^j G_2^j + \left[v_p - (t_p y^2 + p_1)\right] G_1^j (1 - G_2^j) \\ &\quad + \left[v_p - \left((1-y)^2 t_p + p_2\right)\right] G_2^j (1 - G_1^j)\,, \end{aligned} \tag{C.3.1}$$

where $G_i^j = G(\varphi_i^j)$.

The underlying assumption behind the above derivation is that before listening to the station the consumer knows about the potential existence of two competing brands. She does not know, however, about the exact features of the brands or their prices unless she hears commercials about their existence.

Given the advertising level on both stations, it follows from (8) that the prices of the products are determined as follows:

$$\left(p_i - c\right) = t_p [\tfrac{2}{3} H_j + \tfrac{1}{3} H_i]\text{, where} \tag{C.3.2}$$

$$H_i = \frac{X^1[2 - G(\varphi_i^1)]G(\varphi_i^1) + X^2[2 - G(\varphi_i^2)]G(\varphi_i^2)}{X^1 G(\varphi_i^1) G(\varphi_j^1) + X^2 G(\varphi_i^2) G(\varphi_j^2)}\,.$$

Hence the product market benefit can be written as follows:

$$B_p^j = \begin{cases} G_1^j\left[(v_p - c) - t_p y^2 - (p_1 - c)\right] + G_2^j (1 - G_1^j)\left[(v_p - c) - t_p (1-y)^2 - (p_2 - c)\right] & \text{if } y \le \dfrac{1}{2} + \dfrac{H_1 - H_2}{6} \\ G_2^j\left[(v_p - c) - t_p (1-y)^2 - (p_2 - c)\right] + G_1^j (1 - G_2^j)\left[(v_p - c) - t_p y^2 - (p_1 - c)\right] & \text{if } y > \dfrac{1}{2} + \dfrac{H_1 - H_2}{6}, \end{cases} \tag{C.3.3}$$

where p_i's are expressed by (C.3.2).

Assuming that the consumer learns about her exact type y only after listening to commercials, it is possible to obtain the ex-ante expected benefit of the consumer by integrating over the two different regions specified in (C.3.3) as follows:

$$EB_p^j = (v_p - c)\left[G_1^j + G_2^j - G_1^j G_2^j\right] - \tfrac{t_p}{3}\left[G_1^j(2H_2 + H_1 + 1) + G_2^j(2H_1 + H_2 + 1)\right]$$
$$- \tfrac{1}{2} G_1^j G_2^j \left[3(H_1 + H_2) + \tfrac{7}{2} + \tfrac{1}{6}(H_1 - H_2)^2\right].$$

In the media market, advertising is considered to be a nuisance since it interrupts the regular programming of the station. Hence the net benefit derived by the consumer in the product and media market combined can be specified, for instance, as follows:

$$V^j = EB_p^j + v_s - t_s x^2 - \gamma\left(\varphi_1^j + \varphi_2^j\right),$$

where γ is a nuisance parameter reflecting the aversion of the consumer to program interruptions and x is the distance of the consumer from station j. The optimal level of advertising of product i on station j from the perspective of the consumer solves the following equation:

$$\frac{\partial V^j}{\partial \varphi_i^j} = \frac{\partial EB_p^j}{\partial \varphi_i^j} - \gamma = 0\,. \qquad \text{(C.3.4)}$$

Note that (C.3.4) yields an identical condition for the two products on a given station. Specifically, $\varphi_1^j = \varphi_2^j$.

The above derivation implies that there is a certain level of aggregate advertising on each station that is desired by the consumer. Designate this level as $\Phi^* \equiv \varphi_1^j + \varphi_2^j$, where φ_i^j is the solution to (C.3.4).

At the time a consumer selects a station, she only knows the aggregate level of advertising of each station and not the distribution of ads across products. Given that the consumer cannot diversify her time between the stations, we modify (1) appropriately and incorporate the newly derived optimal level of aggregate advertising Φ^* as follows:

$$U^j(x) = \begin{cases} v_s - t_s x^2 - \gamma(\Phi^* - (\varphi_1^j + \varphi_2^j)) & \text{if } \varphi_1^j + \varphi_2^j \le \Phi^* \\ \\ v_s - t_s x^2 + \gamma(\Phi^* - (\varphi_1^j + \varphi_2^j)) & \text{if } \varphi_1^j + \varphi_2^j > \Phi^*. \end{cases} \qquad \text{(C.3.5)}$$

