Heuristic approach for the integrated inventory-distribution problem

Abstract

We study the integrated inventory distribution problem. We consider an environment in which the demand of each customer is relatively small compared to the vehicle capacity, and the customers are located closely such that a consolidated shipping strategy is appropriate. The model considers inventory holding, backorder, and transportation costs. We develop a heuristic procedure to obtain an approximate solution for this NP-hard problem and demonstrate its effectiveness through computational experiments.

1. Introduction

Recent decades have seen fierce competition in local and global markets, forcing manufacturing enterprises to streamline their logistic systems, as they comprise an important component of the final cost of goods. The major components of logistic costs are transportation costs, representing approximately one third, and inventory costs, representing one fifth (Buffa and Munn, 1989). The transportation and inventory cost reduction problems have been thoroughly studied separately; while, the integrated problem has recently attracted more interest in the research community as new ideas of centralized supply chain management systems, such as vendor managed inventory (VMI), have gained acceptance in many supply chain environments.

The integration of transportation and inventory decisions is represented in the literature by a general class of problems referred to as dynamic routing and inventory (DRAI) problems. As

defined by Baita *et al*. (1998), this class of problems is "characterized by the simultaneous vehicle routing and inventory decisions that are present in a dynamic framework such that earlier decisions influence later decisions." They classify the approaches used for DRAI problems into two categories. The first category operates in the frequency domain where the decision variables are replenishment frequencies, or headways between shipments. Examples in the literature include the work of Blumenfeld *et al*. (1985), Hall (1985), Daganzo (1987), and Ernst and Pyke (1993) (for more references see Daganzo, 1999).

The second category, referred to as the time domain approach, determines the schedule of shipments. With discrete time models, quantities and routes are decided at fixed time intervals. Within this category the most famous problem is the inventory routing problem (IRP), which arises in the application of the distribution of industrial gases. The main concern for this kind of application is to maintain an adequate level of inventory for all the customers and to avoid any stockout. In the IRP, it is assumed that each customer has a fixed demand rate and the focus is on minimizing the total transportation cost; while inventory costs are generally not of concern. Examples of this application in the literature include Bell *et al*. (1983), Golden *et al*. (1984), Dror *et al*. (1985), Dror and Ball (1987) and recently Campbell *et al*. (2002).

In this paper, we consider a DRAI problem that addresses the integrated inventory and vehicle routing decision problem in the time domain. This problem, referred to as the integrated inventory distribution problem (IIDP), considers multiple planning periods, both inventory and transportation costs, and a situation in which backorders are permitted. The kind of application that permits backorders is, of course, different from the distribution of industrial gas, where no shortage is allowed. Backorder decisions are generally justified in two cases. The first is when

there is insufficient vehicle capacity to deliver to a customer. The second case is when there is transportation cost saving that is higher than the incurred backorder cost by a customer.

In the literature, the integration between vehicle routing and inventory decisions with the consideration of inventory costs in the time domain approaches of the DRAI problems has taken different forms. In a few cases a single period planning problem has been addressed as found in Federgruen and Zipkin (1984) and Chien *et al*. (1989). In the multi-period problem, the decisions are conducted for a specific number of planning periods, or the problem is reduced to a single period problem by considering the effect of the long term decisions on the short term ones. Examples include Dror and Ball (1987), Trudeau and Dror (1992), Viswanathan and Mathur (1997), and Herer and Levy (1997).

Other researchers take into consideration various forms such as distributing perishable products (Federgruen *et al*., 1986), and the consideration of the time value of money for long-term planning (Dror and Trudeau, 1996). Some work focused on different structures of the distribution network such as Bard *et al*. (1998) in the case of satellite facilities, Chan and Simchi-Levi (1998) in the case where warehouses act as transshipment points in a 3-level distribution network, and Hwang (1999 and 2000) in the case of a multi-depot problem. Two papers dealt with the integrated production-distribution-inventory problem: Chandra and Fisher (1994) and Fumero and Vercellis (1999).

