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Correlation Measurement in a Turbulent Flow using High-Speed Computing Methods

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Third-order time-correlations downstream of a grid were measured with a hot-wire anemometer using high-speed computing methods. The nonlinear response of the hot-wire to the fluctuations of velocities is taken into account as well as the effect of transverse velocities. It is found that the correlations $R_t^{2,1}(h) = \overline{u'^2(t)u'(t+h)}/(\overline{u'^2})^{3/2}$ and $R_t^{1,2}(h) = \overline{u'(t)u'^2(t+h)}/(\overline{u'^2})^{3/2}$ are substantially different from previous results and demonstrate that the assumption of isotropy is not adequate for these correlations downstream of a grid. The nonlinear response does not significantly affect the difference $\frac{1}{2}(R_t^{2,1} - R_t^{1,2})$. Since previous conclusions concerning the nature of third-order correlations were based on the measurements of such differences they masked the effects of nonlinearity on the individual correlations. Correlations of fifth-order are also presented and their relations to the third-order correlation are discussed. Although the nonlinear corrections are quite important for odd-order correlations they are negligible for correlations of even-order.

1. INTRODUCTION

THE turbulent flow downstream of a grid has been the object of extensive studies since it has been considered to approximate reasonably well a field of homogeneous and isotropic turbulence. Consequently measurements of grid turbulence have served not only to obtain basic experimental results for comparison of statistical characteristics of a turbulent field with theory, but have been quite important in the very formation and the development of theoretical ideas concerning turbulence phenomena. It has been customary to evaluate the assumption of isotropy by measuring such quantities as the longitudinal and transverse intensities of turbulence as well as the second-order correlations. However, a rigorous evaluation of isotropy requires a more detailed study of the validity of this fundamental assumption.

To-date measurement of third-order correlations has been made using constant-current hot-wire anemometry,[1,2] which has a nonlinear response to the fluctuating turbulent velocities. It was preferable therefore, to determine third-order correlations by measuring the differences of correlation coefficients rather than their individual values, and thus minimize the errors due to the nonlinear response. To obtain the individual third-order correlation from such differences the assumption of isotropy was required. In order to adequately assess the validity of this assumption, as far as the third-order correlations are concerned, the effect of the nonlinear response must be determined. In the past this correction would have been difficult to make since it would have required a knowledge of higher-order correlations which were not available. However, an estimate[2,3] of the magnitude of possible errors showed that a significant error could be made even at moderate turbulent intensities and emphasized the necessity for such a correction. The successful application of high-speed computing methods[4] makes possible the direct measurement of third-order correlations and enables one to make the corrections for the nonlinear response when necessary. It should be noted that the use of constant-temperature-linearized hot-wire anemometry, which has a linear response to the fluctuating velocities, would avoid this type of difficulty.

In the present paper high-speed computing methods are applied to a re-evaluation of the third-order correlations. Our results demonstrate the inadequacy of the assumption that the turbulent field is isotropic, and the quantitative values for the third-order correlations are quite different from those obtained previously. Some additional results are also given for other statistical characteristics of turbulence including the fifth-order correlations.

2. EXPERIMENTAL PROCEDURE

The measurement of the turbulence characteristics was made at a distance $x = 48.5$ meshes

* Formerly named David Taylor Model Basin.
[1] R. W. Stewart, Proc. Cambridge Phil. Soc. 47, 146 (1951).
[2] R. R. Mills, Jr., A. L. Kistler, V. O'Brien, and S. Corrsin, NACA Technical Note 4288 (1958).
[3] G. Comte-Bellot, presented at the Colloque sur la contribution de l'électronique au traitement statistique des mesures en physique, Grenoble, France (1966).
[4] F. N. Frenkiel and P. S. Klebanoff, Phys. Fluids 10, 507 (1967).

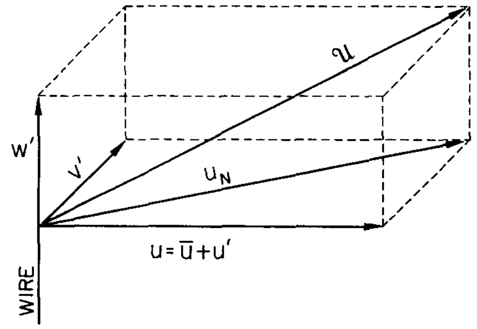

FIG. 1. Illustration of hot-wire response to the instantaneous normal component of velocity for a hot-wire positioned normal to the mean velocity, $\bar{u}$. $\mathfrak{U}$ is the instantaneous velocity and u_N is the effective instantaneous velocity.

downstream of a square mesh grid having a mesh-length $M = 2.54$ cm and woven of rods 0.5 cm in diameter. The wind velocity U as measured with a Pitot-static tube was 15.4 m/sec. No attempt was made to compensate for the boundary-layer growth on the wind tunnel walls. Consequently there was a slight increase in velocity with distance downstream from the grid, $U^{-1}\, dU/dx = 0.038$/m. However, this is considered to be negligible from the point of view of introducing a mean-velocity-gradient effect.[5] A constant current hot-wire anemometer was used, and the measured voltage, representing the longitudinal component of the turbulent velocity $u'(t)$, was recorded on magnetic tape. The root-mean-square of this component was approximately 1.8% of the mean velocity and the Reynolds number, based on U and the mesh length of the grid, was 25 600. The analog recordings were digitized at a frequency of 12 800 readings per second using samples of 12.5-sec duration. Correlation coefficients were then determined by applying high-speed computing methods as described in Ref. 4. Four samples of recorded data obtained under the same flow conditions were used for the analysis and the average value for the correlations was determined.

The hot-wire was a platinum wire of 2.5-μ diameter and 0.75-mm length and was mounted perpendicularly to the mean wind velocity. Under these conditions the end-loss correction and the correction for wire length are considered negligible. "Overheat" ratios for the wire varied from 0.4 to 0.5 and for this range the hot-wire heat loss rate is taken to be linear with temperature rise. Since the turbulence level was sufficiently small (1.8%), it is felt that the compensation for the hot-wire thermal lag was quite satisfactory. Consequently, the remaining major factors are the nonlinear response of the voltage versus velocity characteristic of the hot-wire and the effective velocity to which it responds.

For a wire mounted normal to the mean wind velocity, $\bar{u}$, (Fig. 1) the effective instantaneous velocity is given by the expression

$$u_N = \bar{u}_N + u'_N = [(\bar{u} + u')^2 + v'^2]^{\frac{1}{2}}, \qquad (1)$$

where u' and v' are the longitudinal and transverse turbulent velocity components, respectively, with $\overline{u'} = 0$ and $\overline{v'} = 0$, and where there is no effect of the component w'. We first consider the effect of the nonlinear response taking v' to be negligible as compared with $u = \bar{u} + u'$. It should be noted that we are distinguishing here by different symbols the *wind velocity as measured with a Pitot-static tube,* U, and the true mean velocity, $\bar{u}$.

3. NONLINEAR RESPONSE OF HOT-WIRES

The fundamental relation used to characterize the behavior of hot-wires is the King equation,[6] which for constant current can be written in the form

$$r/(r - R_a) = D + Fu^{\frac{1}{2}}, \qquad (2)$$

where u is the instantaneous velocity (assuming v' to be negligible), r is the instantaneous resistance of the wire, R_a is the resistance of the wire at ambient air temperature (taken here to be constant) and where D and F are coefficients obtained by an appropriate calibration. It should be noted that Eq. (2) refers here to the instantaneous velocity and not to the mean velocity as it is used most often when referring to the King equation.

In order to determine the nonlinear correction, it is convenient to express Eq. (2) in the inverted form, from that which is customarily used,

$$u = f(e) = \left[\frac{(D-1)e - DE_a}{F(e - E_a)}\right]^2, \qquad (3)$$

where $E_a = iR_a$, and where the instantaneous voltage

$$e = \bar{e} + e' = ir, \qquad \overline{e'} = 0,$$

with $\bar{e}$ being the mean voltage, e' the fluctuating voltage, and i the current.

