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History Match and Associated Forecast Uncertainty Analysis—Practical Approaches Using Cluster Computing

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Abstract

This paper presents practical approaches to deal with the complex problem of the uncertainty assessment in the performance forecast using reservoir simulation models with extensive production history. The complexity and difficulty of this type of problem arises mainly from the necessity of finding a large number of simulation models that are consistent not only with the geological data but also with the observed production history. In simpler terms this means finding an appropriate number of multiple solutions to the history match problem that can be used to estimate uncertainty in the forecasts. The rigorous solution[1,2] to this kind of problem involves the application of methods based on Monte Carlo simulation; but they are not routinely applied because of the computational cost associated to the necessary large number of simulations for real field problems.

Advances in computing technology in recent years, especially in the areas of CPU speed and of high performance computing affordability with medium to large CPU clusters, indicate that now is, probably the appropriate time to explore and revisit the practical aspects of performing a more comprehensive history match and forecast uncertainty analysis with Monte Carlo simulation methods.

The approaches presented in this work take advantage of the availability of a medium size 256 CPU Linux cluster that allowed the coupling of distributed high performance computing with efficient sampling techniques to solve the history match and the associated forecast uncertainty problem under a probabilistic inverse problem framework, and to present the results of both history match and forecast in the form of probability density functions (PDF). Prior probabilistic model information is incorporated in the process.

The tests performed with data from a real field indicated that our approaches provide one practical way to address, more comprehensively than current existing approaches, the non-uniqueness issue of the history matching problem and the associated uncertainties in performance forecasts in real fields. Since the results are accomplished in a very short time, significant changes in reservoir management paradigms may result.

Introduction

History matching is an inverse problem to calibrate reservoir simulation models to the observed production history, and it is a critical and necessary step in optimizing reservoir management decisions associated to the subsurface of oil and gas reservoirs. It is recognized that, because of the nature of the geological and production data, and the limitations of the numerical models to properly represent the true physics of real reservoirs, it is not possible to resolve uniquely and deterministically the underlying reservoir description. Thus the uncertainty in the resolution of the subsurface model translates into the uncertainty in the flow predictions (forecasts), which are one of the critical inputs to the reservoir management decision making processes.

The importance of constraining the reservoir models to the observed production data (history match) is that it reduces the uncertainty in the reservoir model description and consequently it reduces the uncertainty in the forecasts. Unfortunately incorporating production data information into the reservoir model is not a simple task. It increases the complexity and difficulty of reservoir property estimation over the case of using geological data alone.

The history match problem in reservoir simulation is not new; researchers and practitioners have been working in this area for at least 40 years and this has resulted in an extensive literature. Most of the published work relates to the use of automatic/assisted history match algorithms[3-31] with the goal of finding efficiently a single good solution to the history match problem. History matching methods based on implementations of gradient based search algorithms[32] proved to be very successful. However, history match methods with the goal of identifying multiple solutions, which are actually what is needed to make probabilistic forecasts, were not extensively explored, most likely because of the assumption that the computational cost would make them impractical to deal with the real field problems. Recent work in academics and industry[33-36] is beginning to address the practical issues of dealing with the non-uniqueness in real field history match problems.

The development of rapid, efficient and accurate computational methods and of associated computer infrastructure is then necessary to facilitate multiple realizations in history matching, so that one can capture the uncertainty in the reservoir parameters. Fast simulation techniques utilizing high performance computing are particularly needed with the expected availability of an avalanche of high quality real-time data from novel sensors.

Probabilistic History Match

The probabilistic inverse problem theory[1,2] provides the mathematical framework to resolve in a comprehensive way the inverse problem involving the use of incomplete and uncertain data to make inferences of the parameters defining a physical model. When applied to the reservoir simulation history match case, the theory indicates that the parameters defining a reservoir model should not be deterministically estimated. Instead a probabilistic description of them should be sought. The multiplicity of solutions to the history match problem is thus expressed as a probabilistic description of the reservoir model, and this means that it is necessary to estimate the probability distribution function (PDF) of the reservoir model parameters. The solution of inverse problems such as reservoir simulation history matching under a probabilistic framework requires the utilization of algorithms based on Monte Carlo simulation.

Monte Carlo methods are well known and fully developed. Their application implies the need to compute many thousands of times the forward model associated with the inverse problem of interest. For the case of history match this translates into thousands of flow simulations. Since current medium size simulation models have a "running" time in the orders of hours, a simple math calculation indicates that the rigorous probabilistic methods are not easily applicable and some approximations have to be done to make the problem tractable. In order to maximize the business impact of production data collected at the field, that is to take advantage of the production data to reduce forecast uncertainty, the level of such approximations to the inverse problem theory framework have to be evaluated taking into account the technology and resources that are available at the time of solving a specific real field problem. Both technology and resource availability are continuously evolving with time.

