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Control rules for dispatching trains on general networks with multiple train speeds

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Abstract

The most popular dispatching rule for double-track segments is to dedicate one track for trains traveling in one direction. However, today passenger trains are able to travel at a much faster speed compared to freight trains, but the limitation of the rail infrastructure makes it more cost effective to allow the passenger trains to share some portions of the railway tracks with the slower freight trains. The drawback of this dedicated rule is that a fast train can be caught behind a slow train and experience significant knock-on delay. The new dispatching rules enable the fast train to pass the slow train by using the track travelled by trains in the opposite direction if the track is empty. Simulation experiments show that a switchable policy can reduce the average train delay by 21% over a dedicated track policy and with the existence of crossovers the reduction in the average train delay can be as high as 41.9%.

Keywords: rail scheduling ; dispatching ; headway control.

Résumé

La règle la plus populaire pour répartir l'usage d'un tronçon de ligne ferroviaire à double voie est d'affecter une voie à chaque sens de circulation regroupant tous les trains allant dans la même direction. Cependant, aujourd'hui les trains de passagers sont capables de circuler à une vitesse bien supérieure à celle des trains de fret, mais les limites de capacité d'infrastructure rendent plus efficace de permettre aux trains de passagers de partager certaines portions de voie avec des trains de marchandises plus lents. L'inconvénient de cette règle d'expédition est qu'un train rapide peut se retrouver retenu derrière un train lent et subir un retard conséquent. De nouvelles règles d'expédition permettent au train rapide de doubler le train lent en utilisant la voie de sens opposée si la voie est libre. Des simulations expérimentales montrent qu'une telle solution de voies partagées peut réduire le retard moyen des trains de 21% par rapport à la solution de voies entièrement dédiées, et dans le cas de croisements ces réductions peuvent atteindre jusqu'à 41,9%.

Mots-clés: planification du rail ; répartition ; contrôle d'interdistance.

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1. Introduction

Railway has always been an effective mode to transport both people and goods. Freight trains are about four times more fuel efficient than trucks and passenger trains are popular because they can comfortably transport people to their destinations on time at a low cost. Nowadays, with the advancement of technology, passenger trains are able to travel at a much faster speed compared to freight trains. However the limitation of the rail infrastructure makes it more practical and cost effective to allow passenger trains to share some portions of the railway tracks with slower freight trains. If a faster passenger train catches up with a freight train on the railway track and there are no crossover junctions, the nature of the railway transportation determines that the passenger train has to follow the freight train at the speed of the freight train while keeping a safety headway from the freight train.

The detailed schedules of the trains are not always available before their movement (i.e. it is common to see freight trains travel without a pre-determined schedule). Also, normally the schedules only contain rough time windows for train departures and arrivals and the detailed movements of the trains need to be determined by the dispatchers in real time. In this case, we need simple dispatching rules to efficiently guide the trains through the network and a method to accurately predict the delay time of trains under such dispatching rules.

Mu and Dessouky (2013) propose a flexible switchable dispatching policy which can significantly reduce the delay of the fast trains over a dedicated track dispatching policy by reducing the likelihood of the fast trains catching up with the slow trains. In a dedicated track policy, the tracks can only be used by trains traveling in a designated direction. In a switchable policy, the tracks can be used by trains traveling in both directions. In order to make their analytical delay functions computationally tractable, they only consider two possible train speeds in the network (fast and slow trains) and ignore the effects of train length and safety headways between the trains. However in reality, trains can travel at multiple levels of speed in the network (i.e., the passenger train company might provide train service with different travel times for the same train route at different train fares). This paper proposes dispatching rules to specifically address the situation where trains travel at multiple levels of speed. Simulation models are used to accurately predict the delay of our proposed policies. The simulation models also consider the train lengths and safety headways so that the predicted results match more closely with real world operations.