Referring to the Taylor expansion we can express the instantaneous velocity as

$$u = f(\bar{e}) + ae' + be'^2 + ce'^3,$$

where

$$a = \frac{df(\bar{e})}{d\bar{e}}, \qquad b = \frac{1}{2}\frac{d^2f(\bar{e})}{d(\bar{e})^2}, \qquad c = \frac{1}{6}\frac{d^3f(\bar{e})}{d(\bar{e})^3},$$

[5] G. Comte-Bellot and S. Corrsin, J. Fluid Mech. **25**, 657 (1966).

[6] L. V. King, Phil. Trans. Roy. Soc. (London) **A214**, 373 (1914).

and where the higher-order terms were omitted thus considering that the hot-wire behavior can be represented by a third-order curve. Taking the mean value we find

$$\bar{u} = f(\bar{e}) + b\overline{e'^2} + c\overline{e'^3},$$

and

$$u' = u - \bar{u} = ae' + b(e'^2 - \overline{e'^2}) + c(e'^3 - \overline{e'^3}). \quad (4)$$

Taking the square and averaging we find

$$\overline{u'^2} = \overline{e'^2}\{a^2 + 2ab(\overline{e'^2})^{1/2}\overline{\mathcal{E}^3} + [(b^2 + 2ac)\overline{\mathcal{E}^4} - b^2]\overline{e'^2} + 2bc(\overline{\mathcal{E}^5} - \overline{\mathcal{E}^3})(\overline{e'^2})^{3/2} + c^2[\overline{\mathcal{E}^6} - (\overline{\mathcal{E}^3})^2](\overline{e'^2})^2\}, \quad (5)$$

where $\mathcal{E} = e'(\overline{e'^2})^{-\frac{1}{2}}$. If one assumes that the dynamic response of the hot-wire to fluctuating velocity, and the static response to a change in mean velocity are equivalent, the coefficients a, b, c, can be determined by an appropriate calibration to obtain D and F, and

$$a = \frac{2(D-1)E_a\bar{e} - 2DE_a^2}{F^2(\bar{e} - E_a)^3},$$

$$b = \frac{(2D+1)E_a^2 - 2(D-1)E_a\bar{e}}{F^2(\bar{e} - E_a)^4}, \quad (6)$$

$$c = \frac{2(D-1)E_a\bar{e} - 2(D+1)E_a^2}{F^2(\bar{e} - E_a)^5},$$

In principle a, b, c should be determined from a calibration in a laminar flow, i.e., with no turbulence. However, the calibration was performed in the turbulent flow under the same conditions at which the measurements were made. Figure 2 represents a typical set of calibration curves where the ratio $\bar{e}(\bar{e} - E_a)^{-1}$ is given as a function of the square root of the wind velocity as suggested by the King equation. The wind velocity U, was measured with a

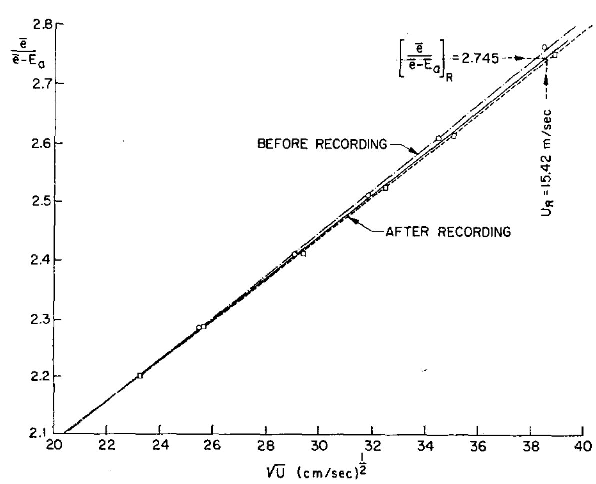

FIG. 2. Typical calibration of a hot-wire normal to the wind velocity as represented by King's equation.

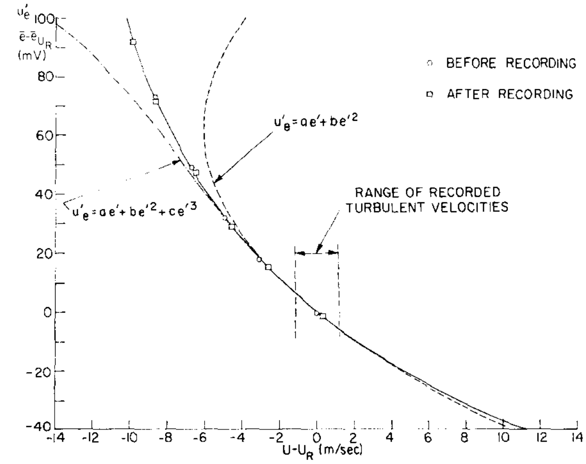

FIG. 3. Comparison of second-degree and third-degree approximations to the nonlinear response of the hot-wire with its calibration.

Pitot-static tube which at the turbulence level of the present experiment may be considered to be equal to $\bar{u}$. Calibrations were made before and after recording of the turbulence signal. In addition, the wind velocity U_R, and the ratio $[\bar{e}(\bar{e} - E_a)^{-1}]_R$ were determined at the time of recording. In general there was a shift between the calibration before and after recording, and the procedure adopted was a proportional interpolation of the coefficients D and F obtained from the two calibrations and based on the measured value of $[\bar{e}(\bar{e} - E_a)^{-1}]_R$ at the time of recording. As an example of the order of magnitude of the coefficients a, b, and c we find for the calibration given in Fig. 2 that $a(\overline{e'^2})^{\frac{1}{2}} = -27$ cm sec^{-1}; $b(\overline{e'^2}) = 0.304$ cm sec^{-1}; $c(\overline{e'^2})^{3/2} = -0.0029$ cm sec^{-1} with $(\overline{e'^2})^{\frac{1}{2}} = 1.40$ mV.

It can be shown that the correction resulting from determining a, b, and c by calibrating in the turbulent field is on the order of c in the measurement of the odd-order moments. A computation with and without the coefficient c showed that its effect is negligible, and since the nonlinear effect as will be shown later is negligible for the even-order moments the effect of calibrating in the turbulent field rather than in the laminar flow is considered negligible *under the present conditions*.

The calibration data of Fig. 2 are presented in Fig. 3 in a more convenient form in terms of the difference $\bar{e} - \bar{e}_{U_R}$ as a function of $U - U_R$, where $\bar{e}_{U_R}$ is the mean voltage corresponding to the wind velocity U_R at the time of recording. The solid curve represents the interpolated calibration curve and the open circles and open squares represent the "before" and "after" calibrations, respectively. The calibration curve can be compared with the approxi-

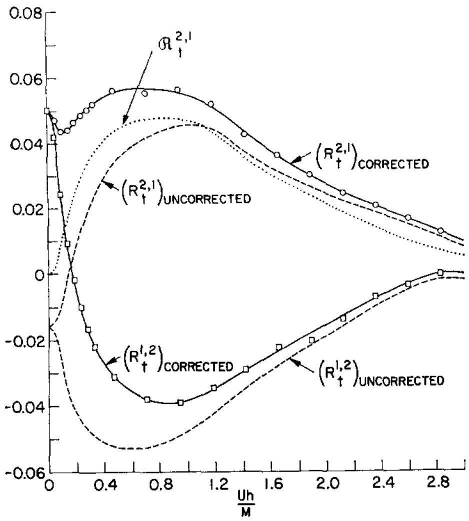

FIG. 4. Comparison between the uncorrected third-order correlations $R_t^{2,1}(h)$ and $R_t^{1,2}(h)$ as measured by a constant current hot-wire and the correlations corrected for the nonlinear response. The curves for the uncorrected and corrected difference $\Re_t^{2,1}(h) = \frac{1}{2}[R_t^{2,1}(h) - R_t^{1,2}(h)]$ are identical and are represented by the dotted curve.

mate expression for the turbulent velocity given by Eq. (4), by determining the numerical values for

$$u_e' = u - f(\bar{e}_{UR}) = ae' + be'^2 + ce'^3.$$

Figure 3 presents the curve corresponding to the above approximation as well as to the approximation by a second-degree curve

$$u_e' = ae' + be'^2.$$

The degree to which these approximations are required to fit the calibration curve can be determined by noting the range within which the turbulent velocities are observed to fluctuate. Both approximations cover the range of fluctuating velocities very satisfactorily as can be seen in Fig. 3. Equation (4) can now be used to correct the recorded fluctuating voltage $e'(t)$ and thus to obtain the turbulent velocity $u'(t)$. It should be noted that in some recent experimental studies[7] on the hot-wire response a variation of King law was proposed where the hot-wire heat loss varies as $u^{0.45}$ instead of $u^{0.5}$. In this connection it should be emphasized that the nonlinear coefficients, a, b, c, as determined by our procedure are based on the experimental calibration and Eq. (2) is merely a convenient intermediary in obtaining Eqs. (6). In fact, the resulting numerical values of the nonlinear coefficients obtained directly by numerical differentiation of the voltage–velocity calibration curve agreed very well with those obtained from Eqs. (6).