The approaches of solving the history match problem and associated forecast uncertainty with gradient methods implicitly assume that the critical limitation is the availability of computational power. It is recognized that the major shortcoming of gradient based methods is that they provide a single solution to the history match problem, and it is not guaranteed that such a solution satisfies the maximum likelihood criteria, the algorithms will stop at the first local minima they find. Uncertainty analysis based on gradient information[38-39] is limited to a region in the parameter space in the vicinity of the solution found. Gradient based methods are very elegant from the mathematical point of view and most importantly they are still the most practical and computationally efficient methods when only one CPU is available, and thus they should not be disregarded.

Advances in computing technology in recent years have resulted in a significant increase of CPU speed and the simultaneous reduction of costs, making high performance computing with medium-large size CPU clusters (100-2,000 CPU's) economically viable. This trend is expected to continue for the foreseeable future. It is then not necessary to constrain the history match analysis to methods developed to work with limited computational resources. This paper explores the practicality of one approach that takes advantage of the availability of medium-large cluster computing to provide not only better quality but also faster solutions to the history match and uncertainty problem for real field problems.

Description of Approach for Probabilistic History Match and Uncertainty Assessment using High Performing Computing

The approaches we developed here are based on previous work on history match and uncertainty estimation[36] that was adapted[40] to take advantage of the availability of medium size CPU clusters with the purpose of computing history match solutions approximating the probabilistic inverse problem theory framework[1,2]. Our approaches draw mainly from the work of Pan and Horne[41] that relied on the use of organized sampling techniques and high quality proxies as a substitute for CPU intensive numerical reservoir simulation in applications for field development optimization problems.

Our goal is not only to obtain higher quality results than existing approaches but also to obtain them in a timely manner, which in our work means in a period of time short enough to be practical for dealing with real fields. Quality, speed and the efficient use of computational resources are then necessary to have a positive impact on the use of production history information in reservoir management.

A brief and general description of the procedures in our approaches follows.

1. First an objective function $\{E_{HM}\}$ is defined to quantify the quality of the match between the simulated data and the observed production history. General forms of this function are shown in **Eqn. 1-4** which are all commonly used in automatic history match. From the probabilistic point of view, E_{HM} in **Eqn. 2 and 3** is linked to the Bayesian Likelihood function L $\{ \; L \; \alpha \; exp(-E_{HM}) \; \}$.

$$E_{HM}(\vec{\alpha}) = \tfrac{1}{2}\left(\vec{d}^{\,obs} - \vec{d}^{\,calc}\right)^T C_d^{-1}\left(\vec{d}^{\,obs} - \vec{d}^{\,calc}\right) + \tfrac{1}{2}\left(\vec{\alpha}_{HM} - \vec{\alpha}_{HM}^{\,prior}\right)^T C_\alpha^{-1}\left(\vec{\alpha}_{HM} - \vec{\alpha}_{HM}^{\,prior}\right) \quad \textbf{(1)}$$

$$E_{HM}(\vec{\alpha}) = \tfrac{1}{2}\left(\vec{d}^{\,obs} - \vec{d}^{\,calc}\right)^T C_d^{-1}\left(\vec{d}^{\,obs} - \vec{d}^{\,calc}\right) \quad \textbf{(2)}$$

$$E_{HM}(\vec{\alpha}) = \sum_{i=1}^{i=n\_data} \frac{\left|d_i^{obs} - d_i^{calc}\right|}{\sigma_i} \quad \textbf{(3)}$$

$$E_{HM}(\vec{\alpha}) = \sum_{i=1}^{i=n\_data} w_i\left(d_i^{obs} - d_i^{calc}\right)^2 \quad \textbf{(4)}$$

2. Based on an analysis of the field problem, a threshold in the objective function is defined. Simulations in models that result in computed objective functions

below this threshold are considered "acceptable" solutions.

3. An initial sample scheme of the history match parameter space is generated. The sampling scheme depends, among other things, on the number of CPU's available and the actual time to compute one simulation. Simulations are then performed and the values of the objective function are computed and stored in a database.
4. The parameter space is resampled following a second sampling scheme with an interpolation algorithm that acts as a proxy for the reservoir simulator. This step provides the estimated values of the objective function in the second sampling locations. Recall that the objective function E_{HM} being modeled is related to the Bayesian Likelihood function.
5. Analyses of the results from the secondary sampling are then used to generate a new sampling scheme to sample with real numerical simulation.
6. Steps 4 to 5 are repeated until a predetermined stop point is reached. Intermediate steps are taken during the process to improve the reliability of the interpolation algorithms (proxies). This further enhances the efficiency of our approach.
7. By the end of this process a large number of "acceptable" solutions should have been identified. The acceptable solutions are then augmented[36] with realizations of the parameters which have been found to have none or little effect in history match but to have effects in the forecasts, and then new simulations are run.
8. Depending on the number of identified acceptable simulations and the sampling schemes used during the history match period, the forecasts computed following this procedure could be enough to compute the statistics that quantify the forecast uncertainty. Another more general and preferred alternative[36] is to use proxies to estimate the statistics for forecast uncertainty. Estimations of the forecasts at a large number of sample locations are computed as a function of the forecasts generated by the sampling with real numerical flow simulation.
9. Both history match parameters and forecasts results are presented in the form of approximate probability density functions (PDF's), which are obtained by processing the output generated by the proxies during the history match and forecasting stages of the process.