To the best of our knowledge the consideration of backorder and shortage costs in the multi-period planning problem is found only in one case in Herer and Levy (1997). However, they did not explicitly take backorder decisions into consideration in their solution approach as they impose a constraint that would never allow a delivery to a customer to be made after its inventory is consumed. In addition, they assume that customers with high demand rates are

treated separately. We introduce a new heuristic approach for solving the problem with backorders and benchmark it against lower and upper bounds found by a commercial software package, CPLEX, and a simple no inventory heuristic.

The rest of the article is organized as follows. In Section 2 we formulate the problem as a mixed integer program. The proposed heuristic is presented in Section 3. In Section 4 the experimental results are presented followed by the conclusion and directions for future research in Section 5.

2. Problem Description and Mixed Integer Programming Formulation

In the IIDP, we study a distribution system consisting of a depot, denoted 0, and geographically dispersed customers, indexed $1,\ldots,N$. Each customer i faces a different demand d_{it} per time period t (day/week). As traditionally considered, a single item does not restrict the problem to the case of a single product distribution, as the word 'item' can refer to a unit weight or volume of the distributed products and each customer can be viewed as a consumption center for packages of unit weight or volume (Daganzo, 1999). We consider the case in which the demand of each customer is relatively small compared to the vehicle capacity, and the customers are located closely such that a consolidated shipping strategy is appropriate. Deliveries to customers $1,\ldots,N$ are to be made by a capacitated heterogeneous fleet of V vehicles, each with capacity q_v starting from the depot at the beginning of each period. Each customer i maintains its own inventory up to capacity C_i and incurs inventory holding cost of h_i per period per unit and a backorder penalty of π_i per period per unit on the end of period inventory position. We assume that the depot has sufficient supply of items that can cover all customers' demands throughout the planning

horizon. The planning horizon considers T periods. Transportation costs include f_t a fixed usage cost per vehicle, which depends on the period t, and c_{ij} a variable transportation cost between i and j, which satisfies the triangular inequality. The objective is to minimize the overall transportation, inventory holding and shortage costs incurred over a specific planning horizon. We consider an integer variable x^v_{ijt}, which equals 1 if vehicle v travels from i to j in period t, and 0 if it does not. The amount transported on that trip is represented by y^v_{ijt}. At customer i, the inventory at time t is I_{it} and the backorder at time t is B_{it}. The following is a mixed integer programming formulation for the problem.

[IIDP] – *Integrated inventory distribution problem*

$$\text{Min} \sum_{t=1}^{T}\left[\sum_{j=1}^{N}\sum_{v=1}^{V} f_t x^v_{0jt} + \sum_{i=0}^{N}\sum_{\substack{j=0 \\ j\neq i}}^{N}\sum_{v=1}^{V} c_{ij} x^v_{ijt} + \sum_{i=1}^{N}\left(h_i I_{it} + \pi_i B_{it}\right)\right]$$

subject to:

$$\sum_{\substack{j=0 \\ j\neq i}}^{N} x^v_{ijt} \leq 1 \qquad\qquad i = 0, \ldots, N, \; t = 1, \ldots, T \text{ and } v = 1, \ldots, V \quad (1)$$

$$\sum_{\substack{k=0 \\ k\neq i}}^{N} x^v_{ikt} - \sum_{\substack{l=0 \\ l\neq i}}^{N} x^v_{lit} = 0 \qquad\qquad i = 0, \ldots, N, \; t = 1, \ldots, T \text{ and } v = 1, \ldots, V \quad (2)$$

$$y^v_{ijt} - q_v x^v_{ijt} \leq 0 \qquad i = 0, \ldots, N, \; j = 0, \ldots, N, \; i\neq j, \; t = 1, \ldots, T \text{ and } v = 1, \ldots, V \quad (3)$$