4. THIRD-ORDER CORRELATIONS

The correction for the nonlinear response can be obtained by first computing the turbulent velocities $u'(t)$ from the digital values according to Eq. (4) and then determining the correlations

$$R_t^{2,1}(h) = \frac{\overline{u'^2(t)u'(t+h)}}{(\overline{u'^2})^{\frac{3}{2}}} \qquad R_t^{1,2}(h) = \frac{\overline{u'(t)u'^2(t+h)}}{(\overline{u'^2})^{\frac{3}{2}}}$$

directly from the corrected values using high-speed computing methods. In Fig. 4 the corrected values of the third-order correlations obtained in this manner and shown as solid curves, are compared with the uncorrected correlations represented by the dashed curves. The difference between the corrected and uncorrected third-order correlations is particularly significant at the smaller values of h. In the case of isotropic turbulence $R_t^{2,1}(h)$ should be equal to $-R_t^{1,2}(h)$, and $\overline{u'^3}/(\overline{u'^2})^{3/2}$ should be equal to zero. The nature of the corrected third-order correlations emphasizes the lack of isotropy since

$$R_t^{2,1}(h) \neq -R_t^{1,2}(h),$$

and

$$R_t^{2,1}(0) = R_t^{1,2}(0) = \overline{u'^3}/(\overline{u'^2})^{\frac{3}{2}} \neq 0.$$

Particularly noticeable is the fact that the corrected value of the third-order moment $\overline{u'^3}/(\overline{u'^2})^{3/2}$ is relatively large, and is not only different in magnitude but also of opposite sign than the uncorrected value. Thus, it is apparent that the assumption of isotropic turbulence does not adequately represent the behavior of the third-order correlations in grid turbulence even for a small time delay or for small spatial separation as has been considered the case.

It is interesting that the difference $(R_t^{2,1} - R_t^{1,2})$ for the corrected correlations and the difference $(R_t^{2,1} - R_t^{1,2})$ for the uncorrected correlations are practically indistinguishable from one another, and, therefore,

$$\Re_t^{2,1}(h) = \tfrac{1}{2}[R_t^{2,1}(h) - R_t^{1,2}(h)]$$

shown in Fig. 4 manifests the same behavior as observed by others.[1,2] However, it should be emphasized that obtaining the individual third-order correlations $R_t^{2,1}$ and $R_t^{1,2}$ by measuring the difference and assuming isotropy is not justified.

The use of high-speed computing methods makes it possible to correct the velocity fluctuations for the nonlinear response of the hot-wire directly. This digital method of correction can, therefore, be re-

[7] D. C. Collis and M. J. Williams, J. Fluid Mech. 6, 357 (1959).

garded as carrying out the same function as the constant-temperature-linearized hot-wire[8] in the analog method which often has the inconvenience of a higher noise to signal ratio than the constant-current equipment.

Instead of using the nonlinear correction of the fluctuating voltage $e'(t)$ to obtain the turbulent velocity $u'(t)$ from which the correct correlations are determined, one can first determine the uncorrected correlations and then apply the relations derived from Eq. (4) to obtain the corrected correlations. This latter method may be of particular interest when a constant current hot-wire anemometer is used without appropriate high-speed computing equipment or when previously obtained uncorrected data (or data obtained making an insufficiently justified assumption of isotropy) are to be corrected for the nonlinear response. Introducing the nondimensional variables

$$v_1 = \frac{u'(t)}{(\overline{u'^2})^{\frac{1}{2}}}, \qquad v_2 = \frac{u'(t+h)}{(\overline{u'^2})^{\frac{1}{2}}},$$

$$\mathcal{E}_1 = \frac{e'(t)}{(\overline{e'^2})^{\frac{1}{2}}}, \qquad \mathcal{E}_2 = \frac{e'(t+h)}{(\overline{e'^2})^{\frac{1}{2}}},$$

and noting that $\overline{v^2} = 1$; $\overline{\mathcal{E}^2} = 1$, from Eq. (4) we find

$$v_1 = \mathcal{A}\mathcal{E}_1 + \mathcal{B}\mathcal{E}_1^2 - \mathcal{B}, \quad v_2 = \mathcal{A}\mathcal{E}_2 + \mathcal{B}\mathcal{E}_2^2 - \mathcal{B}, \quad (7)$$

where

$$\mathcal{A} = \frac{(\overline{e'^2})^{\frac{1}{2}}}{(\overline{u'^2})^{\frac{1}{2}}}\, a, \qquad \mathcal{B} = \frac{\overline{e'^2}}{(\overline{u'^2})^{\frac{1}{2}}}\, b.$$

The coefficients $\mathcal{A}$, $\mathcal{B}$, are determined from the calibration coefficients a, b. Equations (7) can then be used to determine the correlations between the turbulent velocity components

$$R_t^{m,n}(h) = \frac{\overline{u'^m(t)u'^n(t+h)}}{(\overline{u'^2})^{(m+n)/2}} = \overline{v_1^m v_2^n},$$

as functions of the correlations between the fluctuating voltages

$$R_e^{m,n}(h) = \frac{\overline{e'^m(t)e'^n(t+h)}}{(\overline{e'^2})^{(m+n)/2}} = \overline{\mathcal{E}_1^m \mathcal{E}_2^n}.$$

Thus we find the third-order correlations

$$R_t^{2,1} = \mathcal{A}^3 R_e^{2,1} + \mathcal{A}^2\mathcal{B}[R_e^{2,2} + 2R_e^{3,1} - (2R_e^{1,1} + 1)],$$
$$R_t^{1,2} = \mathcal{A}^3 R_e^{1,2} + \mathcal{A}^2\mathcal{B}[R_e^{2,2} + 2R_e^{1,3} - (2R_e^{1,1} + 1)], \quad (8)$$

which, for $h = 0$, give

$$\overline{v^3} = \mathcal{A}^3\overline{\mathcal{E}^3} + 3\mathcal{A}^2\mathcal{B}(\overline{\mathcal{E}^4} - 1). \quad (9)$$

Using the definition

$$\mathcal{R}^{m,n}(h) = \tfrac{1}{2}[R^{m,n}(h) - R^{n,m}(h)]$$

with the appropriate subscripts we find from Eqs. (12),

$$\mathcal{R}_t^{2,1} = \mathcal{A}^3\mathcal{R}_e^{2,1} + 2\mathcal{A}^2\mathcal{B}\mathcal{R}_e^{3,1}. \quad (10)$$

Thus, we find that the nonlinear correction for the third-order correlation can be made by using the measurements made with a constant-current hot-wire so long as we measure the correlations between the voltages of the second, third, and fourth orders.