The a priori information is incorporated into the uncertainty estimates using Bayes theorem which allows the updating of the prior probabilistic model parameter description with the incorporation of new information, the production history. That is: *prior PDF* + *production data* $\rightarrow$ *posterior PDF*.

It is clear from the description of the process that the PDF's we compute are not the "true" PDF's. We obtain an approximation of them using the proxy modeling procedure.

One of the differences with the probabilistic inverse theory is that in our approach equal likelihood is assigned to models with associated objective function values below the "acceptability" threshold and zero if above the threshold. This is a minor departure from the theory and arises from the decision[36] to consider on "equal basis" all the models that result in simulated data within an error band around the observed data.

Although our process is conceptually simple, care should be taken to obtain an efficient implementation, especially in the area of sampling and proxy modeling. For example, gradient information can be incorporated[36] in the process, both in the sampling and proxy construction.

Field Case Application

Preliminary Information

Our approaches were tested with data from a real field subject of a published field case study[37]. A 3D structural view of the field is shown in **Fig. 1**.

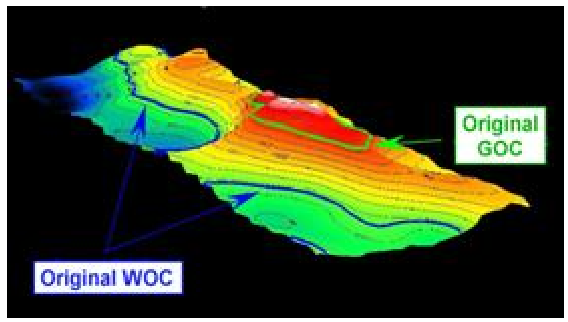

Fig. 1 3D structural view of the field (From ref. 37)

This is a large oil field that after more than 30 years of oil production and some gas injection is being considered as a place for storing the associated gas produced in neighboring fields. The feasibility study requires the use of full-field numerical reservoir simulation to estimate, in a probabilistic way, the volume of gas that can be injected into the two reservoirs that make up this depleted field.

The earlier real field study[37] was used as the starting point and data input source for this test. In the earlier study 60 "acceptable" solutions to the history match problem were found after running one batch of 600 simulations that were arranged to sample the parameter space with the special procedure described by Pan and Horne[41]. The 600 flow simulations were performed using a small Linux cluster.

Our test aimed to use efficiently high performance computing to obtain high quality estimation of the uncertainty in the rates and volumes of gas to inject. The forecast uncertainty was to be constrained to:

a) A predefined simulation model constructed on geological and geostatistical data.
b) A parameterization scheme of the simulation model predefined by the earth scientists familiar with the field.
c) A predefined development plan for new wells and surface facilities.
d) The observed 30 years production history.

From the constraints enumerated above it is clear that the uncertainty estimated in this work is only a component of the total true uncertainty.

The parameterization scheme of the simulation model for history match and forecast is shown in **Table 1**.

Table 1 Parameter definition for history match and forecast uncertainty analysis

#	Parameters	Min	Max
1	Water Oil Contact (WOC)	Base-50'	Base+50'
2	Gas Oil Contact (GOC)	Base-50'	Base+50'
3	Fault Transmissibility Multiplier	0.00	1.00
4	Global K_h Multiplier	0.10	2.00
5	Global K_v Multiplier	0.01	2.00
6	Fairway Y-Perm Multiplier	0.75	4.00
7	Fairway K_v Multiplier	0.75	4.00
8	Critical Gas Saturation	0.02	0.04
9	Kv Multiplier Between Reservoirs	0.00	5.00
10	Skin @ New Gas Injection Wells	0.00	30.00

The parameters in **Table 1** are indicated in our plots in a normalized scale of [0 , 1];

that is $p_{norm} = (p - p_{min})/(p_{max} - p_{min})$.

Two different prior states of model information were studied. By prior we mean the probabilistic description of the model parameters in **Table 1** before taking in consideration the production history data. By posterior we mean the probabilistic description of the same model parameters after the addition of the production history data. In one case the prior information was represented in the form of independent and uniform distributed PDF's for all the parameters defined in **Table 1**. In the second case the prior information consisted of triangular PDF's for the parameters 1 and 2 in **Table 1** (WOC and GOC respectively – **Fig. 14b**) and uniform distribution PDF for all the other model parameters. The triangular PDF distribution represents a more certain state of the prior knowledge of the model parameters than the uniform PDF distribution.

To quantify uncertainty in the gas injection forecast it is thus necessary to find different combinations of the parameters in **Table 1** that when applied to the simulation model result in good matches to the historical data. These combinations of parameters (multiple solutions to the history match problem) are used later to estimate the uncertainty in the forecasts. Parameters #9 and #10 on **Table 1** deserve additional explanation. Parameter #9 relates to the vertical communication between the two reservoirs that make up the field. Parameter #10 relates to the skin damage factor of the new gas injector wells to be drilled as specified in the field development plan, and it is obvious that this parameter has no effect whatsoever to the history match but it may have a significant effect in the forecasts.