$$\sum_{\substack{k=0 \\ k\neq i}}^{N} y^v_{ikt} - \sum_{\substack{l=0 \\ l\neq i}}^{N} y^v_{lit} \leq 0 \qquad\qquad i = 1, \ldots, N, \; t = 1, \ldots, T \text{ and } v = 1, \ldots, V \quad (4)$$

$$I_{it-1} - B_{it-1} - I_{it} + B_{it} + \sum_{v=1}^{V}\left(\sum_{\substack{l=0 \\ l\neq i}}^{N} y^v_{lit} - \sum_{\substack{k=0 \\ k\neq i}}^{N} y^v_{ikt}\right) = d_{it} \quad i = 1, \ldots, N \text{ and } t = 1, \ldots, T \quad (5)$$

$$I_{it} \leq C_i \qquad\qquad\qquad\qquad i = 1, \ldots, N \text{ and } t = 1, \ldots, T \qquad\qquad (6)$$

$$I_{it} \geq 0 \qquad\qquad\qquad\qquad i = 1, \ldots, N \text{ and } t = 1, \ldots, T \qquad\qquad (7)$$

$$B_{it} \geq 0 \qquad\qquad\qquad\qquad i = 1, \ldots, N \text{ and } t = 1, \ldots, T \qquad\qquad (8)$$

$y^v_{ijt} \geq 0$ $i = 0, \ldots, N$, $j = 0, \ldots, N, i \neq j$, $t = 1, \ldots, T$ and $v = 1, \ldots, V$ (9)

$x^v_{ijt} = 0$ or 1, $i = 0, \ldots, N$, $j = 0, \ldots, N, i \neq j$, $t = 1, \ldots, T$ and $v = 1, \ldots, V$ (10)

The objective function includes transportation costs and inventory holding and shortage costs on the end inventory position. Constraints (1) make sure that a vehicle will visit a location no more than once in a time period, and constraints (2) ensure route continuity. Constraints (3) serve for two purposes. The first one is to ensure that the amount transported between two locations will always be zero whenever there is no vehicle moving between these locations, and the second is to ensure that the amount transported is less than or equal to the vehicle's capacity. Constraints (4) are necessary to eliminate sub-tours. Constraints (5) are the inventory balance equations for the customers. Constraints (6) limit the inventory level of the customers to the corresponding storage capacity. It is assumed that the amount consumed by each customer in a given period is not kept in the customer's storage location; accordingly, it is not accounted for in constraints (6). Constraints (7) to (10) are the domain constraints.

3. Approximate Transportation Costs Heuristic

The integrated inventory and routing problem IIDP is NP-hard as it includes the vehicle routing problem (VRP). We therefore propose a constructive heuristic that provides a good solution in a reasonable time. Solution heuristics that have been proposed in the literature for the different variations of the integrated inventory-distribution problem, particularly the inventory routing problem, are either based on subgradient optimization of a Lagrangian relaxation (see Bell *et al.*, 1984 and Chien *et al.*, 1989) or a constructive procedure. The constructive heuristics are broadly classified into heuristics that allocate customers to service days and then solve a VRP to generate

vehicle routes for each day (Dror and Ball, 1987); and heuristics that allocate customers to days and vehicles and then solve a traveling salesman problem for every assignment (Dror *et al.*, 1985).

The constructive heuristic we propose here is of this later type. These strategies are mostly used when the inventory routing problem is not allowed backorders and the inventory holding costs are negligible. The consideration of inventory holding and shortage costs in the IIDP demands a modification to this strategy to explore the tradeoffs between transportation, inventory, and backorder costs among customers.

3.1. *Algorithm Description*

Although the IIDP problem in question has two types of capacity constraints, storage limit at the customer and vehicle capacity, the main idea behind the proposed heuristic is inspired by the optimal policies for the uncapacitated lot-sizing problem (Silver *et al.*, 1998). The guiding principles for the uncapacitated case can motivate an effective heuristic for the IIDP, especially when the demand of each customer per period is small relative to the capacities (although the total demand across all customers could exceed the capacity). These guiding principles are (1) deliveries are only made when the customer's inventory reaches zero, and (2) if inventory to a particular customer is carried over to the next period, there will be no delivery in the next period.