A. Approximate Relations for quasi-Gaussian Probability Distribution

We now assume that the measured voltages $e'(t)$ are distributed according to a quasi-Gaussian probability density distribution for which the odd-order correlation coefficients other than the third-order are neglected, and the even-order correlations can be expressed as functions of the second-order correlation according to the relations corresponding to a Gaussian distribution. Therefore, we have the relations

$$R_e^{2,2} = 1 + 2(R_e^{1,1})^2, \qquad R_e^{3,1} = R_e^{1,3} = 3R_e^{1,1},$$

which applied to Eqs. (8) gives

$$R_t^{2,1} = \mathcal{A}^3 R_e^{2,1} + 2\mathcal{A}^2\mathcal{B}[(R_e^{1,1})^2 + 2R_e^{1,1}],$$
$$R_t^{1,2} = \mathcal{A}^3 R_e^{1,2} + 2\mathcal{A}^2\mathcal{B}[(R_e^{1,1})^2 + 2R_e^{1,1}], \quad (11)$$

The curves in Fig. 5 represent the third-order correlations corresponding to the quasi-Gaussian

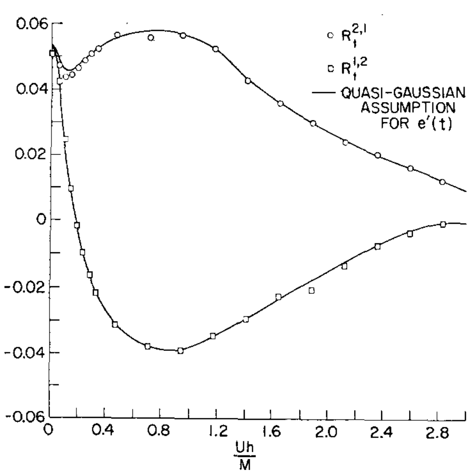

FIG. 5. Comparison of third-order correlations corrected for the nonlinear response of the hot-wire using the high-speed computing method with that obtained by a simplified procedure assuming the fluctuating voltage to be Gaussian.

[8] The nonzero value of $\overline{u'^3}/(\overline{u'^2})^{\frac{3}{2}}$ and the inequality $R_t^{2,1}(h) \neq -R_t^{1,2}(h)$ have been confirmed by recent experiments of V. G. Harris and S. Corrsin (Johns Hopkins University) using constant-temperature linearized hot-wire anemometry.

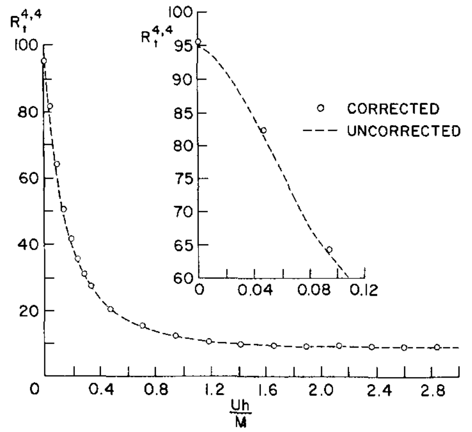

FIG. 6. Comparison between corrected and uncorrected correlations of eighth-order, $R_t^{4,4}(h)$, illustrating the negligible effect of the nonlinear response of the hot-wire on measurements of even-order correlations.

assumption obtained from Eqs. (11) and the open circles and open squares represent the correct values of $R_t^{2,1}$ and $R_t^{1,2}$, respectively. As can be seen these approximate equations represent the third-order correlations quite well except for a relatively small departure for the small values of the time interval h.

For $h = 0$ we find from Eqs. (11)

$$\overline{v^3} = \mathcal{A}^3\overline{\mathcal{E}^3} + 6\mathcal{A}^2\mathcal{B}. \quad (12)$$

The above equation can also be obtained by introducing the Gaussian value $\overline{\mathcal{E}^4} = 3$ in Eq. (9).

5. OTHER CORRELATIONS

Other correlations than those of third-order can be determined using similar methods. The nonlinear correction is performed easily by using the high-speed computing after the fluctuating voltage is multiplied by the appropriate calibration coefficients. If the nonlinear correction is to be made after the correlation for the non-corrected fluctuating voltage is obtained, then we can determine the expressions for $R_t^{m,n}$ as functions of $R_e^{m,n}$. Thus, for the second-order correlations we find

$$R_t = \mathcal{A}^2R_e + \mathcal{A}\mathcal{B}(R_e^{2,1} + R_e^{1,2}). \quad (13)$$

Here we will not give the expressions for $R_t^{2,2}$, $R_t^{3,3}$ and $R_t^{4,4}$ explicitly; however, it should be noted that these correlations are very close to the corresponding correlations $R_e^{2,2}$, $R_e^{3,3}$, and $R_e^{4,4}$. The differences between the corrected and uncorrected correlations are too small to be represented by different curves except possibly for the uncorrected correlation $R_e^{4,4}$ represented on Fig. 6 by dashed curves and compared with the open circles which represent the corrected $R_t^{4,4}$.

Since the only appreciable difference between $R_t^{m,m}$ and $R_e^{m,m}$ for even values of $(m + n)$ appears to be at small time intervals h, the following approximate expressions for $\overline{v^{2m}} = R_t^{m,m}(0)$ are given:

$$\begin{aligned}\overline{v^4} &= \mathcal{A}^4\overline{\mathcal{E}^4} + 4\mathcal{A}^3\mathcal{B}(\overline{\mathcal{E}^5} - \overline{\mathcal{E}^3}),\\ \overline{v^6} &= \mathcal{A}^6\overline{\mathcal{E}^6} + 6\mathcal{A}^5\mathcal{B}(\overline{\mathcal{E}^7} - \overline{\mathcal{E}^5}), \quad (14)\\ \overline{v^8} &= \mathcal{A}^8\overline{\mathcal{E}^8} + 8\mathcal{A}^7\mathcal{B}(\overline{\mathcal{E}^9} - \overline{\mathcal{E}^7}).\end{aligned}$$

While the even-order correlations for turbulent velocities are quite close to the corresponding correlations for voltages this is not the case for the odd-order correlations as has already been seen for the third-order correlations. Thus, in the case of the fifth-order moment $\overline{v^5}$, we find the approximate relation

$$\overline{v^5} = \mathcal{A}^5\overline{\mathcal{E}^5} + 5\mathcal{A}^4\mathcal{B}(\overline{\mathcal{E}^6} - \overline{\mathcal{E}^4}) \quad (15)$$

in which the first term on the right hand side may be considerably smaller than the second term. Thus, we find that the fifth-order moment for the turbulent velocities will be quite different from the corresponding fifth moment for the voltage.

6. NON-GAUSSIAN PROBABILITY DISTRIBUTION

Now let us assume that the turbulent velocities $u'(t)$ are distributed according to the non-Gaussian joint-probability density distribution

$$P(v_1, v_2) = P_0(v_1, v_2) \sum_{0}^{j+k=4} A_{j,k}H_{j,k}(v_1, v_2), \quad (16)$$

where P_0 is a Gaussian joint-probability density distribution, $H_{j,k}(v_1, v_2)$ are the Hermite polynomials of two variables and $A_{j,k}$ are coefficients defined in terms of the higher-order correlations $R_t^{m,n}$ according to the distribution law suggested by J. Kampé de Fériet.[9] Under this assumption the correlations of fifth order can be expressed as functions of the measured (and corrected for the nonlinear response) correlations of lower orders. Previously[4] we gave the appropriate relations for $R_t^{3,2}$ and $R_t^{2,3}$ and we list them here together with the remaining correlations of fifth order

$$\begin{aligned}R_t^{3,2} &= R_t^{3,0} + 6R_tR_t^{2,1} + 3R_t^{1,2},\\ R_t^{2,3} &= R_t^{0,3} + 6R_tR_t^{1,2} + 3R_t^{2,1},\\ R_t^{4,1} &= 4R_tR_t^{3,0} + 6R_t^{2,1}, \quad (17)\\ R_t^{1,4} &= 4R_tR_t^{0,3} + 6R_t^{1,2},\\ R_t^{5,0} &= R_t^{0,5} = 10R_t^{3,0} = 10R_t^{0,3}.\end{aligned}$$

[9] J. Kampé de Fériet, David Taylor Model Basin, Report 2013 (1966); also see Ref. 4, p. 514.

Figure 7 presents the curves for $R_t^{3,2}$ and $R_t^{2,3}$, and Fig. 8 the curves $R_t^{4,1}$ and $R_t^{1,4}$ obtained by the above equations from the measured correlations of lower order. These curves are compared with the corresponding correlations as measured directly and corrected for nonlinear response.