Two major techniques have been applied to study the delay of railway operations in the literature: simulation model and queueing theory. Frank (1966) studies delay on a single track. He assumes a single train speed and deterministic travel times. He also studies the cycle time of trains and number of trains needed to meet a certain level of demand for transportation. Greenberg et al. (1988) use queueing models to predict dispatching delays on single-track, low speed rail segments. They calculate the train delay based on a simple dispatching rule on a single-track segment. They extend the analysis on one single-track segment to the entire railway network. Chen et al. (1990) study the delay for different types of trains on a single track. The mean and variance of train delay are approximated by solving a system of nonlinear equations. Harker et al. (1990) extend Chen's model to partially double-track railway segments. Ozekici et al. (1994) apply Markov chain techniques to study the knock-on delays. They study different arrival patterns of passengers and dispatching policies. The average passenger waiting time is derived explicitly. Huisman et al. (2001) study the delay of railway comprising heterogeneous trains. The delay times of trains are obtained by solving a system of differential equations. Huisman et al. (2002) propose a solvable queueing network model to predict network-wide train delays. Their analytical results closely match with real delay data of the Netherlands railway system. Yuan et al. (2007) optimize the capacity utilization of stations by using a stochastic model of train delay propagation, which is associated with pre-determined timetables, whereas the delay in this paper is referring to simple knock-on delay.

Dessouky et al. (1995) and Lu et al. (2004) model train movements through complex railway networks. Their model simulates multiple train speeds, headways, train lengths and acceleration and deceleration rates of trains. Hooghiemstra et al. (1998) build a simulation model for transportation planners to study network level dynamics under different timetables. Yalçınkaya and Bayhan (2009) use a simulation modelling and a response surface methodology approach to determine the optimal settings such as headway to minimize passenger waiting times. Medanic and Dorfman (2002) propose a discrete event model using a travel-advance strategy (TAS) to schedule trains on a single line and is extended by Dorfman and Medanic (2004) for general networks and incorporated in a simulation model of an actual real system. Li et al. (2008) present an alternative simulation model that implements an algorithm that makes use of global information of the system state to schedule trains. In general, the simulation models can be at different scales and closely imitate real situations.

In this paper, we develop three heuristic dispatching rules for the control of trains travelling on double-track railway segments with heterogeneous traffic and experimentally compare the performance of the heuristic rules using simulation. The first switchable policy is similar to the policy proposed in Mu and Dessouky (2013). This policy switches the fast train, if it has potential delay on its designated track, to the designated track for the opposite direction if that track is empty. The earlier paper only considered the case of two possible train speeds while in this paper we generalize to consider multiple speeds the train can travel. The second policy has a smarter condition to dispatch the trains by considering the speed of the attempting switching train. The last policy also considers the blocking time in the dispatching rule. That is, fast trains are switched to the opposite direction designated track if they do not extend the current busy period on the opposite direction designated track for too long. We also study the improvement of the efficiency of the dispatching rules obtained by placing crossovers in the segment.

The rest of the paper is organized as follows. Section 2 introduces the dispatching polices for double-track segments with trains travelling at multiple speeds. Section 3 extends the dispatching policy to better utilize crossovers in the railway segment. Section 4 summarizes the findings of this paper.

2. Dispatching rules for double-track segments with multiple train speeds

2.1 Descriptions of dispatching policies

Mu and Dessouky (2013) study the dispatching policies for a double-track segment when there are only two train speeds. They focused on developing an analytical model to predict the expected delay with a switchable policy. To keep the analytical model tractable they assumed the train length and safety headways were both zero. In this paper, we extend the study on double-track to the case where trains are travelling at multiple speeds and consider the train length and safety headways in our analysis.

Figure 1 shows a typical double-track railway segment between two major intersections. The length of the track segment is denoted by D. There are multiple types of trains travelling on the track segments. Each type of train is identified by its speed and we assume that the arrival of each train type at each end of the double-track segment is an independent Poisson Process. The upper and lower tracks of the segment can be traveled in both directions. The free running time of the train is defined to be the minimum traveling time of the train assuming there is no other traffic in the network. The delay time of the train can be calculated as:

Delay = Completion time – Arrival time at the segment – Free running time

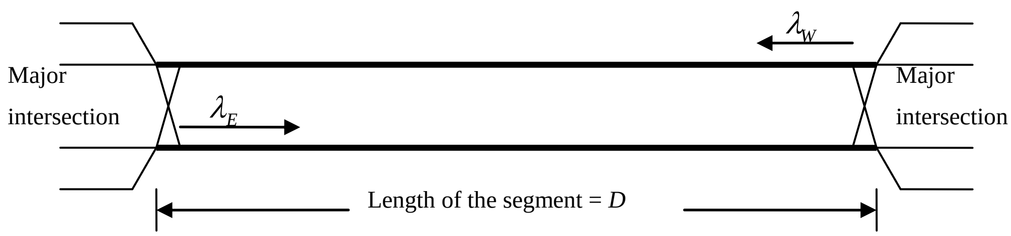

Figure 1. Double-track railroad segment

A delay can occur (1) when a train tries to keep the required headway length with another train traveling ahead of it, (2) when a faster train catches up with a slower train traveling in the same direction so that the faster train has to travel at the speed of the slower train, while keeping the required headway, or (3) when a train arrives at the track segment and it has to wait for the track to be cleared from being occupied by trains traveling in the reverse direction.

Probably the easiest dispatching policy for a double-track segment is to dedicate one track of traffic to one direction (i.e., all eastbound trains travel on the lower track while all westbound trains travel on the upper track). We refer to this policy as a dedicated policy. The drawback of the dedicated policy is that it can be likely for a fast train to catch up with a slower train on its dedicated track. If the fast train catches up with a slower train, it has to keep a safety distance between the slower train and travel at the speed of the slower train. Thus, the fast train can experience a significant delay if there is a large difference in train speeds.

Next we introduce three dispatching rules that allow trains to travel in either track segments. Without loss of generality, for the following dispatching rules, let the lower track be the designated track for trains traveling eastbound and let the upper track be the designated track for trains travelling westbound. We refer to the three dispatching rules as Switchable2-I, Switchable2-II and Switchable2-III policy. The Switchable2-I policy considers the potential delay of the arriving train when deciding to switch the train. Besides the potential delay, the Switchable2-II policy also considers the speed of the arriving train when deciding to switch the train. The faster the speed is, the more likely the train will switch. To increase the switching frequency, the Switchable2-III policy switches the train to the other track in cases where the other track is not completely empty of trains.

<u>Switchable2-I policy</u>

1. Upon arrival, if the designated track is occupied by trains travelling in the opposite direction, the arriving train waits for the opposing moving trains to finish traveling on the segment before proceeding on its designated track.

2. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if the potential length of delay of the arriving train traveling on its designated track is less than ω, the arriving train will start traveling on its designated track.

3. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if the potential length of delay of the arriving train traveling on its designated track is greater than ω, the arriving train will attempt to switch to the other track. The arriving train will use the reverse direction track only if it is empty. If the reverse direction track is occupied by trains travelling in either direction, the arriving train will use its designated track.

The Swtichable2-I policy is similar to the switchable policy described in Mu and Dessouky (2013), where they derive analytical equations to measure the delay assuming a no headway requirement since they assume the trains are infinitesimally small. The Switchable2-I policy will attempt to switch an arriving train if it will catch up with a slower train on its designated track and its potential delay is greater than ω. The optimal value of ω ranges from 0 to the time difference between the free running times of the slowest and fastest train. If a fast train catches up with a slower train near the end of its designated track, the potential delay of the fast train might not be significant enough for the policy to switch the fast train to the other track, since usage of the other track might block the traffic in the other direction. When a train attempts to switch, the policy only allows it to switch when the reverse direction track is empty. If the reverse direction track is occupied by another switched train which is traveling in the same direction as the train attempting to switch, the policy prohibits the train from switching so not to extend the reverse direction busy period for the other track.

With many different train speeds, even a slow train can catch up with another slower train and experience delay on its designated track. If we only consider the potential delay on the designated track as the criterion to switch the train, we might tend to switch some slow trains. The switched slow trains will block the other track for a long time, which is not desired for the traffic in the other direction. Intuitively, if both a relatively fast and a relatively slow train have the same potential delay on the designated track. The relatively fast train should be switched because the fast train can finish traveling on the reverse direction track in a shorter amount of time. Thus a higher potential delay and a higher train speed should lead to a higher chance to switch. Let D_p denote the potential length of delay an arriving train will experience on its designated track. Let S_{ar} denote the speed of the arriving train. In the Switchable2-II policy, instead of having $D_p \geq \omega$ as the criterion to switch the arriving train, a more complicated criterion $\alpha D_p + \beta S_{ar} \geq \delta$ is used, where α, β and δ are parameters. A good assignment of the values of α, β and δ can be obtained by discretizing them and enumerating the parameters in multiple simulation runs.