Note that if the aggregate level of advertising on station j falls short of the aggregate desired level, the utility of the consumer increases when additional commercials are aired on the station. The new specification results in different expressions for the market shares of stations as follows:

$$X^j = \frac{1 + d_j - d_k}{2} + \frac{\gamma}{2t_s(1 - d_1 - d_2)}\left[(\varphi_1^j + \varphi_2^j)(-1)^{\delta_j} + (\varphi_1^k + \varphi_2^k)(-1)^{\delta_k + 1}\right], \qquad \text{(C.3.6)}$$

$$\delta_j = \begin{cases} 0 & \text{if } \varphi_1^j + \varphi_2^j \le \Phi^* \\ 1 & \text{if } \varphi_1^j + \varphi_2^j > \Phi^*, \end{cases} \qquad j,k = 1,2;\ j \ne k .$$

In particular, if both stations advertise at aggregate levels falling short of Φ^*, (C.3.6) implies that the market share of each station increases when its own advertising level is higher or its competitor's level is lower. Such an outcome is the exact reverse of the results derived in part (ii) of Lemma 1.

Combining the new market share expressions from (C.3.6) with the assumption that $T(\varphi) > 0$ it is easy to show that producers find it optimal to advertise at levels that result in an aggregate level per station beyond Φ^*. To explain this outcome, suppose that at the equilibrium a station advertises an aggregate level less than Φ^*. If a given producer increases its advertising with this station, two favorable effects on its profits are implied. First, since $T(\varphi) > 0$ the benefit from reaching additional consumers exceeds the extra cost the producer will have to pay to advertise with the station. Second, since the market share of this station will increase as a result of the increased advertising, the station becomes a more effective medium to reach consumers. Both effects imply that the producer should increase its advertising with the station, thus pushing the aggregate level of advertising beyond Φ^*.

A similar argument applies also if consumers can diversify their viewing experience between the two stations. Since aggregate advertising falls in the region where consumers consider it a nuisance, the utility formulation in (1) can be made without any loss of generality (i.e. the extra term $\gamma\Phi^*$ in (C.3.5) is just a fixed term that can be interpreted as included in the willingness to pay parameter v_s in equation (1).)

C.4 (Additional Sources of Station Revenue)

In this section we consider the case when stations receive revenue, for example license fees from cable providers, in addition to advertising sales. We show that if these fees are tied to the number of viewers, then the incentive for stations to provide identical programming remains. In particular, the conditions assumed in the main text are sufficient to guarantee that equilibrium profits, in the presence of license payments, are increasing in the location d.

Formally, suppose station j receives a *license payment*, denoted $L(X^j)$, which depends positively on the size of the set of listeners, X^j, so that $L'(X^j) > 0$. This license payment could represent, for example, a license fee paid by a cable provider based on the number of cable subscribers. We then write station j's payoff as

$$C_j = a_1^j \varphi_1^j + a_2^j \varphi_2^j + L(X^j). \tag{C.4.1}$$

Producer i's payoff remains the same as in the original model. (See (4).) We can then derive the payment for advertising space on station j, as determined by negotiations, as

$$a_i^j \varphi_i^j = \frac{(p_i - c)}{2}\left[X^j D_i^j + \frac{\gamma \varphi_i^j D_i^r}{2t_s(1-d_1-d_2)}\right] - \tfrac{1}{2}\left[L(X^j) - L(\tilde{X}^j)\right] \quad i, j, r = 1,2;\ r \neq j$$

where

$$\tilde{X}^j = \frac{1 + d_j - d_r}{2} + \frac{\gamma[(\varphi_1^r + \varphi_2^r) - \varphi_l^j]}{2t_s(1-d_1-d_2)} \quad r \neq j;\ l \neq i.$$

Since $\tilde{X}^j > X^j$, $\Delta L^j \equiv L(X^j) - L(\tilde{X}^j) < 0$. The term ΔL^j represents the loss in the license payment that results from a smaller share of listeners on station j caused by producer i's advertising. As determined through negotiations, station j receives the additional payment $-\tfrac{1}{2}\Delta L^j > 0$, as compared with (6), which reflects station j's improved bargaining position as a result of an outside source of revenue.

Assuming symmetry among producers, we can write a producer's first order condition for advertising on station j as

$$\left.\frac{\partial F_i}{\partial \varphi_i^j}\right|_{\text{Sym}} = (p-c)\left\{X^j D^j\left[\frac{G'(\varphi^j)}{G(\varphi^j)} - \frac{1}{2\varphi^j}\right] - \frac{\gamma}{4t_s(1-d_1-d_2)}\left[2D^j - D^r\right]\right\} + \frac{\Delta L^j}{2\varphi^j} = 0 \quad j = 1,2;\ r \neq j. \tag{C.4.2}$$

A producer's first order condition with respect to product price is as given in (11). If we denote the equilibrium level of advertising and product price in the presence of license payments by φ^L and p^L, respectively, then φ^L and p^L simultaneously solve (C.4.2) and (11).