The main steps that our Approximate Transportation Costs Heuristic (ATCH) takes to decide what should be delivered on period t are the following: First, for every customer i that needs delivery in period t and every period $\tau \geq t$ whose demand could be serviced, we construct an estimate of the transportation cost values ($TR_{i,\tau}$). This estimate, which is described in Subsection 3.3, corresponds to the cost reduction obtained by removing a customer from the delivery tour.

Then the values $TR_{i,\tau}$ are compared to the inventory holding and shortage costs that result by adding or subtracting quantities to the day t delivery of each customer. Finally, after deciding the delivery for each customer, a VRP is solved using a savings algorithm (Clarke and Wright, 1964) with these updated delivery amounts. We note that any efficient solution technique for the VRP can be used at the last step.

Note that we only estimate future transportation costs for customers that require a delivery in period t. Thus, the solution we obtain will only consider deliveries to clients that have an inventory that reaches zero in the beginning of that period (and has positive demand), in agreement with our first guiding principle. If we assume that an individual customer demand is small compared to vehicle capacity, it is rare that satisfying a future demand to a customer will saturate the vehicle capacity. Thus when profitable, the solutions obtained by ATCH will tend to completely satisfy future demand, in agreement with the second guiding principle.

The comparison between the estimated transportation costs $TR_{i,\tau}$ and inventory holding and shortage costs is separated into deciding whether to have backorders on period t and whether to use excess capacity in the vehicles to cover future customer demand.

Backorders can be profitable for two reasons, it is either cheaper to pay the backorder cost than the transportation cost, or there is insufficient capacity in the vehicles to satisfy demand. Let $\delta_{i,t} = d_{i,t} - I_{i,t-1} + B_{i,t-1}$ be the outstanding demand at customer i at the beginning of period t, and let ND be the set of customers that have $\delta_{i,t} > 0$. The following problem decides whether to deliver to customer i in period t or not ($z_i = 1$ or 0 respectively) and the quantity r_i to deliver such that the sum of backorder cost and estimated transportation cost is minimized and vehicle capacity constraints are satisfied.

[SUB1] – *Backorder decision sub-problem*

$$\text{Min} \sum_{i \in ND} \left[\pi_i \left(\delta_{i,t} - r_i \right) + TR_{i,t} \, z_i \right]$$

Subject to:

$$\sum_{i \in ND} r_i \leq \sum_{v=1}^{V} q_v \qquad\qquad\qquad (11)$$

$$r_i \leq \delta_{i,t} \, z_i \qquad\qquad \forall i \in ND \quad (12)$$

$$r_i \geq 0 \qquad\qquad\qquad \forall i \in ND \quad (13)$$

$$z_i = 0 \text{ or } 1. \qquad\qquad \forall i \in ND \quad (14)$$

Constraint (11) ensures that we do not exceed the total vehicle capacity of period t, and Constraint (12) enforces that we deliver at most the outstanding demand and only to clients included in the delivery in period t.

Let us now turn to the problem of deciding whether to allocate extra vehicle capacity in period t to meet future customer demand. Here we only consider meeting future demand of customers that have a delivery in period t, in agreement with the first guiding principle. Consider the integer variable $u_{i\tau}$ to decide whether to deliver customer i's demand for period τ in the current period t. Let DL represent the total remaining vehicle capacity, i.e. DL $= \sum_{v=1}^{V} q_v - \sum_{i \in ND} r_i$, and let TL_i be the latest period where customer's i demand can be satisfied, i.e.

