Laminar Boundary-Layer Oscillations and Stability of Laminar Flow

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ABSTRACT

An account is given of an experimental investigation conducted at the National Bureau of Standards in 1940 and 1941, in which sinusoidal velocity fluctuations were discovered in the laminar boundary layer of a flat plate and in which the characteristics of these fluctuations were studied in detail and found to agree with the characteristics predicted earlier by the Tollmien-Schlichting stability theory. This work confirmed the theory, settling a controversy of many years and throwing new light on the causes of transition from laminar to turbulent flow. The work was done with the cooperation and financial assistance of the National Advisory Committee for Aeronautics and was published originally as an N.A.C.A. Advanced Confidential Report. The present review has been made possible by the declassification of the original report.

The fluctuations are termed "laminar boundary-layer oscillations" to distinguish them from the irregular velocity fluctuations previously observed by other investigators. In earlier investigations these oscillations were not found, probably because of the high turbulence in the wind tunnels in which investigations were conducted. The present investigation was conducted in a wind tunnel of turbulence much less than 0.1 per cent. The oscillations were readily detected and studied by means of the hot-wire anemometer.

A description is given of the methods used to produce and study boundary-layer oscillations. By these methods the oscillations are found to consist of a wave motion in the boundary layer. Amplified, damped, and neutral waves are found, and these characteristics, together with wave velocities and type of wave motion, are in accord with theory. Attention is called to the subsequent theoretical work of Lin, and some of Lin's results are given.

SYMBOLS

x = distance from leading edge of flat plate
y = distance from surface of flat plate
U_0 = mean velocity outside boundary layer
u = instantaneous x-component of fluctuation velocity
v = instantaneous y-component of fluctuation velocity

w = instantaneous fluctuation velocity perpendicular to U_0 and parallel to surface of flat plate

$\left.\begin{array}{l} u' \\ v' \\ w' \end{array}\right\}$ = root-mean-square values of u, v, and w

c_r = wave velocity
β_r = $2\pi f$, where f = oscillation frequency
α = $2\pi/\lambda$, where λ = wave length
ν = kinematic viscosity
δ = boundary-layer thickness
δ^* = boundary-layer displacement thickness

$\left.\begin{array}{l} \delta^* = 1.72\sqrt{\nu x/U_0} \\ \delta^* = 0.341\,\delta \end{array}\right\}$ —for Blasius velocity distribution

R = $U_0\delta^*/\nu$ = boundary-layer Reynolds Number
R_x = $U_0 x/\nu$ = x-Reynolds Number
  $R = 1.72\sqrt{R_x}$ for Blasius velocity distribution
q_x/q_0 = ratio of dynamic pressure just outside boundary layer at any point x to dynamic pressure at arbitrary reference point.

INTRODUCTION

THE STABILITY OF LAMINAR FLOW and the origin of turbulence has attracted widespread interest in recent years with the advent of the laminar-flow airfoil. However, long before laminar flow had assumed the practical importance it has today, transition from laminar to turbulent flow in boundary layers was of great scientific interest and was the subject of many investigations and a great deal of speculation. A number of investigators attempted to solve the theoretical problem of the stability of laminar flow by determining under what conditions small disturbances in the form of velocity variations would increase or decrease with time. A brief account of such theories is given by Prandtl[1] with numerous references to works dating from that of Lord Rayleigh in 1880. This method of attack was recognized as the most logical one but was none too successful because of an oversimplification of the problem to avoid mathematical difficulties.

About 17 years ago Tollmien[2] of Göttingen made important advances in the application of the mathematical

theory and published a theory of the stability of laminar motion in the boundary layer near a thin flat plate in an air stream flowing parallel to the plate. According to this theory, small disturbances in velocity of any wave length lying within a certain region would be amplified, whereas disturbances of shorter or longer wave length would be damped. The calculations were repeated and extended by Schlichting[3,4] in 1933 and 1935. The amplified disturbances were assumed to grow until they caused a breakdown of the laminar flow. This was assumed to be the fundamental process of transition from the laminar to the turbulent state. The theory was universally discredited by scientists and engineers with a practical turn of mind, mainly because no evidence of disturbances with these characteristics had ever been found in boundary layers. All of the experimental evidence up to 1940 supported the view that any kind of disturbance, whether from turbulence in the air stream or irregularities in the surface, would cause transition when the disturbances were large enough or the Reynolds Number was sufficiently high.

When investigators began to use hot-wire anemometers to study velocity fluctuations, they found irregular motions of large amplitude in laminar boundary layers. The reporting of this phenomenon by Dryden at the Fourth International Congress for Applied Mechanics in Cambridge, England, in 1934 attracted considerable attention. A discussion of the subject is given in reference 5. The fluctuations were recognized as disturbances impressed on the layer from the turbulence in the surrounding flow, and, while they eventually caused transition, they appeared to have no connection with the Tollmien theory. They were random and broke into turbulence with no apparent amplification. They were, in fact, additional evidence against the theory that transition resulted from the growth of small disturbances of a particular wave length, and they supported the growing conviction that stability or instability was merely a question of the size or intensity of the disturbance.

The experimental investigation described here clarified this picture by the discovery of harmonic oscillations in the boundary layer associated with waves with properties that completely verified the theory of selective amplification and damping. The work was completed late in 1941 and was published originally in reference 6. The present review has been made possible by the declassification of the original report. The reader should consult reference 6 for a complete description of the experiment and detailed results, as well as a review of stability theory. After this experimental work had been completed, Lin,[7] studying at the California Institute of Technology, undertook a revision of the mathematical theory of the stability of two-dimensional parallel flow and a clarification of some of the features of the Tollmien-Schlichting theory which had been adversely criticized. Some of Lin's results are given here in Fig. 12.