Switchable2-II policy

1. Upon arrival, if the designated track is occupied by trains travelling in the opposite direction, the arriving train waits for the opposing moving trains to finish traveling on the segment before proceeding on its designated track.

2. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if $\alpha D_p + \beta S_{ar} < \delta$, the arriving train will start traveling on its designated track.

3. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if $\alpha D_p + \beta S_{ar} \geq \delta$, the arriving train will attempt to switch to the other track. The arriving train will use the reverse direction track only if it is empty. If the reverse direction track is occupied by trains travelling in either direction, the arriving train will use its designated track.

Switchable2-III is based on the Switchable2-II policy. For the first two policies, the attempted switching trains will switch if the reverse direction track is empty. The idea being that if another train is allowed to travel on the reverse direction track when one is already traveling on it may cause a significant amount of time that segment is blocked for a train traveling on its designated direction. However, in the case of multiple speeds, if the reverse direction track is occupied by a switched train, it will do no significant harm if we switch another faster train to the reverse direction track, given the latter switched train can catch up with the former switched train. To extend this idea, in the Switchable2-III policy, if the reverse direction track is occupied by a switched train, a newly arriving train can switch to the reverse direction track only if the arriving train will extend the busy period on the reverse direction track by no longer than μ time units. A good value of μ can be found by discretizing it and enumerating in multiple simulation experiments.

Switchable2-III policy

1. Upon arrival, if the designated track is occupied by trains travelling in the opposite direction, the arriving train waits for the opposing moving trains to finish traveling on the segment before proceeding on its designated track.

2. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if $\alpha D_p + \beta S_{ar} < \delta$, the arriving train will start traveling on its designated track.

3. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if $\alpha D_p + \beta S_{ar} \geq \delta$, the arriving train will attempt to switch to the other track. The arriving train will use the reverse direction track if it is empty. If the reverse direction track is occupied by trains travelling in the same direction as the arriving train and if the arriving train extends the current busy period on the reverse direction by no longer than μ time units, the arriving train will switch to the reverse direction track. Otherwise, the arriving train travels on its designated track.

2.2 Numerical experiments

Numerical experiments are conducted to compare the performance of the three dispatching policies. Simulation is used to compute the average delay of the various dispatching policies. The simulation model is built using Arena (Kelton et al., 2009). In the basic settings of the experiments, the arrival rate of the trains in one direction is 0.16 trains per minute and the length of the track segment is eight miles. There are five possible speeds for the trains. The speed of each arriving train is equally likely to be 50 m/h, 70 m/h, 90 m/h, 120 m/h and 140 m/h. The lengths of the trains travelling at speed of 50 m/h and 70 m/h are 5,000 feet and 6000 feet, respectively. The trains travelling at speed of 90 m/h, 120 m/h, and 140 m/h have lengths of 1000 feet. The reason the faster trains are shorter is that they are more likely to be passenger trains instead of freight trains. The safety headway between two consecutive trains is set to be one mile. For the numerical experiments, all the distance units are in miles and the time units are in minutes.

For the Switchable2-I policy, the optimal value of ω needs to be determined for each scenario. The possible values for ω range from 0 to $D / S_{sl} - D / S_{fa}$, where S_{sl} and S_{fa} denote the slowest and fastest train speeds possible on the track, respectively. In the numerical experiment, we discretize the value of ω into steps of 0.1 and enumerate all the possible values. The value of ω which gives the smallest average delay for all the trains is used in the Switchable2-I policy.