The historical data to match consisted of observed water and gas field production rates. **Fig. 3** shows the historical field water production rate. **Fig. 4** displays a field cumulative water production plot. The history match was executed to match rates, not cumulative production. Display of the historical gas production data is omitted for brevity. During the history match period the flow simulation was constrained to the observed well oil production rate. The simulation model has Cartesian grid of 21 x 83 x 47 cells, it is a black-oil type formulation, and the average total running time in a single CPU is about 3 hours.

The quality of the match was determined using an objective function defined following **Eqn. 2**.

The threshold in the objective function used to accept-reject models based on the quality of the match was determined by taking into account not only an assessment of the measurement errors but also from practical experience in history matching, and the insight gained in the earlier study[37].

Field Case - Discussion of Results

During this test a total of 3,159 flow simulations were performed following the process outlined earlier. And 285 (confirmed by simulation) acceptable solutions were found. **Fig. 2** shows a simplified diagram of the process.

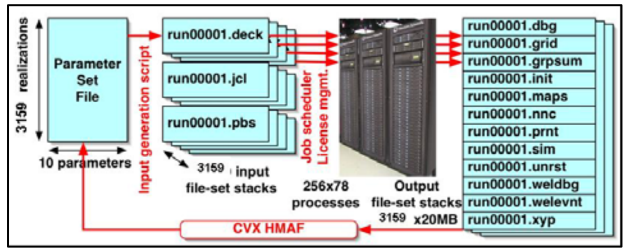

Fig. 2 the 256 CPU Linux cluster at USC used for History Match using High Performance Computing (HPC) – from ref. 40.

A medium size 256 CPU Linux cluster[40] was used. The simulations were sent to the Linux cluster in 7 batches of input decks. As explained earlier, the sample locations for each batch is determined by analysis of the sampling of the parameter space using a proxy for the reservoir simulator. The proxy for the flow simulator in this work consisted in kriging interpolation. After 7 batches (3,159 simulations) the process was stopped. Total effective computational time was about 2 days. As a simple comparison, should this work have been performed using 1 CPU it would have taken more than a year of computation time.

Figures 3 – 15 illustrate the quality of the results and give the indication at a glance of the value that can be created in just 2 days of computer work. A description and comments follows.

The quality of the match to the observed data can be appreciated in **Fig. 3**. It shows how the simulated field water production rates corresponding to the 285 "acceptable" solutions compare with the historical data. Recall that one of the targets was to find multiple combinations of parameters within the ranges specified in **Table 1** that would produce simulated data within a narrow band around the observed data (historical data). An easier way to visualize the same information carried by **Fig. 3** is by plotting "cumulative production" data as shown in **Fig. 4**, nonetheless it should be reminded that the history match was performed on production rates - not on cumulative production, and thus the "acceptability" band observed in **Fig. 3** becomes a "fan" shape band in the cumulative plot of **Fig. 4**.

The quality of the match to the other production data, that is the field gas production rate, was of the same quality as in the case of field water production rate. The corresponding plots are omitted only for brevity.

An estimation of the wide range of the dynamic behavior of the model unconstrained to the production data is shown in **Fig. 5** where the field cumulative water production curves corresponding to the 3,159 flow simulations are plotted.

Information related to the uncertainty in the volume of gas to inject, which was the focus of this field study, is shown in **Figs. 6 – 7**. The field cumulative gas injection corresponding to the 285 solutions is shown in **Fig. 6**. An estimation of the range in the volume of gas to inject unconstrained to the production history is shown in **Fig. 7**. The uncertainty in the forecast is quantified by sampling the parameter space with a proxy and is presented in the form of a histogram (**Fig. 8**) and/or as a Cumulative Density Function - CDF (**Fig. 9**).

The values taken by the parameters water-oil and gas-oil contacts (WOC and GOC) in the 285 "acceptable" solutions are plotted in **Fig. 10**. This plot is the 2-dimensional projection of a higher dimensional parameter space defined in **Table 1**. For comparative purposes we have included in **Fig. 10** the history match solution found in a previous study using the traditional trial and error method.

Probabilistic descriptions of the model parameters after incorporating the production data were estimated using proxies. **Figure 11** shows the multiple solutions estimated for two different error thresholds and it gives a preliminary idea of the posterior PDF describing the reservoir model. **Figures 12** to **14** formalize such a probabilistic model description, a single thresholds was used in the estimation process. **Figure 12** shows an estimation of the joint marginal posterior PDF corresponding to parameters # 1 and #2 (Water-Oil and Gas-Oil Contacts respectively) assuming uniform PDF's as prior model information.

The effect of the prior probabilistic information in model inversion (history match) is shown in **Figs**. **13** and **14**. **Figure 13** shows an estimation of the joint marginal posterior PDF corresponding to parameters # 1 and #2 (Water-Oil and Gas-Oil Contacts respectively) for two different prior information scenarios (a) prior PDF is uniform distribution, and (b) prior PDF is triangular distribution for WOC and GOC. **Figure 14** shows the posterior marginal PDF for WOC for the two prior states of model information described before.