First Evidence of Amplified Oscillations

In 1940 a research program was undertaken at the National Bureau of Standards, with the cooperation and financial assistance of the National Advisory Committee for Aeronautics, to investigate the effectiveness of damping screens in reducing wind-tunnel turbulence and, at the same time, to investigate transition on a flat plate with different degrees of stream turbulence down to the lowest level attainable. The investigation was conducted in the Bureau's 4½-ft. wind tunnel shown in Fig. 1. A flat aluminum plate, ¼ in. thick, 4½ ft. wide, and 12 ft. long with a pointed leading edge (see Fig. 9), was placed vertically in the working chamber of the tunnel. The pressure gradient along the direction of mean flow was reduced to zero, on the average, by adjustable side walls near the tunnel walls. The position of transition was determined in the customary manner by a small pitot tube in contact with the surface of the plate by which the dynamic pressure was determined very near the surface at different distances from the leading edge. The damping screens were placed in the settling chamber of the tunnel as indicated in Fig. 1, the number and type being varied over a wide range. For each screen arrangement the turbulence of the free stream opposite the plate was measured with the usual
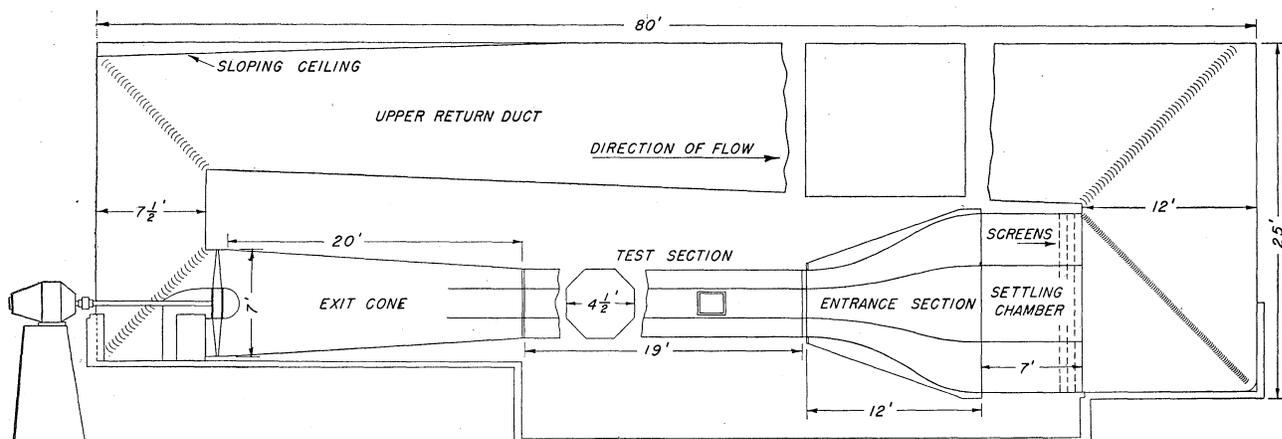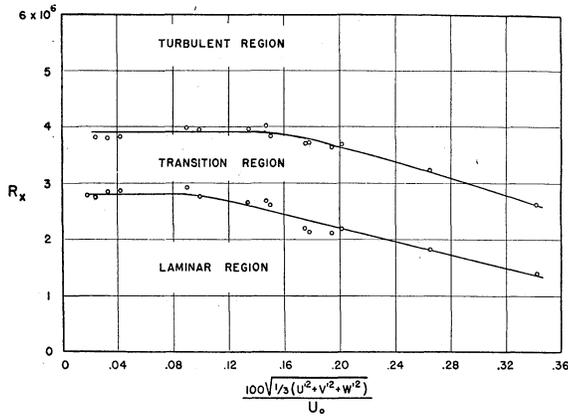

Fig. 1.   Elevation view of 4½-ft. wind tunnel.

FIG. 2.   Effect of turbulence on x-Reynolds Number of transition.   Flat plate, zero pressure gradient.

turbulence-measuring equipment, employing special hot-wire anemometers for obtaining all three components of the fluctuations.

The results of these measurements are shown in Fig. 2.   As the turbulence of the free stream was reduced by increasing the number or the solidity of damping screens, the transition region moved progressively to larger values of the x-Reynolds Number until a value of the turbulence of about 0.08 per cent was reached. Further reduction in turbulence had no effect on the position of transition.

When the stream turbulence had been reduced to a value in the neighborhood of 0.03 per cent, it was decided to observe what, if any, fluctuations were present in the laminar layer.   It was felt that these would be much smaller than the fluctuations observed by Dryden because of the greatly reduced stream turbulence.   For making this observation the hot-wire pickup was arranged in the form of a "bug" as shown in Fig. 3.   This arrangement slides fore and aft along the surface by remote control and places the hot wire at any desired
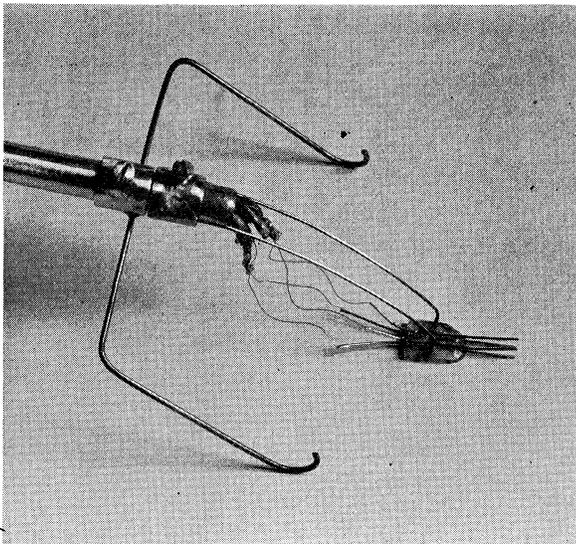
value of x.   A small celluloid block or sled, which rests directly on the surface, carries one or more wires at fixed distances from the surface on the forward extending prongs.   The wires were parallel to the surface and normal to the mean direction of flow so as to be sensitive to the u-component of the fluctuations.   This arrangement of pickup was used to avoid relative motion between the wire and the surface which might give false fluctuations due to vibration.   The hot wires were connected to a compensated amplifier, just as for a turbulence measurement, and the output of the amplifier was connected to a cathode-ray oscillograph, as well as to the usual thermocouple and microammeter, so that the wave form of the fluctuations could be observed as well as the mean-square value.