7. EFFECT OF TRANSVERSE VELOCITIES ON THE RESPONSE OF A HOT-WIRE NORMAL TO THE MEAN VELOCITY

In the previous sections we have taken into account the nonlinear response of hot-wires assuming that the effect of the transverse component of the turbulent velocity $v'(t)$ can be neglected in comparison with the instantaneous component of the velocity parallel to the mean velocity $u(t)$. Now we determine the effect that the transverse component of the velocity has on the results obtained with a hot wire normal to the direction of the mean velocity. First let us note that the nonlinear correction which we have used remains the same if we replace the expressions for the correlations between the longitudinal components of the turbulent velocities

$$R_t^{m,n}(h) = \frac{\overline{u'^m(t)u'^n(t+h)}}{(\overline{u'^2})^{(m+n)/2}} = \overline{v_1^m v_2^n},$$

by correlations between the components u_N' defined by Eq. (1):

$$R_{N,t}^{m,n}(h) = \frac{\overline{u_N'^m(t)u_N'^n(t+h)}}{(\overline{u_N'^2})^{(m+n)/2}}. \tag{18}$$

In order to determine the correction due to the effect now considered we express the correlations

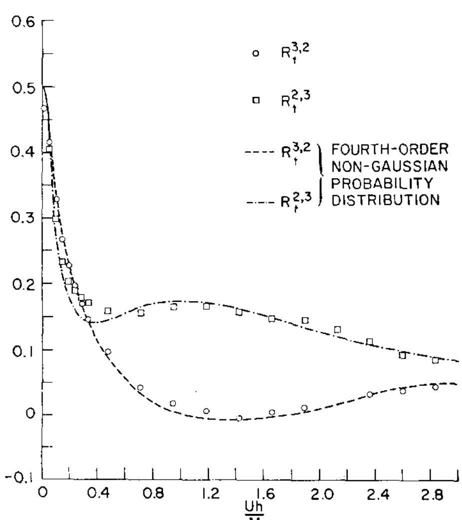

FIG. 7. Measurements of fifth-order correlations, $R_t^{3,2}(h)$ and $R_t^{2,3}(h)$, corrected for the nonlinear response of the hot-wire compared with curves obtained for the fourth-order non-Gaussian probability distribution.

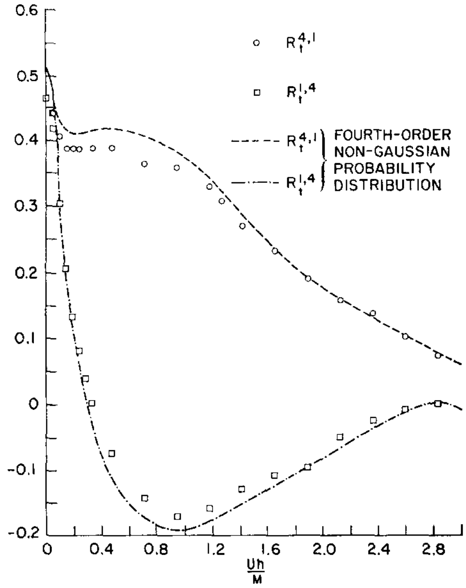

FIG. 8. Measurements of fifth-order correlations, $R_t^{4,1}(h)$ and $R_t^{1,4}(h)$, corrected for the nonlinear response of the hot-wire compared with curves obtained for the fourth-order non-Gaussian probability distribution.

$R_{N,t}^{m,n}$ in terms of the correlations $R_t^{m,n}$. However, the effect of transverse velocities on the hot-wire response are independent of the nonlinear effects, and therefore, they also apply to the linearized constant-temperature hot wires.

Expanding (1) and neglecting terms of a higher order than the fifth we find

$$\frac{u_N}{U} = 1 + \frac{u'}{U} + \frac{1}{2}\frac{v'^2}{U^2} - \frac{1}{2}\frac{u'v'^2}{U^3} + \frac{1}{2}\frac{u'^2v'^2}{U^4} - \frac{1}{8}\frac{v'^4}{U^4} - \frac{1}{2}\frac{u'^3v'^2}{U^5} + \frac{3}{8}\frac{u'v'^4}{U^5}.$$

Taking the average and subtracting it from the above equation we find

$$\begin{aligned}\gamma = T[\upsilon &+ \tfrac{1}{2}T\kappa^2(\alpha^2 - 1) - \tfrac{1}{2}T^2\kappa^2(\upsilon\alpha^2 - \overline{\upsilon\alpha^2}) \\ &+ \tfrac{1}{2}T^3\kappa^2(\upsilon^2\alpha^2 - \overline{\upsilon^2\alpha^2}) - \tfrac{1}{8}T^3\kappa^4(\alpha^4 - \overline{\alpha^4}) \\ &- \tfrac{1}{2}T^4\kappa^2(\upsilon^3\alpha^2 - \overline{\upsilon^3\alpha^2}) + \tfrac{3}{8}T^4\kappa^4(\upsilon\alpha^4 - \overline{\upsilon\alpha^4})],\end{aligned} \tag{19}$$

where

$$\gamma = \frac{u_N'}{U}\,;\; T^2 = \frac{\overline{u'^2}}{U^2}\,;\; \upsilon^2 = \frac{u'^2}{\overline{u'^2}}\,;\; \alpha^2 = \frac{v'^2}{\overline{v'^2}}\,;\; \kappa^2 = \frac{\overline{v'^2}}{\overline{u'^2}}.$$

Let us define

$$\gamma_1 = \frac{u_N'(t)}{U} \qquad \gamma_2 = \frac{u_N'(t+h)}{U},$$

and thus the correlation measured with a hot-wire normal to the mean velocity will be given by

$$R_{N,t}^{m,n}(h) = \frac{\overline{\gamma_1^m\gamma_2^n}}{(\overline{\gamma^2})^{(m+n)/2}}. \tag{20}$$

Equation (19) will give for the second-order moment

$$\begin{aligned}\overline{\gamma_1\gamma_2} = T^2\{&\overline{v_1v_2} + \tfrac{1}{2}T\kappa^2(\overline{v_1\alpha_2^2} + \overline{v_2\alpha_1^2})\\
&- \tfrac{1}{2}T^2\kappa^2(\overline{v_1v_2\alpha_1^2} + \overline{v_1v_2\alpha_2^2}) + \tfrac{1}{4}T^2\kappa^4(\overline{\alpha_1^2\alpha_2^2} - 1)\\
&+ \tfrac{1}{2}T^3\kappa^2(\overline{v_1^2v_2\alpha_1^2} + \overline{v_1v_2^2\alpha_2^2}) - \tfrac{1}{8}T^3\kappa^4[2(\overline{v_1\alpha_1^2\alpha_2^2} + \overline{v_2\alpha_1^2\alpha_2^2})\\
&+ (\overline{v_1\alpha_2^4} + \overline{v_2\alpha_1^4}) - 4\overline{v\alpha^2}] - \tfrac{1}{16}T^4\kappa^2(\overline{v_1^3v_2\alpha_1^2} + \overline{v_1v_2^3\alpha_2^2})\\
&+ \tfrac{1}{8}T^4\kappa^4[3(\overline{v_1v_2\alpha_1^4} + \overline{v_1v_2\alpha_2^4}) + 2(\overline{v_1^2\alpha_1^2\alpha_2^2} + \overline{v_2^2\alpha_1^2\alpha_2^2})\\
&+ 2\overline{v_1v_2\alpha_1^2\alpha_2^2} - 4\overline{v^2\alpha^2} - 2(\overline{v\alpha})^2]\\
&- \tfrac{1}{16}T^4\kappa^6[(\overline{\alpha_1^2\alpha_2^4} + \overline{\alpha_1^4\alpha_2^2}) - 2\overline{\alpha^4}]\}. \qquad (21)\end{aligned}$$