The effect of the prior probabilistic information in the forecasts is shown in **Fig. 15** in the form of PDF's. The CDF corresponding to **Fig. 15a** was already shown in **Fig. 9**. The CDF corresponding to **Fig. 15b** is omitted for brevity.

The effect on the posterior PDF corresponding to the history match parameters 1 and 2 that arises from changing the prior information from uniform to triangular distribution could be more or less anticipated in this case. The effect on the forecasts (**Fig. 15**) is not trivial.

Figures 13 – 15 show also that a) the posterior uncertainty in the model description and the forecasts were reduced by improving the prior belief in the model description, and b) the value of incorporating production history data in the model description to reduce model uncertainty.

We have limited the description to parameters 1 and 2 for brevity. The other model parameters can be analyzed the same way.

Obtaining the kind of information shown in **Fig. 13** to **15** was within our technical goal, which is to obtain probabilistic descriptions of the model parameters and a probabilistic estimation of the forecasts. The practicality of our approaches was proved by the fact that the effective computational time to obtain the results was less than two days (**Table 2**).

A comparison of the computational cost between our approach and other alternatives is presented in **Table 2**. The numbers of acceptable solutions confirmed with flow simulation for the case of probabilistic history match with 1 and 5 CPUs were left unreported since it is difficult to estimate them because we need to follow a sampling strategy different from the one used in this work. The table includes an actual test performed using a gradient based approach for history match, an implementation[25] of the Gauss-Newton algorithm. There are other gradient search alternatives to the Gauss-Newton, they would also produce a single solution and the amount of CPU time may be of the same order of magnitude. The number of "Model Simulations" for the Gauss-Newton option in **Table 2** is the number of iterations.

Table 2 Comparison of history match methods – Estimated CPU time

Number of CPU's	Method	Number of Model Simulations	Effective Simulation* CPU time	Number of Acceptable Solutions
1	Trial & Error	n/a	n/a	1
1	Gauss-Newton (Numerical Derivatives)	7	8.8 days	1
1	Gauss-Newton (Analytical Derivatives)	7	2.8 days	1
1	Probabilistic History Match	3,159	394.9 days	n/a
5	Probabilistic History Match	3,159	79.9 days	n/a
256**	Probabilistic History Match	3,159	1.5 days	285

(*) Assumed 3 hours average running time
(**) This study

Although it is clear that, to obtain quality results, it is necessary to make a large number of simulations, that condition is not enough. The efficiency of the sampling schemes used in this work is compared with the theoretical ones calculated for standard Monte Carlo random sampling in **Fig. 16**, where we plot $\{\sigma_{est}/n^{0.5}{}_{accepted\_confirmed}\}$ vs. {*total number of simulations runs*}. Our organized sampling procedure reduces the forecast uncertainty much more rapidly than the standard Monte Carlo sampling would predict as a function of the number of forward simulation runs. Simple Monte Carlo sampling and high performance computing alone are not enough to obtain results in a time frame to make this technology practical, and it is necessary to perform an organized sampling such as the one used in this work.

Conclusions

This work shows that a high quality solution to the history match and uncertainty forecast problem can be obtained in a time frame of just days by the efficient use of high performance computing with medium size CPU clusters. The high quality solution comes from the probabilistic history match approach we used to approximate the solution proposed by the Probabilistic Inverse Problem theory. The model parameters are described in the form of probability density functions and their effects in the forecasts are quantified in the form of probabilistic estimation of forecast uncertainty. The practicality of our approach is given by the fact that the high quality results can be generated in matter of days. The results from this work also indicate that the combination of intelligent sampling and proxy modeling are the critical factors to reach practicality.

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Nomenclature

C = covariance matrix
CDF = cumulative density function
d = production data
E = objective function
GOC = gas oil contact
K = permeability
L = likelihood function
n_data = number of production data observations
PDF = probability density function
W = Data Weight Matrix
WOC = water oil contact
w = data weight
α = reservoir parameters

Subscripts

d = data
HM = history matching
h = horizontal
i = observation index
v = vertical
α = model parameter

Superscripts

-1 = inverse matrix
$calc$ = calculated with numerical model
est = estimated
obs = observed – historical data
$\rightarrow$ = *vector*

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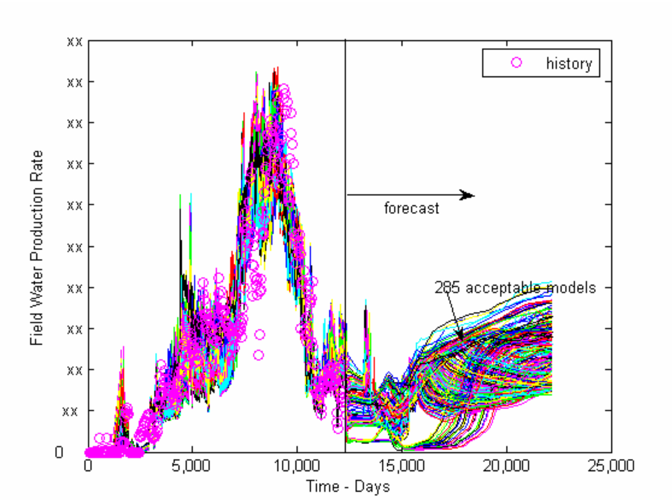