Irregular fluctuations were almost nonexistent as expected, but, as the pickup was moved downstream, an almost pure sinusoidal oscillation appeared, weak at first, but with increasing amplitude as the distance downstream increased.   Just ahead of transition, bursts of extremely large amplitude occurred, and, at the initial point of transition, these bursts were accompanied by a breaking into irregular high-frequency fluctuations characteristic of turbulence.   These phenomena are illustrated in Fig. 4, which shows a set of film records made by photographing the oscillograph screen with a moving-film camera.   Fig. 5 shows the oscillations at a lower wind speed and greater distance from the leading edge.   Both figures show the progression of events leading up to transition.

Since the occurrence of the oscillations was unexpected, a number of tests were made to make certain that the oscillations were not merely some effect of vibration.   The possibility of such effects was soon
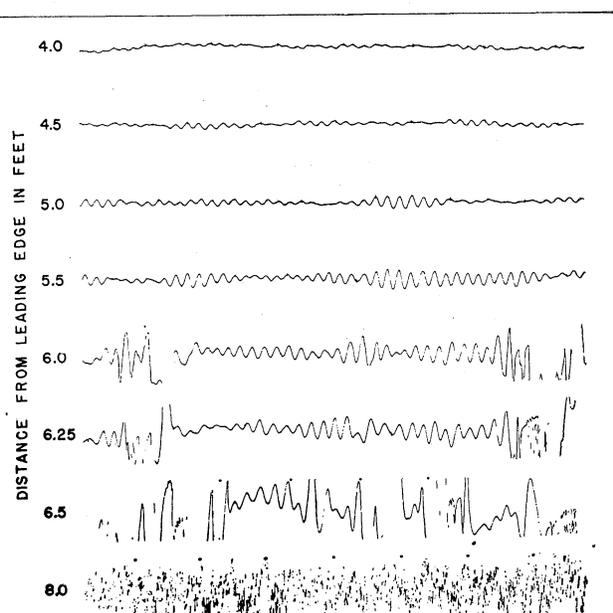

FIG. 3.   Hot-wire pickup for studying velocity fluctuations in boundary layer.

FIG. 4.   Oscillograms showing laminar boundary-layer oscillations in boundary layer of flat plate.   Distance from surface $=$ 0.023 in.; $U_0 = 80$ ft. per sec.;   time interval between dots $=$ $1/_{30}$ sec.

FIG. 5. Oscillograms showing laminar boundary-layer oscillations in boundary layer of flat plate. Distance from surface = 0.023 in.; U_0 = 53 ft. per sec.; time interval between dots = $1/30$ sec.
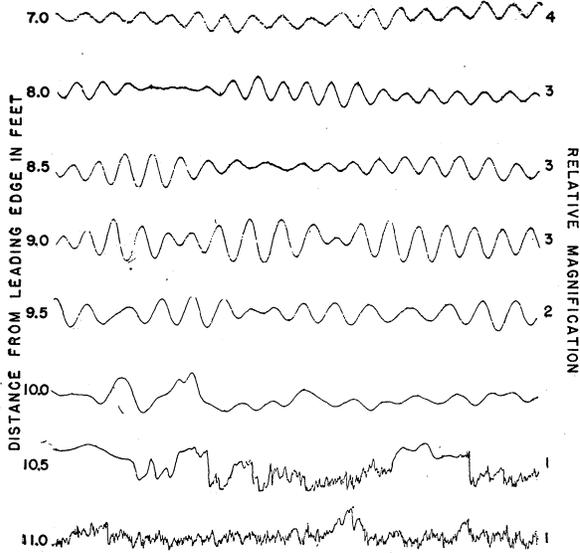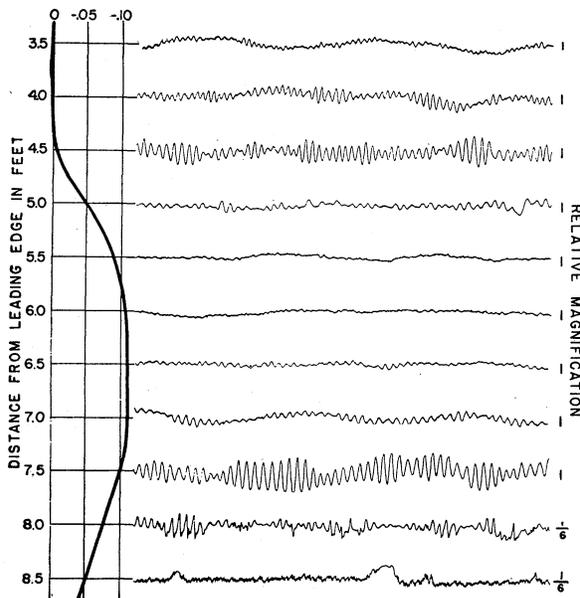

FIG. 6. Effect of falling and rising pressure on laminar boundary-layer oscillations. Scale at upper left is ratio of pressure change to free-stream dynamic pressure. Distance from surface = 0.021 in.; U_0 = 95 ft. per sec.; time interval between dots = $1/30$ sec.

ruled out. In the course of the observations it was noted that the oscillations were definitely connected with transition, since the zone in which they occurred always preceded transition and moved with it fore and aft along the plate as the wind speed was varied. This was not absolute proof that the oscillations were the cause of transition, since there existed the possibility that the boundary layer became shock-excited by transition occurring a short distance downstream, giving rise to an oscillation that was possibly the result of transition rather than the cause. This was ruled out by re-

moving transition by means of an abrupt pressure drop and yet leaving the oscillations totally unchanged at an upstream position. The results of this experiment are shown in Fig. 6, which also serves to illustrate how a falling pressure damps out the oscillations.