For the Switchable2-II policy, the values of α, β and δ need to be determined. Without loss of generality, the value of α can be fixed at 1 and a good assignment of values of β and δ can be obtained by discretization and enumeration. There are no obvious upper bounds for β and δ. In the experiment, the upper bound of β is set to 2 and the upper bound of δ is set to $1(D/S_{sl} - D/S_{fa}) + 2S_{fa}$. The best values of β and δ are always found to be far below their upper bounds. The best values of β and δ which produce the lowest average train delay are used by the Switchable2-II policy.

The values of α, β and δ in the Switchable2-III policy are determined the same way as in the Switchable2-II policy. The extra parameter μ has an upper bound of $(D + 1.136) / S_{sl}$ (where 1.136 accounts for the longest train length in unit of miles) and a lower bound of 0. The parameter μ is also discretized and enumerated together with the other two parameters.

Table 1 shows the average delays of the four policies when the arrival rate is 0.16 trains per minute. The average is based on 10 simulation runs. By choosing a good switching threshold value, the Switchable2-I policy is able to significantly reduce the average delay from the dedicated policy. However, the more complex switching condition function of Switchable2-II policy reduces the delay some more and the Switchable2-III policy is able to further reduce the average train delay. Compared to the dedicated policy, the best switchable policy, the Switchable2-III policy reduces the average train delay by 21%.

Table 1. Comparisons of the four policies

Arrival rate = 0.16	Average train delay	Standard deviation
Dedicated policy	0.9666	0.0013
Switchable2-I	0.8202	0.0011
Switchable2-II	0.7923	0.0012
Switchable2-III	0.7623	0.0012

Table 2 shows the delay of the fastest trains (e.g. high priority passenger trains) under different dispatching policies on the 8-mile long track segment. The Switchable2-III policy is able to reduce the delay under the dedicated policy by 0.484 minutes. Considering that the normal route length of the passenger train is much longer than 8 miles (e.g., in downtown Los Angeles area, the route length of passenger trains can be as high as 40 miles), the potential reduction of delay for passenger trains over their entire routes could be significant.

Table 2. Comparisons of four policies (fastest train delay)

Arrival rate = 0.16	Average train delay	Standard deviation
Dedicated policy	1.7178	0.0013
Switchable2-I	1.3855	0.0011
Switchable2-II	1.3313	0.0014
Switchable2-III	1.2335	0.0013

Figure 2 shows the performance of the four policies as the arrival rate varies. The relative performances between the four policies remain the same as seen in Table 1. As expected, the delay increases as the arrival rates increase but the gap between the dedicated policy and the switchable policy reduces at the increased rates since there is less opportunity for switching when there are more trains in the network. Figure 3 shows the performance of the four policies as the track length varies. As the figure shows, the average delay relationship with the track segment length is similar to the arrival rates since there is less opportunity for switching with longer segments assuming no crossovers within the segment.