Fig. 3 - Field Water Production Rate: historical data and multi-model match

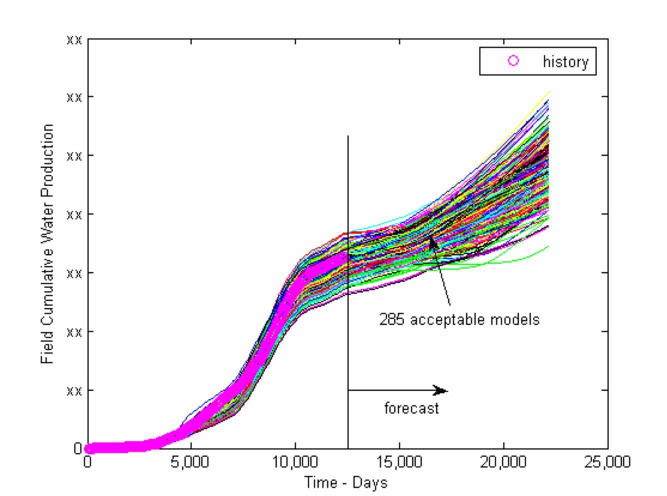

Fig. 4 - Field Cumulative Water Production: historical data and multi-model match

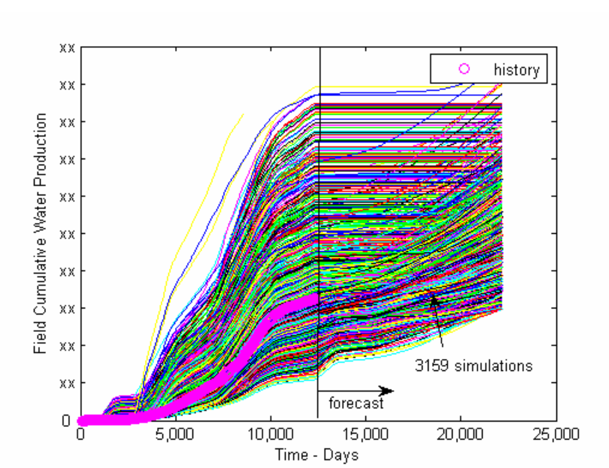

Fig. 5 - Field Cumulative Water Production: historical data and 3,159 simulations performed using High Performance Computing.

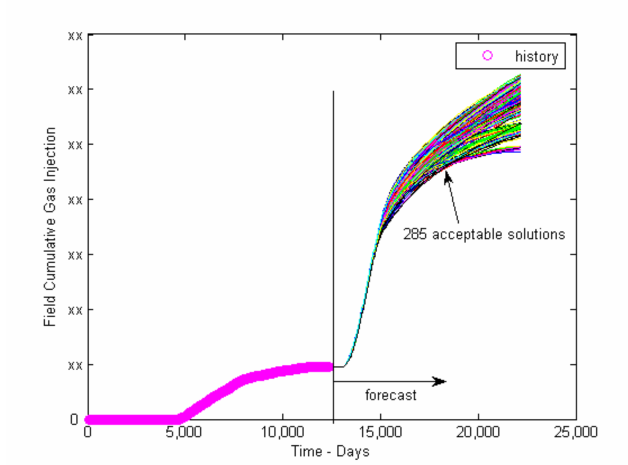

Fig. 6 - Field Cumulative Gas Injection Volumes: historical data and forecast constrained to historical data (285 confirmed acceptable models)

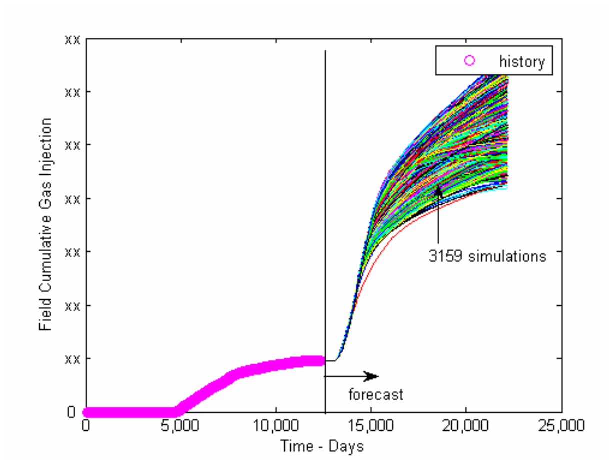

Fig. 7 - Field Cumulative Gas Injection Volumes: Uncertainty in Forecasts (unconstrained to production history)