All evidence pointed to the conclusion that these oscillations had some connection with the Tollmien-Schlichting stability theory. Accordingly, the theoretical diagram shown in Fig. 7, taken from Schlichting's treatment,[3] was used as a test. This theoretical diagram applies to a Blasius velocity distribution and could be applied here, since the velocity distribution along the plate with zero average pressure gradient was found to be a Blasius distribution to within the accuracy of velocity measurements. As the oscillation frequencies were determined from numerous oscillograms, such as those shown in Figs. 4 and 5, $\beta_r \nu / U_0^2$ and the corresponding R were calculated and plotted on the diagram. As shown in Fig. 7, the experimental points all fell along Branch II of the curve.

The interpretation of this diagram is as follows: Within the loop all disturbances are amplified, while in all other regions the disturbances are damped. Since the curve divides the amplified from the damped, the curve itself is called the neutral curve. According to theory, and later found to be true experimentally, all disturbances travel in a downstream direction with a velocity given by their frequency and wave length. Small disturbances consisting of many frequencies (random disturbances, for example), originating in some upstream position of the boundary layer, all advance in the direction of increasing R. As they advance, they are first damped, then amplified as they traverse the enclosed region, and then are again damped when they pass beyond Branch II. They are therefore most highly amplified when they reach Branch II. A frequency observation was always made at some particular value of R. Taking the theory at face value, calculations show that from a band of frequencies arriving at a given value of R, the one most highly amplified
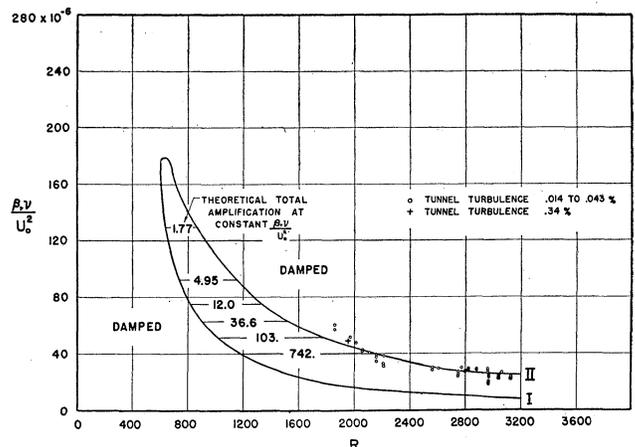

FIG. 7. Theoretical neutral curve and amplification zone according to Schlichting including experimental points from frequency of oscillations on oscillograms. Theoretical total amplification from Branch I to Branch II is indicated.

must fall on Branch II. Therefore, because of the position of the points, it is concluded that the observed frequencies were the ones most highly amplified. The harmonic character of the records, or the purity of the frequency, may be accounted for by the fact that at a particular value of R all frequencies not falling on Branch II must lie either above the curve where they have undergone some damping or below the curve where they have not been fully amplified. This amounts to a filtering process.

Thus the observed oscillations could at last be fully accounted for on theoretical grounds. They were, according to theory, the amplified components of some initial disturbance, probably coming from the small amount of turbulence still remaining in the tunnel stream. The reason why such phenomena were observed in this experiment and not in previous ones was answered by producing a tenfold increase in the stream turbulence and again looking for the oscillations. They were found, but, if the turbulence had been much higher, they would have been difficult to identify because of the near coincidence of the point at which they appeared and the transition point. The one observed point taken at the higher turbulence is shown in Fig. 7 in good agreement with the other points. In all known previous experiments the stream turbulence was of the order of 1 per cent. Under these conditions transition was produced by the initial disturbances, and the laminar layer was destroyed before any observable selective amplification could occur.

In the experiment so far, the theory was used to interpret the results. It still remained to find out whether amplified neutral and damped oscillations or waves were a reality and whether the theory was correct. An investigation was therefore begun with a view to setting up waves in the laminar layer under controlled conditions, rather than being left to random and accidental disturbances.

METHODS OF PRODUCING OSCILLATIONS AND RESULTS

In the search for schemes to excite oscillations in the boundary layer, a number of devices were tried before completely satisfactory results were obtained. Methods using sound, both pure notes and random noise, were none too satisfactory because of resonance effects and the complexity of the wave pattern in the tunnel. The experiments are worth mentioning, however, because sound is an unavoidable type of disturbance both in wind tunnels and in flight with power-driven aircraft.

The first scheme tried was to place a 25-watt loud-speaker in the tunnel and "fill" the tunnel with sounds of various intensity and frequency. The sound from the speaker was, of course, superimposed on the general background noise of the tunnel. Boundary-layer oscillations could be induced at will in a variety of positions along the plate by choosing the right combination of frequency and wind speed. Also, transition could be
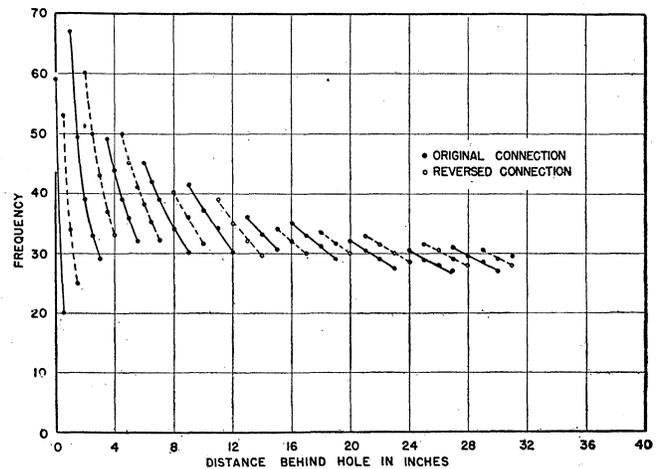

FIG. 8. Frequency of oscillations excited by sound through hole in plate. $U_0 = 23$ ft. per sec.; frequency expressed in cycles per second.

moved 1 or 2 ft. ahead of its normal position. Random noise from the loud-speaker produced similar results, although the control was poorer. In general, the quantitative results were similar to those shown in Fig. 7.