Comparing (C.4.2) with (9) and (10), a given producer faces a higher marginal cost of advertising since she must partially compensate station j for the loss in license payment due to advertising. Producers advertise less intensely with each station, as a result. This is formally expressed in the following proposition.

Proposition C.4.1

(i) The equilibrium level of advertising in the presence of license payments satisfies the condition

$$T(\varphi^L) = \frac{-\Delta L}{\frac{t_p}{2}\varphi^L[2-G(\varphi^L)]^2} + \frac{\gamma}{2t_s(1-2d)}, \qquad \text{(C.4.3)}$$

where $\Delta L = \Delta L^j\big|_{\varphi^1=\varphi^2=\varphi^L} < 0$.

(ii) Introducing license payments reduces the equilibrium level of advertising:

$$\varphi^L < \varphi^*.$$

(iii) If $\frac{\partial}{\partial\varphi}(\Delta L/\varphi) \le 0$ then the symmetric equilibrium level of advertising is decreasing in d.

Proof

(i) Follows from (C.4.2) and (11).

(ii) The negativity of ΔL and (13) imply

$$T(\varphi^L) > \frac{\gamma}{2t_s(1-2d)} = T(\varphi^*).$$

The result follows since $T(\varphi)$ is decreasing in φ.

(iii) Implicitly differentiate (C.4.3) with respect to d to obtain

$$T'(\varphi^L)\frac{d\varphi^L}{d(d)} = \left\{-\frac{2}{t_p}\left[2-G(\varphi^L)\right]^{-2}\frac{\partial}{\partial\varphi^L}\left(\frac{\Delta L}{\varphi^L}\right) - \frac{\Delta L}{\varphi^L}\frac{4G'(\varphi^L)}{[2-G(\varphi^L)]^3}\right\}\frac{d\varphi^L}{d(d)}$$
$$+\frac{\gamma}{t_s(1-2d)^2}\left\{1+\frac{L'(\tilde{X})}{\frac{t_p}{2}\varphi^L[2-G(\varphi^L)]^2}\right\}$$

and solve for the derivative $d\varphi^L / d(d)$. This derivative is negative since $\Delta L < 0$, $T'(\varphi) < 0$ and $\frac{\partial}{\partial\varphi}(\Delta L/\varphi) < 0$. Q.E.D.

The condition in (iii) of the proposition implies that the license payment does not increase at a rate more than linearly with advertising. For example, if the license payment is on a *per subscriber* basis, such as $L(X) = wX$, $w > 0$, then

$$\Delta L = w\left[\frac{1}{2} - \left(\frac{1}{2} + \frac{\gamma\varphi}{2t_s(1-2d)}\right)\right] = \frac{-w\gamma\varphi}{2t_s(1-2d)},$$

for which $\frac{\partial}{\partial\varphi}(\Delta L/\varphi) = 0$, thereby satisfying the condition in (iii).

Using (C.4.1) and (11), equilibrium station profits in the presence of license payments is given by

$$C^L = \frac{t_p[2-G(\varphi^L)]^2}{4}\left[1 + \frac{\gamma\varphi^L}{t_s(1-2d)}\right] + L\left[\frac{1}{2} + \frac{\gamma\varphi^L}{2t_s(1-2d)}\right]. \qquad \text{(C.4.4)}$$

The following proposition states that under conditions assumed in the main text (the conditions of Corollary 1) and the condition in (iii) of Proposition C.4.1, C^L is increasing in d.

Proposition C.4.2

If the elasticity of the outreach probability $G(\varphi)$ is non-increasing and $\frac{\partial}{\partial\varphi}(\Delta L/\varphi) \leq 0$, then stations improve profits by symmetrically locating closer to the center of the media market (i.e. increasing d toward ½).

Proof

It suffices to show that the expression

$$1 + \frac{\gamma\varphi^L}{t_s(1-2d)},$$

occurring in bracketed expressions of (C.4.4), is increasing in d. From (C.4.3), this expression can be rewritten as

$$1 + \varphi^L T(\varphi^L) + \frac{\Delta L \varphi^L}{\frac{t_p}{2}[2-G(\varphi^L)]^2}. \qquad \text{(C.4.5)}$$

This expression is decreasing in φ^L since the second term is non-increasing in φ^L (see proof of Corollary 1) and the last term is negative and increasing in absolute value in φ^L. (Recall that $\Delta L < 0$.) The conclusion follows by part (iii) of Proposition C.4.1. Q.E.D.

As a result of this proposition, we conclude that incentive for stations to minimally differentiate their programming is preserved in the presence of license payments, such as those paid by cable providers.