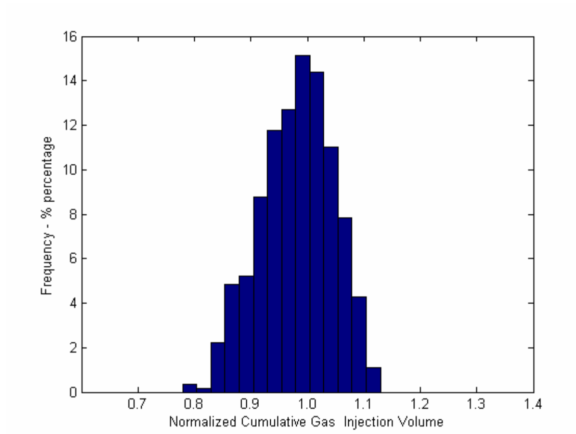

Fig. 8 - Field Cumulative Gas Injection Volume: Uncertainty in Forecasting expressed in the form of Histogram. Prior model PDF is uniform distribution for all parameters.

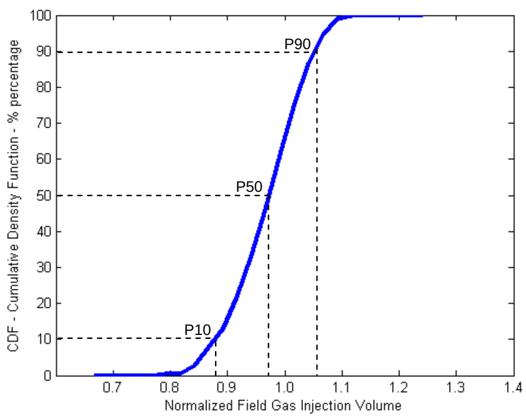

Fig. 9 - Field Cumulative Gas Injection Volume: Uncertainty in Forecasting expressed in the form of CDF. Prior parameters PDF are uniform distribution.

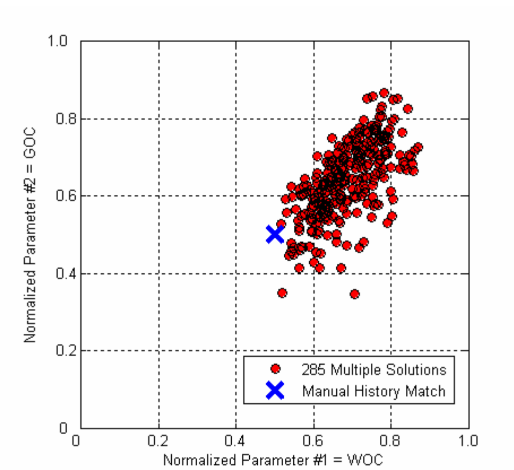

Fig. 10 - Combinations of WOC and GOC that result in 285 confirmed history matches.

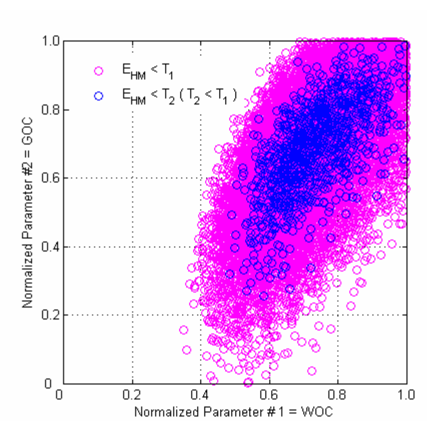

Fig. 11 - Estimated combinations of WOC and GOC that resulted in misfit functions values below two thresholds – E_{HM} ($T_2 < T_1$).

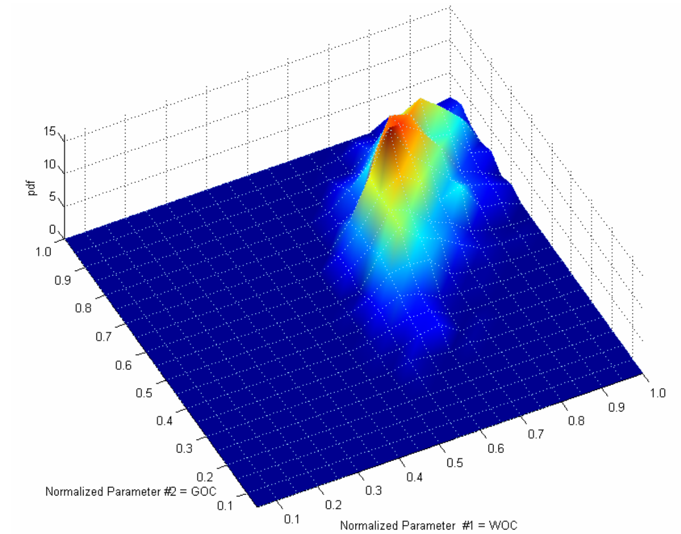

Fig.12 - Estimated posterior joint marginal PDF for parameters #1 and #2 (WOC & GOC). Prior PDF is uniform distribution in all the parameters.