The second scheme tried was an attempt to localize the sound field by bringing sound into the boundary layer through a small hole in the plate. A small head-phone was mounted on the back side of the plate opposite a $^1/_8$-in. hole. A survey of possible types of measurements showed that this method could be used only for obtaining the type of result given in Fig. 8. The hot-wire pickup was arranged to move upstream or downstream in the boundary layer downstream from the hole. The amplified output of the hot wire was fed back into the headphone, causing the system to oscillate with a frequency depending on the distance between the hot wire and the hole, as shown in Fig. 8. Best results were obtained for a wind speed of 40 ft. per sec. or less. The explanation of this performance is that proper phase relations had to exist for regeneration, and the frequency automatically decreased as the path length between the hot wire and hole increased in order to keep an integral number of waves between the source and the pickup. There existed a preferred frequency band, determined by the characteristics of the wire, electrical and acoustic circuits, and boundary layer. Jumps occurred when one additional wave length was necessary to keep the frequency near the center of the preferred band. Reversing connections on the headphone changed the phase by 180° and resulted in the dashed curves. The distance between curves of any one type along any line of constant frequency is thus the wave length corresponding to that frequency. This showed for the first time the wave nature of the oscillations but otherwise conveyed little new information.

Since the theory was known only for two-dimensional disturbances in a boundary-layer type of flow, what was really wanted was a source of two-dimensional disturb-
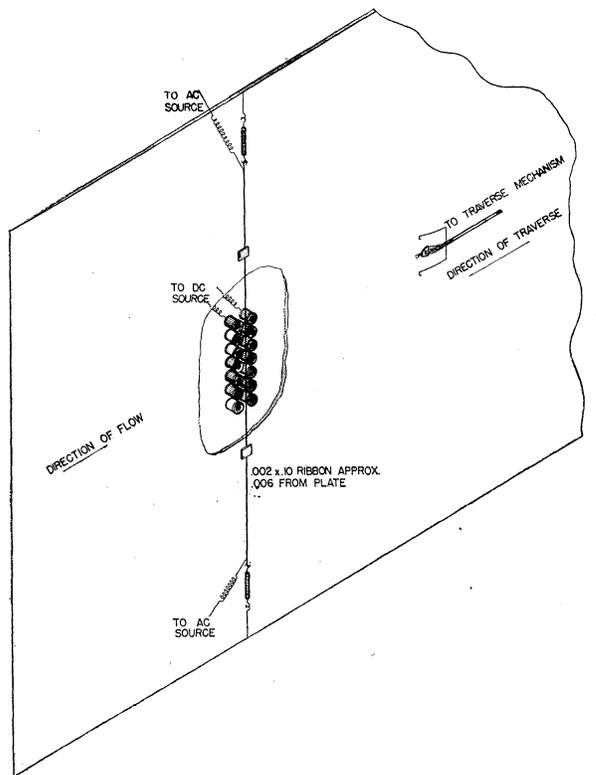
FIG. 9.   Setup for exciting oscillations in laminar boundary layer by vibrating ribbon.

ance.   This was found in the vibrating-ribbon method illustrated in Fig. 9.   A ribbon of phosphor bronze, 0.002 in. thick and 0.1 in. wide, was mounted under tension on the plate as shown.   Spacers were placed under the ribbon producing a 12-in. segment near the center uniformly parallel to the plate and about 0.006 in. from the surface.   When an alternating current was passed through the ribbon in the presence of a constant magnetic field from the electromagnets on the opposite side of the plate, the 12-in. segment vibrated to and from the surface in a single loop.   The vibration set up a wave with the frequency of the ribbon which traveled downstream through the boundary layer and passed by the hot-wire pickup.   The current through the ribbon was supplied from an oscillator, and the frequency could be varied through wide limits.   The effect of the ribbon on the mean flow was exceedingly small, and what effects there were could not be detected 2 in. downstream.   Neither the interference effects nor the results produced were critical to the dimensions and position of the ribbon.   The ribbon was placed at various locations on the surface to cover the desired range of Reynolds Numbers.

Two hot wires, one 0.010 in. and the other 0.110 in. from the surface, both sensitive only to the u-component, were mounted on the prongs of the "bug" so that it would be possible to study the wave properties at two fixed distances from the surface at various distances downstream from the ribbon along the centerline of the plate.   Both hot wires were within the

layer, and the velocity fluctuations introduced into the boundary layer could be picked up by either wire. Generally, only one wire was used at a time, depending on which gave the stronger response.   Distance from the ribbon, wind speed, and frequency could be varied independently.   The wave form showed some distortion for an inch or so behind the ribbon but became nearly sinusoidal at greater distances.   Amplified, damped, and neutral waves were found at once, and it only remained to make quantitative measurements of amplitude, frequency, and wave length to obtain the necessary data for checking the theory.

The usual procedure was to measure the root-mean-square value of the u-component of the fluctuations, denoted by u', at various distances from the ribbon for some fixed wind speed U_0 at first one frequency,
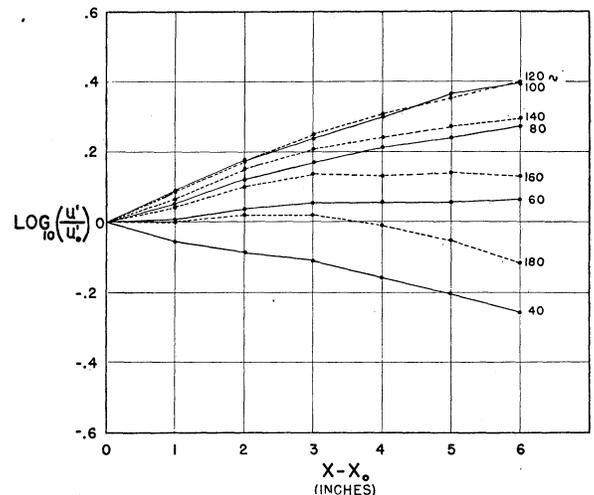