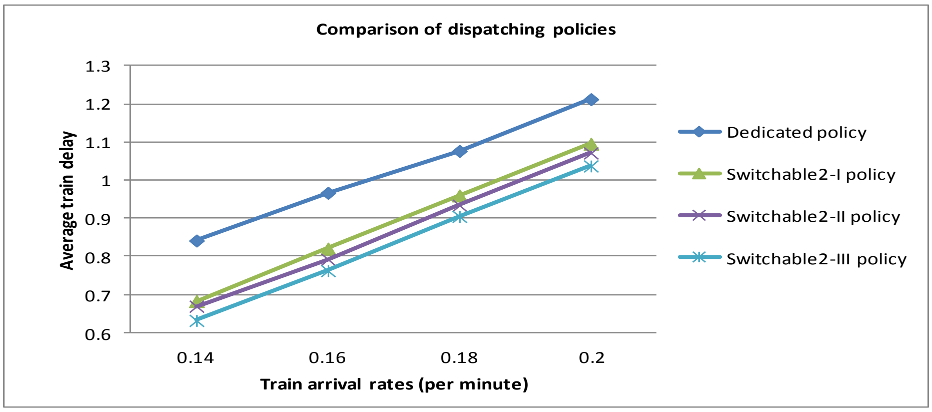

Figure 2. Varying arrival rates

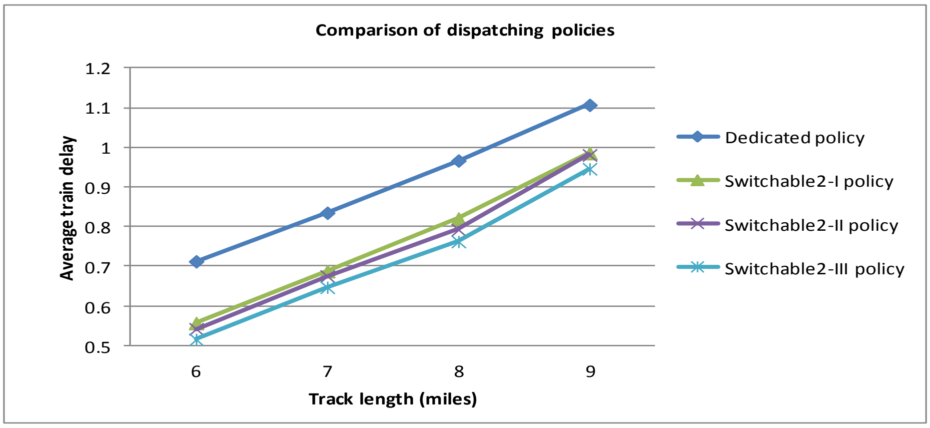

Figure 3. Varying track lengths

3. Dispatching rules for double-track segments with crossovers

3.1 Description of dispatching rule

Now suppose the double-track segment has a crossover in the middle of the segment. The introduction of the crossover at the middle can significantly increase the effectiveness of the switchable policy. With the help of the crossover, trains can switch to the other track at the beginning of the track and then switch back in the middle. Also, trains can switch to the other track in the middle of the track. In both ways, the switched trains do not have to travel through the entire segment of the other track. Thus the double track segment could be better utilized. Next, we are going to describe a switchable policy (namely, Switchable2-w/cross) which is based on the Switchable2-III policy. Treating the crossover in the middle of the segment as a station connecting two halves of the segment, the Switchable2-w/cross policy dispatches trains almost the same as what the Switchable2-III policy will do for those two double-track segments connected together. The Switchable2-w/cross policy is designed to dispatch trains with multiple speeds on a double-track segment with crossovers in the middle. The description of the Switchable2-w/cross policy below focuses on the eastbound trains and uses the notations in Figure 4. Let D_p^{EB1} denote the potential delay of the arriving train on track segment EB1. Let D_p^{EB2} denote the potential delay of the train on track segment EB2 as it arrives at EB2.

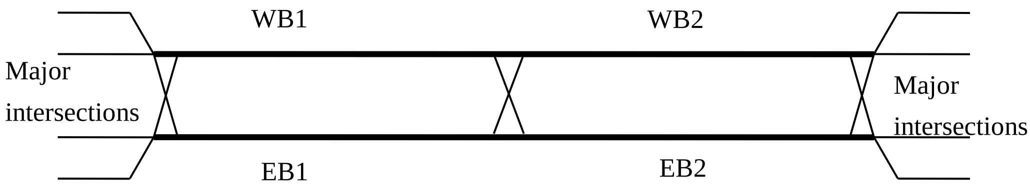

Figure 4. Double-track railroad segment with crossovers

<u>Switchable2-w/cross policy</u>

1. Upon arrival, if EB1 is occupied by trains travelling in the opposite direction, the arriving train waits for the opposing moving trains to finish traveling on the segment before proceeding on EB1.

2. Upon arrival, if EB1 is not occupied by trains travelling in the opposite direction, and if $\alpha_1 D_p^{EB1} + \beta_1 S_{ar} < \delta_1$, the arriving train will start traveling on EB1.

3. Upon arrival, if EB1 is not occupied by trains travelling in the opposite direction, and if $\alpha_1 D_p^{EB1} + \beta_1 S_{ar} \geq \delta_1$, the arriving train will attempt to switch to WB1. The arriving train will use WB1 if it is empty and if EB2 is not occupied by westbound trains. If WB1 is occupied by eastbound trains and if the arriving train extends the current busy period on WB1 by no longer than μ_1 time units, the arriving train will switch to WB1. Otherwise, the arriving train travels on EB1.