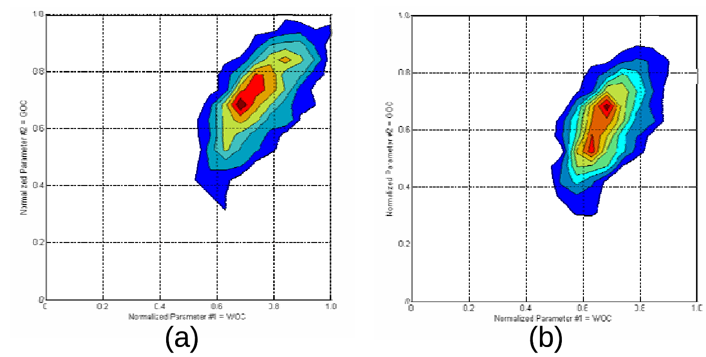

Fig. 13 - Estimated posterior joint marginal PDF for parameters #1 and #2 (WOC & GOC). (a) Prior PDF is uniform distribution in all the parameters (same as Fig. 12). (b) Prior PDF is triangular distribution for parameters 1 & 2 and uniform for all the other parameters.

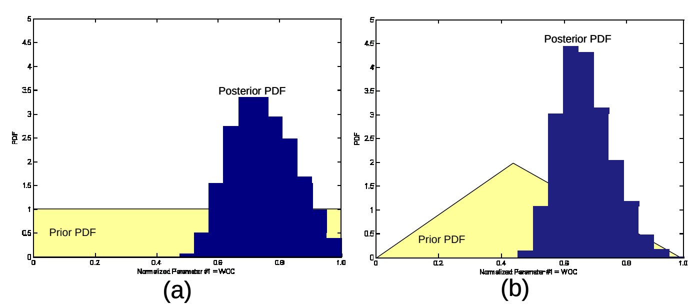

Fig. 14 - Estimated posterior marginal PDF for parameter #1 (WOC). (a) Prior PDF is uniform distribution. (b) Prior PDF is triangular distribution.

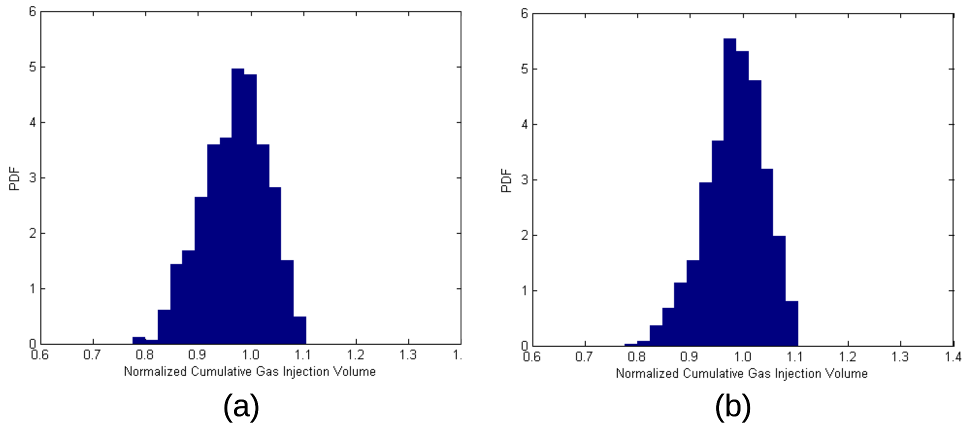

Fig. 15 - PDF for the Normalized Field Cumulative Gas Injection Volume after including production history data; (a) a priori information consisting of uniform PDF for all model parameters; (b) a priori information consisting of triangular distributions for parameters 1 and 2, and all other parameters have uniform distribution.

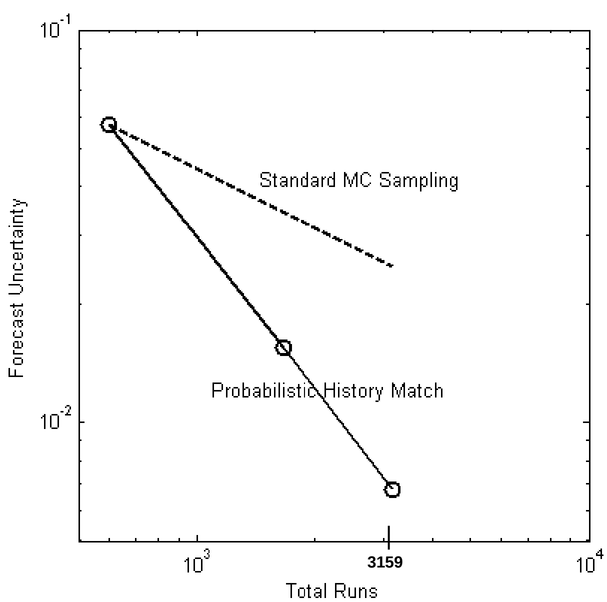

Fig. 16 - Efficiency of proposed approach: Comparison of the proposed approach with theoretical standard Monte Carlo sampling (also in ref. 40).