FIG. 10.   Growth and decay of u-component of oscillations produced by vibrating ribbon 4 ft. from leading edge of plate x_0 is 2 in. downstream from ribbon; $U_0 = 64$ ft. per sec.; frequency expressed in cycles per second.
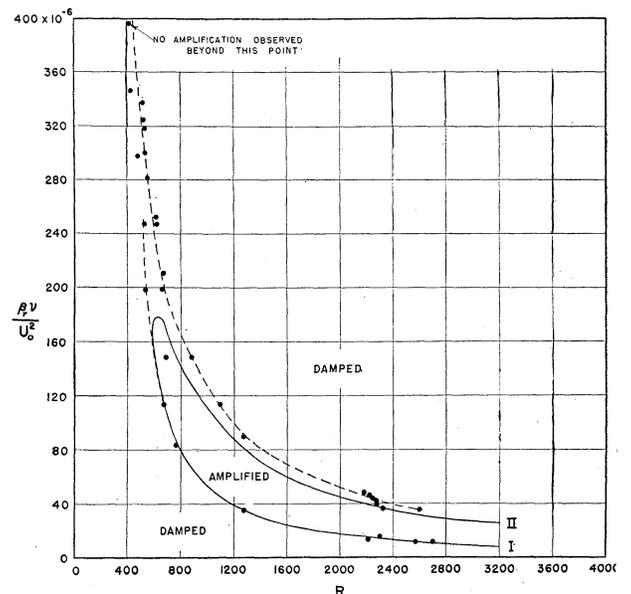

FIG. 11.   Neutral curves (dashed) obtained from observed neutral frequencies compared to theoretical curve according to Schlichting (solid).

then another. In this way results similar to those illustrated in Fig. 10 were obtained. In this figure, x_0 denotes a position 2 in. behind the ribbon and u_0' is the root-mean-square velocity fluctuation at x_0. The curve for 40 cycles per sec. shows damping throughout. As the frequency is increased, amplification takes place, reaches a maximum at 120 cycles per sec., and then decreases until 180 cycles per sec. again shows damping. In more accurate terms, positive slopes at any value of x mean amplified waves, negative slopes mean damped waves, and zero slopes mean neutral waves. From sets of curves like these, not only could the neutral waves be found for outlining the amplification zone but amplification and damping coefficients could also be determined for the entire region. The reader is referred to reference 6 for the complete results. The experimentally outlined neutral curves are shown in Fig. 11, again compared to Schlichting's theoretical curve.

The wave length and wave velocity were determined in the following simple manner: The input to the ribbon from the oscillator was connected to one pair of plates of the cathode-ray oscillograph, and the output from the hot-wire pickup was connected to the opposite pair. A stationary Lissajou figure consisting of a single loop was obtained, since the frequencies of both input and output were the same. As the spacing between the ribbon and the hot wire was changed, the phase change could easily be seen. Since a phase change of 180° could be detected accurately, the distance intervals corresponding to one-half wave length were measured to determine the wave length. The wave velocity was then obtained by multiplying the wave length by the frequency.

Wave lengths and wave velocities were measured for the neutral oscillations, and these furnished additional experimental data for comparison with theory. Fig. 12 shows the neutral curves on a wave-length basis ($\alpha\delta^*$ $= 2\pi\delta^*/\lambda$, where λ is the wave length). Recently, Lin's theoretical curve became available from reference 7, and this is shown in Fig. 12 in addition to Schlich-

FIG. 13. Neutral curves (dashed) obtained from observed velocity of neutral waves compared to theoretical curve of Schlichting (solid).

ting's curve. The experimental points are in a little better agreement with Lin's curve than with Schlichting's. On the whole the agreement between theory and experiment is good.

Fig. 13 shows the neutral curves on a wave-velocity basis. Comparison is again made with Schlichting's theoretical curve. It will be seen that the neutral waves are propagated downstream through the boundary layer, with a velocity definitely related to the free-stream velocity but only about one-third as fast.

When these experiments were being performed, each check with theory was a stimulating experience. There was nothing so unusual about setting up a wavy disturbance in the boundary layer, but finding that this waviness really constituted a unique wave phenomenon with properties determined by the boundary-layer flow was out of the ordinary. Clearly, the wave was a motion superimposed on the mean flow, but what kind of motion was this? According to Schlichting's theory in reference 4, when the longitudinal motions were in one direction near the surface they were in the opposite direction farther out. In other words, the boundary layer was not disturbed as a whole, but in any one section the oscillation consisted of speeds increasing in a certain interval of y while simultaneously decreasing in another.