In a similar way we obtain for the third-order moment

$$\begin{aligned}\overline{\gamma_1^2\gamma_2} = T^3\{&\overline{v_1^2v_2} + \tfrac{1}{2}T\kappa^2(\overline{v_1^2\alpha_2^2} + 2\overline{v_1v_2\alpha_1^2} - 2\overline{v_1v_2} - 1)\\
&- \tfrac{1}{2}T^2\kappa^2[\overline{v_1^2v_2\alpha_2^2} + 2\overline{v_1^2v_2\alpha_1^2} - (2\overline{v_1v_2} + 1)\overline{v\alpha^2}]\\
&+ \tfrac{1}{4}T^2\kappa^4[2\overline{v_1\alpha_1^2\alpha_2^2} + \overline{v_2\alpha_1^4} - 2(\overline{v_1\alpha_2^2} + \overline{v_2\alpha_1^2}) - 2\overline{v\alpha^2}]\\
&+ \tfrac{1}{2}T^3\kappa^2[2\overline{v_1^3v_2\alpha_1^2} + \overline{v_1^2v_2^2\alpha_2^2} - (2\overline{v_1v_2} + 1)\overline{v^2\alpha^2}]\\
&- \tfrac{1}{8}T^3\kappa^4[4(\overline{v_1^2\alpha_1^2\alpha_2^2} + \overline{v_1v_2\alpha_1^2\alpha_2^2}) + 6\overline{v_1v_2\alpha_1^4} + \overline{v_1^2\alpha_2^4}\\
&- 4(\overline{v_1v_2\alpha_1^2} + \overline{v_1v_2\alpha_2^2}) - 4(\overline{v_1\alpha_2^2} + \overline{v_2\alpha_1^2} + \overline{v\alpha^2})\overline{v\alpha^2}\\
&- 4\overline{v^2\alpha^2} - (2\overline{v_1v_2} + 1)\overline{\alpha^4}]\\
&+ \tfrac{1}{8}T^3\kappa^6[\overline{\alpha_1^4\alpha_2^2} - 2\overline{\alpha_1^2\alpha_2^2} - (\overline{\alpha^4} - 2)]\}. \qquad (22)\end{aligned}$$

and the third-order moment $\overline{\gamma_1\gamma_2^2}$ can be obtained by interchanging the subscripts 1 and 2 in Eq. (22). For the fourth-order moment $\overline{\gamma_1^2\gamma_2^2}$ we find the expression

$$\begin{aligned}\overline{\gamma_1^2\gamma_2^2} = T^4\{&\overline{v_1^2v_2^2} + T\kappa^2[(\overline{v_1v_2^2\alpha_1^2} + \overline{v_1^2v_2\alpha_2^2})\\
&- (\overline{v_1^2v_2} + \overline{v_1v_2^2})] - T^2\kappa^2[(\overline{v_1^2v_2^2\alpha_1^2} + \overline{v_1^2v_2^2\alpha_2^2})\\
&- (\overline{v_1^2v_2} + \overline{v_1v_2^2})\overline{v\alpha^2}] + \tfrac{1}{4}T^2\kappa^4[(\overline{v_1^2\alpha_2^4} + \overline{v_2^2\alpha_1^4})\\
&+ 4\overline{v_1v_2\alpha_1^2\alpha_2^2} - 4(\overline{v_1v_2\alpha_1^2} + \overline{v_1v_2\alpha_2^2})\\
&- 2(\overline{v_1^2\alpha_2^2} + \overline{v_2^2\alpha_1^2}) + 2(2\overline{v_1v_2} + 1)]\}, \qquad (23)\end{aligned}$$

and for $\overline{\gamma_1^3\gamma_2}$,

$$\begin{aligned}\overline{\gamma_1^3\gamma_2} = T^4\{&\overline{v_1^3v_2} + \tfrac{1}{2}T\kappa^2[3\overline{v_1^2v_2\alpha_1^2} + \overline{v_1^3\alpha_2^2} - (3\overline{v_1^2v_2} - \overline{v^3})]\\
&- \tfrac{1}{2}T^2\kappa^2[(3\overline{v_1^3v_2\alpha_1^2} + \overline{v_1^3v_2\alpha_2^2}) - (3\overline{v_1^2v_2} + \overline{v^3})\overline{v\alpha^2}]\\
&+ \tfrac{3}{4}T^2\kappa^4[(\overline{v_1v_2\alpha_1^4} + \overline{v_1^2\alpha_1^2\alpha_2^2}) - (2\overline{v_1v_2\alpha_1^2}\\
&+ \overline{v_1^2\alpha_2^2} + \overline{v^2\alpha^2}) + (\overline{v_1v_2} + 1)]\}. \qquad (24)\end{aligned}$$

The fourth-order moment $\overline{\gamma_1\gamma_2^3}$ can be obtained by interchanging the subscripts 1 and 2 in the above equation.

By taking $v = v_1 = v_2$ and $\alpha = \alpha_1 = \alpha_2$ in Eq. (21), we find

$$\begin{aligned}\overline{\gamma^2} = T^2\{&1 + T\kappa^2\overline{v\alpha^2} - T^2\kappa^2\overline{v^2\alpha^2}\\
&+ \tfrac{1}{4}T^2\kappa^4(\overline{\alpha^4} - 1) + T^3\kappa^2\overline{v^3\alpha^2} - \tfrac{1}{4}T^3\kappa^4(3\overline{v\alpha^4}\\
&- 2\overline{v\alpha^2}) - T^4\kappa^2\overline{v^4\alpha^2} + \tfrac{1}{4}T^4\kappa^4[6\overline{v^2\alpha^4} - 2\overline{v^2\alpha^2}\\
&- (\overline{v\alpha^2})^2] - \tfrac{1}{8}T^4\kappa^6(\overline{\alpha^6} - \overline{\alpha^4})\}. \qquad (25)\end{aligned}$$

In the same way from Eq. (22) we find the expression

$$\begin{aligned}\overline{\gamma^3} = T^3\{&\overline{v^3} + \tfrac{3}{2}T\kappa^2(\overline{v^2\alpha^2} - 1) - \tfrac{3}{2}T^2\kappa^2(\overline{v^3\alpha^2} - \overline{v\alpha^2})\\
&+ \tfrac{3}{4}T^2\kappa^4(\overline{v\alpha^4} - 2\overline{v\alpha^2}) + \tfrac{3}{2}T^3\kappa^2(\overline{v^4\alpha^2} - \overline{v^2\alpha^2})\\
&- \tfrac{3}{8}T^3\kappa^4[5\overline{v^2\alpha^4} - 4\overline{v^2\alpha^2} - \overline{\alpha^4} - 4(\overline{v\alpha^2})^2]\\
&+ \tfrac{1}{8}T^3\kappa^6(\overline{\alpha^6} - 3\overline{\alpha^4} + 2)\}, \qquad (26)\end{aligned}$$

and from either Eq. (23) or Eq. (24) we find

$$\begin{aligned}\overline{\gamma^4} = T^4\{&\overline{v^4} + 2T\kappa^2(\overline{v^3\alpha^2} - \overline{v^3}) - 2T^2\kappa^2[\overline{v^4\alpha^2} - \overline{v^3}(\overline{v\alpha^2})]\\
&+ \tfrac{3}{2}T^2\kappa^4[\overline{v^2\alpha^4} - 2(\overline{v^2\alpha^2}) + 1]\}. \qquad (27)\end{aligned}$$

Equations (25), (26), and (27) can, of course, be obtained directly from Eq. (19) as was done by Comte-Bellot.[3] We also find directly from Eq. (19) the expression for the fifth moment

$$\overline{\gamma^5} = T^5[\overline{v^5} + \tfrac{5}{2}T\kappa^2(\overline{v^4\alpha^2} - \overline{v^4})]. \qquad (28)$$

In Eqs. (21)–(28) we have neglected all terms of higher order than T^6. Using similar procedures we can find the appropriate expressions for the correlations $\overline{\gamma_1^m\gamma_2^n}$ of still higher order, except that it may be necessary to include in Eq. (19) terms of higher order than T^5. In fact, this may already be necessary for Eq. (28).

In order to apply Eqs. (21)–(28) it is necessary to estimate the orders of magnitude of the different terms and to determine under what conditions some of these terms may be neglected. For that purpose we refer to the various terms given in Eqs. (25)–(28) for the values of the moments $\overline{\gamma^n}$ and more particularly to the cross correlations between the longitudinal and transverse components of the turbulent velocities at the same point of the field of turbulence. In view of the experimental evidence as to the nonisotropy and non-Gaussianity of the turbulent field, it seems desirable to obtain some measure of these various terms experimentally rather than be limited to the assumptions of isotropy and Gaussianity.