4. When an eastbound train reaches the end of track EB1, if EB2 is occupied by trains travelling in the opposite direction, the train at the end of EB1 waits for the opposing moving trains to finish traveling on EB2 before proceeding on EB2. When an eastbound train reaches the end of track EB1, if EB2 is not occupied by trains travelling in the opposite direction, and if $\alpha_2 D_p^{EB2} + \beta_2 S_{ar} < \delta_2$, the eastbound train will start traveling on EB2. But if $\alpha_2 D_p^{EB2} + \beta_2 S_{ar} \geq \delta_2$, the eastbound train will attempt to switch to WB2. The eastbound train will use WB2 if it is empty. If WB2 is occupied by eastbound trains and if the arriving train extends the current busy period on WB2 by no longer than μ_2 time units, the train at the end of EB1 will switch to WB2. Otherwise, the train travels on EB2.

5. When an eastbound train reaches the end of track WB1, if $\alpha_2 D_p^{EB2} + \beta_2 S_{ar} < \delta_2$, the eastbound train will start traveling on EB2. But if $\alpha_2 D_p^{EB2} + \beta_2 S_{ar} \geq \delta_2$, the eastbound train will attempt to continue to WB2. The eastbound train will use WB2 if it is empty. If WB2 is occupied by eastbound trains and if the train at the end of WB1 extends the current busy period on WB2 by no longer than μ_2 time units, the train at the end of WB1 will continue to WB2. Otherwise, the train travels on EB2.

3.2 Numerical experiments

In this numerical experiment, the length of the track is set to eight miles. The speed and length of each arriving train have the same characteristics as in Section 2.2. The crossover is located in the middle of the eight-mile long track segment. Suitable values of the parameters α_1, β_1, δ_1, μ_1, α_2, β_2, δ_2 and μ_2 in the Switchable2-w/cross policy can be obtained as in the Switchable2-III policy.

Figure 5 shows the average delay under both the dedicated policy and the Switchable2-w/cross policy. The results clearly show that the Switchable2-w/cross policy dominates the dedicated policy as the arrival rates vary. In this numerical experiment, the Switchable2-w/cross policy can reduce the average train delay by as high as 41.9%.

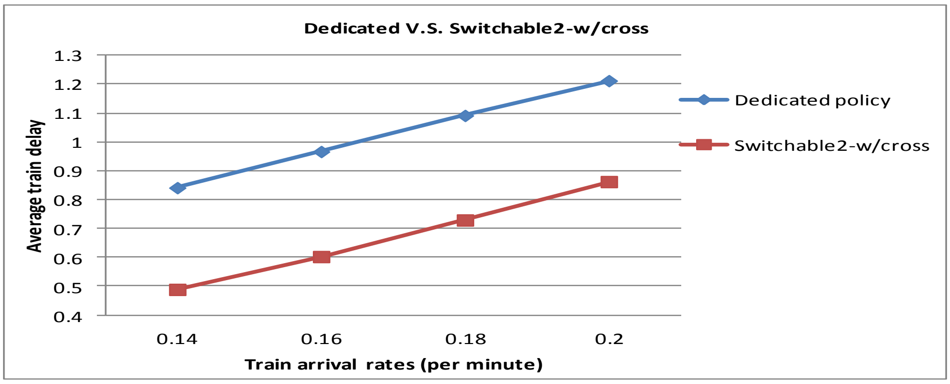

Figure 5. Dedicated vs. Switchable2-w/cross

4. Conclusions

In this paper, we focus on developing dispatching policies for double-track segments. As opposed to a regular dedicated policy, we propose several dispatching policies based on the idea of switching faster trains, that would catch up with slower trains, to the other track. We study three dispatching policies that allow trains to switch tracks when they are caught behind a slow moving train for the situation where trains travel at multiple speeds. The three dispatching policies are compared using simulation. In the simulation, we have considered the train lengths and safety headway length. The best switchable policy reduces the average train delay by 21%. With the existence of crossovers in the middle of the double-track segment, the Switchable2-w/cross policy can reduce the average train delay by as high as 41.9%.

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