This theoretical prediction was accordingly put to the test by getting simultaneous records of the u-fluctuations from one hot wire at a fixed position near the surface and from another wire set at various distances from the surface in the same cross section of the layer. The records are shown in Fig. 14. The numbers on the left show the position of the movable wire, and the upper trace in each set shows the wave picked up by it. As the wire was moved outward, the wave amplitude decreased to zero but remained in phase with the wave near the surface until it had disappeared. At 0.165 in. from the surface there is scarcely any perceptible oscil-
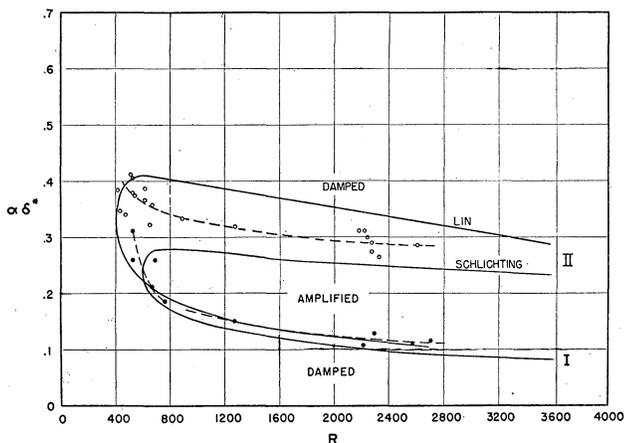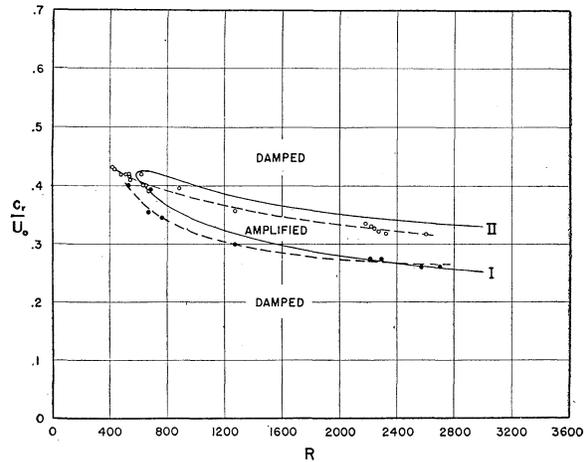

FIG. 12. Neutral curves (dashed) obtained from observed neutral wave lengths compared to theoretical curves (solid). Closed circles—Branch I; open circles—Branch II.
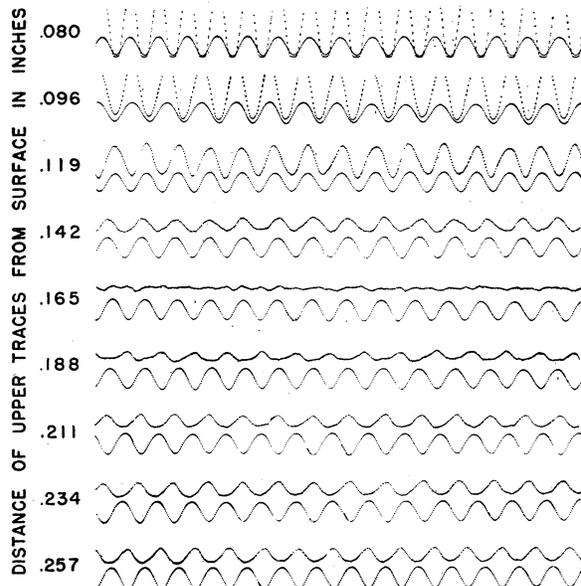
FIG. 14.   Simultaneous records obtained with two hot wires located 1 ft. downstream from vibrating ribbon showing phase reversal in u-component of oscillations.   Lower trace obtained from hot wire 0.055 in. from surface.   Ribbon 3 ft. from leading edge;   frequency 70 cycles per sec.; $U_0 = 42$ ft. per sec.
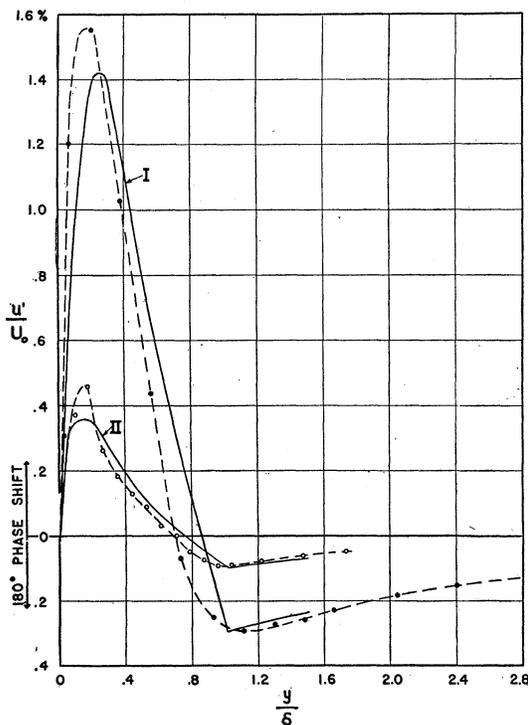

FIG. 15.   Distribution of amplitude of oscillations across boundary layer.   Solid curves are theoretical according to Schlichting;   points and dashed curves are experimental.

lation.   As the distance was further increased, the wave reappeared but now 180° out of phase with the wave near the surface.

This was pleasingly in accordance with theory, not only qualitatively but quantitatively, as shown by Fig. 15.   Schlichting had calculated the distribution of amplitude for two positions on the neutral curve—

namely, $R = 894$ for Branch I and $R = 2,070$ for Branch II.   The measurements shown in Fig. 15 corresponded as nearly as possible to these two positions.

With the measurement of wave length, velocity, phase relationships, damping, and amplification, as shown by the u-component of the fluctuations, the physical characteristics of the waves were fairly well defined.   The v-component, required by continuity, might also have been studied.   However, it was not studied because of experimental difficulties and the belief that little additional information could have been gained by doing so.   For the same reason, no attempt was made to observe the correlation between u and v predicted by theory.   The general applicability of the theories of Tollmein, Schlichting, and Lin can no longer be doubted.   For the latest and most comprehensive treatment of the subject Lin's work is recommended.

An attempt was made to observe the transition into turbulent flow caused by oscillations produced by the vibrating ribbon, and a number of oscillograph records of this phenomenon are presented in reference 6.   The results gave the general impression that transition occurred when the amplitude of the oscillations was sufficiently large.   This goes back to the original idea held before 1940, with the important additional information—namely, that now we know that small outside or initial disturbances are linked to the disturbance that finally causes transition through amplified boundary-layer oscillations.