A. Cross-Correlations between Transverse and Longitudinal Turbulent Velocities

An X-wire arrangement consisting of two mutually perpendicular wires was used with the wires placed in the plane of the u and v' components of the velocity and at 45° to the direction of the mean velocity

(Fig. 9). The voltage sum and voltage difference corresponding to the fluctuating velocities $u'(t)$ and $v'(t)$, respectively, were recorded simultaneously on magnetic tape. Particular attention was given to matching the sensitivities of the individual wires so as to minimize the contamination of one component by the other. A sample of data approximately 5.5 sec in duration was digitized at a rate of 28 800 per sec. The measurements were again made at a mean velocity of 15.4 m/sec and at a distance of 48.5 mesh lengths downstream of the same 2.54-cm mesh grid as for the previous measurements. The intensity of the longitudinal component of turbulence was, as before, 1.8% of the mean velocity, and the ratio of the measured transverse to longitudinal components of turbulence was 0.92. As a first approximation we take $T = 0.018$ and $\kappa = 0.92$ and neglect the correction for the effect of the transverse velocities in the measurements with the X-wire arrangement, which can be expected to be rather small under our experimental conditions. However, we will make the correction for the nonlinear response of the hot-wire since we have seen

TABLE I. Measured cross-correlations $\overline{v^m\alpha^n} = \overline{u'^m v'^n}/(\overline{u'^2})^{m/2}(\overline{v'^2})^{n/2}$ and moments $\overline{v^m} = \overline{u'^m}/(\overline{u'^2})^{m/2}$ and $\overline{\alpha^m} = \overline{v'^n}/(\overline{v'^2})^{n/2}$ at a point in a turbulent field downstream of a grid.

	Not corrected for nonlinearity[a]	Corrected for nonlinearity[a]	Gaussian and isotropic
$\overline{v\alpha}$	−0.102	−0.102	0
$\overline{v^3}$	−0.0115	0.0556	0
$\overline{v^2\alpha}$	0.0048	0	0
$\overline{v\alpha^2}$	−0.0743	−0.0754	0
$\overline{\alpha^3}$	0.0062	0.0062	0
$\overline{v^4}$	2.927	2.927	3
$\overline{v^3\alpha}$	−0.309	−0.309	0
$\overline{v^2\alpha^2}$	0.9903	0.986	1
$\overline{v\alpha^3}$	−0.255	−0.256	0
$\overline{\alpha^4}$	2.935	2.935	3
$\overline{v^5}$	−0.128	0.508	0
$\overline{v^4\alpha}$	0.0136	0.0976	0
$\overline{v^3\alpha^2}$	−0.203	0.273	0
$\overline{v^2\alpha^3}$	0.0618	0.049	0
$\overline{v\alpha^4}$	−0.465	−0.465	0
$\overline{\alpha^5}$	0.0693	0.0693	0
$\overline{v^6}$	13.89	13.80	15
$\overline{v^5\alpha}$	−1.500	−1.504	0
$\overline{v^4\alpha^2}$	2.878	2.848	3
$\overline{v^3\alpha^3}$	−0.800	−0.793	0
$\overline{v^2\alpha^4}$	2.918	2.936	3
$\overline{v\alpha^5}$	−1.176	−1.168	0
$\overline{\alpha^6}$	14.18	14.18	15

[a] The signs depend on the selection of the positive direction for the transverse component α (or v'). A change of direction results in a change in sign for all cross-correlations and moments in which α has an odd exponent.

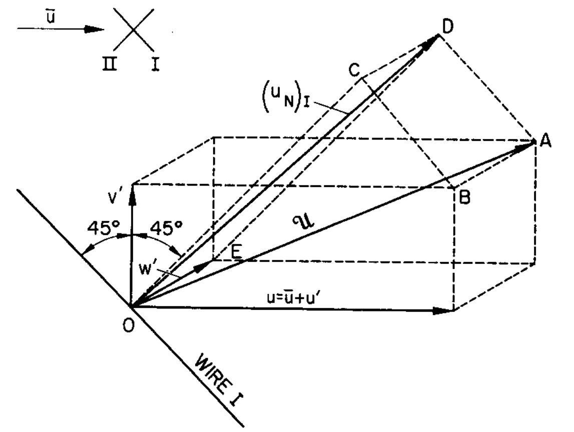

FIG. 9. The X-wire arrangement formed by two wires I and II perpendicular to one another in the plane of the mean velocity $\bar{u}$, with the wires at 45° from the direction of $\bar{u}$. Plane OCDE is perpendicular to the direction of wire I; $\mathfrak{U} = (u^2 + v'^2 + w'^2)^{\frac{1}{2}}$; OC $= (u + v')/\sqrt{2}$ and OD $= [\frac{1}{2}(u + v')^2 + w'^2]^{\frac{1}{2}}$.

that such corrections may be quite important. This nonlinear correction has been applied in the same manner as in the case of the normal hot-wire. However, in the present case the calibration for the voltage difference between the two arms of the X-wire shows that the response of the hot-wire to the v' fluctuations is practically linear within the limits of the fluctuations of this velocity component. The various cross-correlations

$$\overline{v^m\alpha^n} = \frac{\overline{u'^m v'^n}}{(\overline{u'^2})^{m/2}(\overline{v'^2})^{n/2}},$$

and moments

$$\overline{v^m} = \frac{\overline{u'^m}}{(\overline{u'^2})^{m/2}} \quad \text{and} \quad \overline{\alpha^n} = \frac{\overline{v'^n}}{(\overline{v'^2})^{n/2}}$$

were obtained by analysis of the analog data using high-speed computing methods as previously reported. The numerical values for these cross correlations and moments obtained using the sample of recorded data are listed in Table I where both the values before and after correction for nonlinear response are given. It is immediately noticeable and in fact somewhat surprising, that the cross-correlation

$$\overline{v\alpha} = \frac{\overline{u'v'}}{(\overline{u'^2})^{\frac{1}{2}}(\overline{v'^2})^{\frac{1}{2}}}$$

is of the order of 0.1 and is not zero or at least not negligible. We do not wish to emphasize the accuracy of the experimental results since in this particular case they are limited to those from a single sample of recorded data and to measurements made at only one position in the turbulent field. However, this aspect merits further detailed investigation inasmuch as there is no *a priori* reason why, in a decaying field of turbulence with a nonzero value of $\overline{u'^3}$, $\overline{v\alpha}$ may not exist. Similarly, it is premature to assess the significance of the other cross-correlations $\overline{v^m\alpha^n}$ and the moments $\overline{v^m}$ and $\overline{\alpha^n}$ given in Table I and the relations between them. Nevertheless, we feel that the results may be of some interest since no previous data are available for most of the data listed in Table I and since they help to provide a more direct evaluation of the corrections for the effect of transverse velocities.

Let us now consider the case when the turbulent field is isotropic and the probability distribution densities of the velocity components $u'(t)$ and $v'(t)$ are Gaussian [such conditions are also considered by Comte-Bellot[3] in estimating some terms of Eqs. (25)–(27)]. For this case the values are also listed in Table I.

It is seen, at least for this particular sample of data, that the even-order moments of $\overline{v^m}$ and $\overline{\alpha^n}$ are approximated fairly well by the assumption of Gaussianity and isotropy. However, the odd-order moments of $\overline{v^m}$, $\overline{\alpha^n}$, and the odd-order cross-correlations $\overline{v^m\alpha^n}$ as well as even-order correlations $\overline{v^m\alpha^n}$ involving odd powers of v and α are not adequately given by these assumptions.

We can also consider the case of a non-Gaussian joint-probability density distribution of the components v and α similar to the one represented by Eq. (16). The resulting values for the higher-order cross-correlations $\overline{v^m\alpha^n}$ do compare somewhat better with the measured results. Such a comparison will, however, be more appropriate when more extensive results regarding the cross-correlations $\overline{v^m\alpha^n}$ are available.

From Eqs. (25)–(28) we can now estimate the orders of magnitude of the corrections due to the transverse velocities by introducing the values of $\overline{v^m\alpha^n}$ corrected for nonlinearity, as they are listed in Table I as well as the values of T and κ. We find the ratios

$$\frac{\overline{u_N'^2}}{\overline{u'^2}} = 0.99977, \qquad \frac{\overline{u_N'^3}}{\overline{u'^3}} = 1.02;$$

$$\frac{\overline{u_N'^4}}{\overline{u'^4}} = 0.981, \qquad \frac{\overline{u_N'^5}}{\overline{u'^5}} = 1.03.$$

It is, therefore, concluded that the correction for the effect of transverse velocities can be neglected in comparison with the correction for the nonlinear response of the hot wire, at least for moderate intensities of turbulence. At high intensities of turbulence and when $\overline{u'^2}$ is very different from $\overline{v'^2}$, the transverse velocities may have a non-negligible effect. We indicated earlier that the effect of the transverse velocities on the X-wire arrangement can be expected to be rather small under our experimental conditions. For the sake of completeness and since the effect of transverse velocities on measurements with X-wires may become important under different experimental conditions,[3,10,11] it seems, however, desirable to outline the corrections for this effect.