EFFECT OF PRESSURE GRADIENT ON BOUNDARY-LAYER OSCILLATIONS

With but one exception, the foregoing results were all obtained with zero pressure gradient and a Blasius velocity distribution for which the theory was most complete.   The one exception is shown in Fig. 6, where it is seen that a falling pressure decreases the oscillations and an increasing pressure increases them.   This effect was studied in more detail for oscillations produced by the vibrating ribbon with pressures varying along the plate as shown in Fig. 16.   Curve A shows the condition termed zero pressure gradient.   Curves B and C show moderate positive and negative gradients, respectively, and D and E show large gradients.   The results on neutral oscillations obtained with pressure distributions A, B, and C are shown in Fig. 17.   No theoretical curves are shown in this case, the curves labeled "zero pressure gradient" being the experimental curves taken from Fig. 11.   Gradient C had no noticeable effect on the position of the neutral points, whereas gradient B expanded the region of amplification into the previously damped zone above Branch II.

Corresponding figures for distributions D and E could not be obtained because of the large effect of these gradients on the oscillations.   The general impres-

sion given by observing the oscillations was that amplification always occurred in gradient D regardless of the frequency and Reynolds Number, while damping always occurred in gradient E. In other words, for the Reynolds Number range investigated with gradient D, the amplification zone occupied the whole region of a figure such as 17, and for gradient E, the amplification zone lay beyond the highest Reynolds Number reached (maximum Reynolds Number about 2,600). Sufficiently large pressure gradients therefore have a marked effect on the oscillations, negative gradients having a damping effect and positive or adverse gradients having an amplifying effect. This is in agreement with effects commonly observed on transition—namely, that a falling pressure delays or prevents transition whereas a rising pressure brings about early transition. Thus we see, in part at least, why it is that laminar flow is maintained over the region of falling pressure on an airfoil.

Pressure gradient is only one of several factors that affect the stability of boundary layers. Curvature of the surface is another factor that was not touched upon but which has subsequently been investigated by Leipmann[8] at the California Institute of Technology. Liepmann also investigated the effect of small roughness elements on the surface and showed how these may excite the boundary layer into oscillation.

CONCLUDING REMARKS

Perhaps a clearer physical picture of boundary-layer oscillations may be obtained by regarding a laminar layer as a medium in which a particular kind of wave motion is propagated, the propagation constants depending on the nature of the layer. This so-called medium has both positive and negative absorption coefficients. The analogy to a medium should perhaps not be carried too far, but, if a boundary layer is compared to a water surface, it is easy to see how conditions would rarely be so "calm" that even tiny ripples would not exist. Practically, then, some degree of wave motion will always be present. The important question is whether little ripples will grow to sizable waves, eventually producing transition to turbulent flow.

There are, of course, those cases where the initial disturbances are so great—due, for example, to turbulence or surface roughness—that transition occurs at once. There are also those cases frequently met in practice where an adverse pressure gradient is sufficient to cause laminar separation followed almost immediately by a turbulent layer. Engineers are aware of these two cases and avoid them when attempting to preserve laminar flow. The technology of flight has therefore advanced to the stage where amplified waves play an important part in the preservation of laminar flow. It is now important to know where zones of amplification and damping lie and the magnitude of the amplification
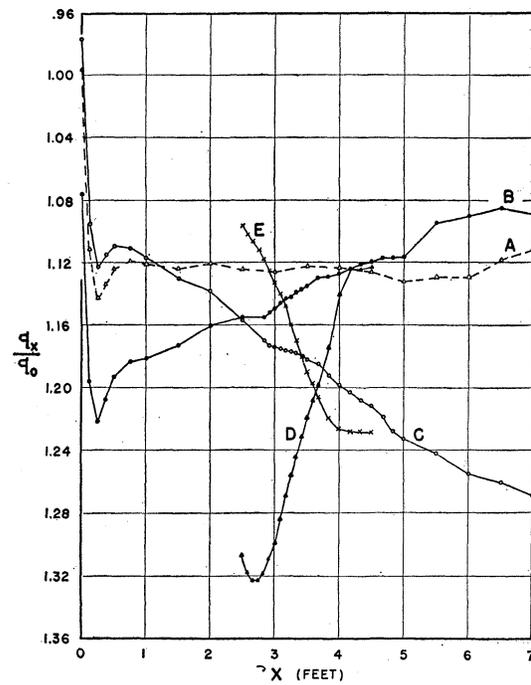

FIG. 16. Distribution of dynamic pressure along plate just outside boundary layer.
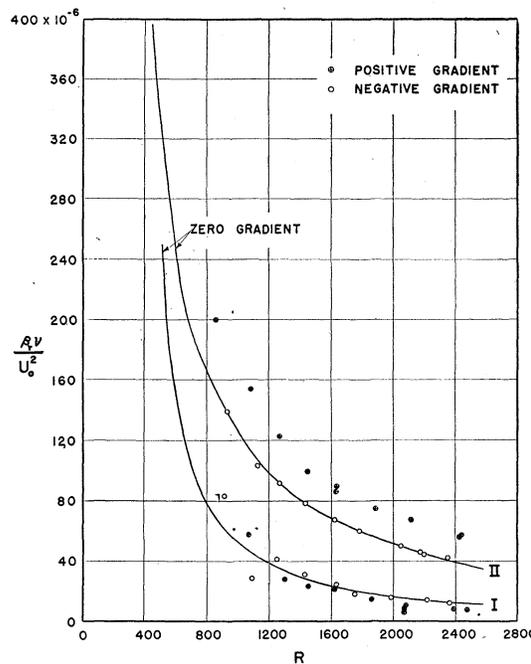

FIG. 17. Experimentally observed points showing neutral oscillations for positive and negative pressure gradients of distributions B and C, respectively, of Fig. 16. Experimental curves for zero pressure gradient shown without points.

and damping in these zones. As we have seen, the zones depend both on the boundary layer and the frequency or wave length of the disturbances. Both are quantities over which some control may be exercised.

So far the theory has been worked out only for the simpler cases. Furthermore, the present experimental work, which is limited to the flow along plane surfaces with zero, positive, and negative pressure gradients, is

little more than a good beginning toward an understanding of the many flow problems involving the stability of boundary layers.

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