B. Effect of Transverse Velocities on X-Wires

For the X-wire arrangement, in contrast to the normal wire, the effect of the w' component must now also be taken into account. These two wires are considered to be sensitive to the components of the instantaneous velocity $\mathfrak{U}$ normal to the direction of the wire. Figure 9 illustrates the relation between the components $u = \bar{u} + u'$, v', w' of the instantaneous velocity $\mathfrak{U}$ and the component of this velocity normal to the wire. Thus, for wire I whose direction makes an angle of $-45°$ with the direction of the mean velocity, we obtain

$$(u_N)_{\mathrm{I}} = [\tfrac{1}{2}(\bar{u} + u' + v')^2 + w'^2]^{\frac{1}{2}},$$

and similarly for wire II, whose direction makes $+45°$ with the direction of the mean velocity, we find

$$(u_N)_{\mathrm{II}} = [\tfrac{1}{2}(\bar{u} + u' - v')^2 + w'^2]^{\frac{1}{2}}$$

Expanding the above two expressions into series and neglecting terms of higher-order than the fifth we find

$$(u_N)_{\mathrm{I}} = \frac{U}{\sqrt{2}}\left[1 + \frac{u'}{U} + \frac{v'}{U} + \frac{w'^2}{U^2} - \frac{u'w'^2}{U^3} - \frac{v'w'^2}{U^3} - \frac{1}{2}\frac{w'^4}{U^4} + \frac{u'^2w'^2}{U^4} + \frac{v'^2w'^2}{U^4} + 2\frac{u'v'w'^2}{U^4} - \frac{u'^3w'^2}{U^5} - \frac{v'^3w'^2}{U^5} + \frac{3}{2}\frac{u'w'^4}{U^5} + \frac{3}{2}\frac{v'w'^4}{U^5} - 3\frac{u'v'^2w'^2}{U^5} - 3\frac{u'^2v'w'^2}{U^5}\right], \qquad (29)$$

and

[10] W. G. Rose, J. Appl. Mech. **29**, 554 (1962).
[11] S. P. Parthasarathy and D. J. Trytten, AIAA J. 1, 1210 (1963).

$$(u_N)_{\text{II}} = \frac{U}{\sqrt{2}}\left[1 + \frac{u'}{U} - \frac{v'}{U} + \frac{w'^2}{U^2} - \frac{u'w'^2}{U^3} + \frac{v'w'^2}{U^3} - \frac{1}{2}\frac{w'^4}{U^4} + \frac{u'^2w'^2}{U^4} + \frac{v'^2w'^2}{U^4} - 2\frac{u'v'w'^2}{U^4} - \frac{u'^3w'^2}{U^5} + \frac{v'^3w'^2}{U^5} + \frac{3}{2}\frac{u'w'^4}{U^5} - \frac{3}{2}\frac{v'w'^4}{U^5} - 3\frac{u'v'^2w'^2}{U^5} + 3\frac{u'^2v'w'^2}{U^5}\right], \quad (30)$$

where the mean velocity $\bar{u}$ is considered to be equal to the measured wind velocity U.

In order to measure the longitudinal component of velocity with an X-wire the voltages from the two wires I and II are added while the transverse component of the velocity is measured by taking the difference of the voltages. Thus, these velocity components with the correction terms for the effect of transverse velocities can be defined, respectively, by the sum

$$u_x = (u_N)_{\text{I}} + (u_N)_{\text{II}},$$

and the difference

$$v_x = (u_N)_{\text{I}} - (u_N)_{\text{II}}.$$

Taking the averages $\bar{u}_x$ and $\bar{v}_x$ and subtracting them from the corresponding instantaneous velocities, we find

$$u_x' = u_x - \bar{u}_x, \quad \text{and} \quad v_x' = v_x - \bar{v}_x.$$

Equations (29) and (30) give

$$\frac{u_x'}{U} = \sqrt{2}\,T\{v + \kappa_w^2T(\beta^2 - 1) - \kappa_w^2T^2(v\beta^2 - \overline{v\beta^2}) - \tfrac{1}{2}\kappa_w^4T^3(\beta^4 - \overline{\beta^4}) + \kappa_w^2T^3(v^2\beta^2 - \overline{v^2\beta^2}) + \kappa_v^2T^3(\alpha^2\beta^2 - \overline{\alpha^2\beta^2}) - \kappa_w^2T^4(v^3\beta^2 - \overline{v^3\beta^2}) + \tfrac{3}{2}\kappa_w^4T^4(v\beta^4 - \overline{v\beta^4}) - 3\kappa_v^2\kappa_w^2T^4(v\alpha^2\beta^2 - \overline{v\alpha^2\beta^2}), \quad (31)$$

and

$$\frac{v_x'}{U} = \sqrt{2}\,\kappa_vT\{\alpha - \kappa_w^2T^2(\alpha\beta^2 - \overline{\alpha\beta^2}) + 2\kappa_w^2T^3(v\alpha\beta^2 - \overline{v\alpha\beta^2}) - \kappa_v^2\kappa_w^2T^4(\alpha^3\beta^2 - \overline{\alpha^3\beta^2}) + \tfrac{3}{2}\kappa_w^4T^4(\alpha\beta^4 - \overline{\alpha\beta^4}) - 3\kappa_w^2T^4(v^2\alpha\beta^2 - \overline{v^2\alpha\beta^2})\}, \quad (32)$$

where

$$\kappa_v^2 = \frac{\overline{v'^2}}{\overline{u'^2}}; \quad \kappa_w^2 = \frac{\overline{w'^2}}{\overline{u'^2}}; \quad \beta^2 = \frac{w'^2}{\overline{w'^2}}.$$

It is interesting to note that even for perfectly matched wires there is in principle an effect of the v' component on the measurement of the u' component seen from the higher-order terms involving α in Eq. (31). Similarly, there is an effect of u' on the measurement of v' in Eq. (32). However, these secondary effects may become negligible since they involve terms of higher orders in T.

The effect of the transverse velocities on the measurement of the cross-correlations

$$\frac{\overline{u'^m(t)v'^n(t)}}{(\overline{u'^2})^{m/2}(\overline{v'^2})^{n/2}}$$

with the X-wire arrangement can thus be obtained from Eqs. (31) and (32) by forming the ratios

$$\frac{\overline{u_x'^m v_x'^n}}{(\overline{u_x'^2})^{m/2}(\overline{v_x'^2})^{n/2}}.$$

Since we are particularly interested in the third-order correlations, we only indicate the corrections for the fourth-order correction term, $\overline{v^2\alpha^2}$, in Eq. (26). Thus, from Eqs. (31) and (32) we find

$$\frac{\overline{u_x'^2v_x'^2}}{U^4} = 4\kappa_v^2T^4\{\overline{v^2\alpha^2} + 2\kappa_w^2T(\overline{v\alpha^2\beta^2} - \overline{v\alpha^2}) - \kappa_w^2T^2[4\overline{v^2\alpha^2\beta^2} - 2(\overline{v^2\alpha})(\overline{\alpha\beta^2}) - 2(\overline{v\alpha^2})(\overline{v\beta^2})] + \kappa_w^4T^2(\overline{\alpha^2\beta^4} - 2\overline{\alpha^2\beta^2} + 1)\}. \quad (33)$$

No experimental information exists with which to properly estimate the value of the correlation involving the products of u', v' and w' in Eq. (33). However, it does not seem probable that they would be of such a magnitude as to introduce a significant correction in the measurement of $\overline{v^2\alpha^2}$ under the present conditions. For example, if one makes the arbitrary assumption that v' and w' are uniquely correlated either positively or negatively, permitting the use of the corrected values given in Table I (for example $\overline{v\alpha^2\beta^2} = \overline{v\alpha^4} = -0.465$) the correction is still only about 1.5%.

In a similar way it can be shown that the corrections for the effect of transverse velocities on the moments and other cross-correlations measured with the X-wire are negligible under our experimental conditions.

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