$TL_i = \min \left\{ \underset{TL}{\arg\max} \left(\sum_{\tau=t+1}^{TL} d_{i\tau} \leq C_i \right), T \right\}$. The following problem decides whether to include future demand for any customer in the current delivery by minimizing the total transportation and inventory costs and satisfying capacity limits. As indicated before, this decision is made only for the customers that need delivery in day t. This part is formulated as follows:

[SUB2] – *Inventory decision sub-problem*

$$\text{Max} \sum_{i \in ND} \sum_{\tau=t+1}^{TL_i} \left[TR_{i,\tau} - (\tau - t) h_i \, d_{i\tau} \right] u_{i\tau}$$

Subject to:

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$$\sum_{i \in ND} \sum_{\tau=t+1}^{TL_i} d_{i\tau} u_{i\tau} \leq DL \tag{15}$$

$$u_{i\tau-1} \geq u_{i\tau} \qquad \tau = t+1,\ldots, TL_i, \quad \forall i \in ND \tag{16}$$

$$u_{i\tau} = 0 \text{ or } 1. \qquad \tau = t+1,\ldots, TL_i, \quad \forall i \in ND \tag{17}$$

Constraint (15) represents both the available vehicle capacity and customers' storage limits. For simplification, the customers' storage limits are represented by the time index (TL_i), which is computed in advance. In addition, the precedence constraints (16) are added to represent the fact that future demand in a certain day is to be considered only if the customer's preceding day demand is fulfilled.

By solving SUB1 and SUB2, the algorithm decides how much to deliver to each customer in day t. The used delivery routes are actually computed by solving a VRP. The flow chart in Fig. 1 summarizes the major steps of the proposed heuristic ATCH.
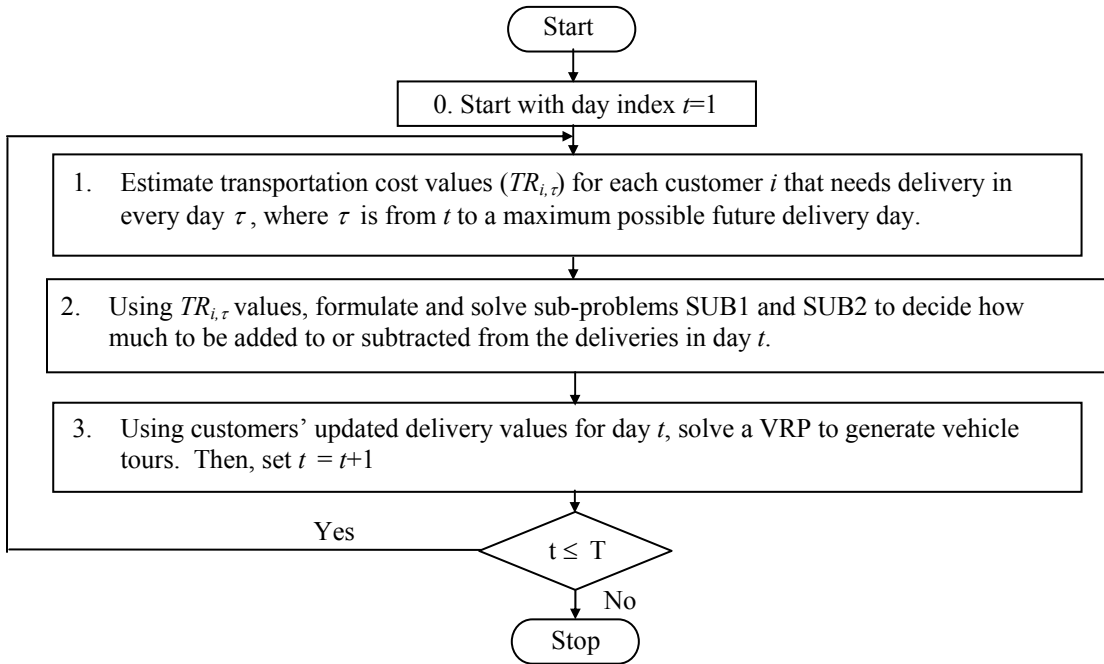

Fig. 1. An outline of ATCH

The following subsection provides the algorithmic solutions for both sub-problems used to decide the amount of delivery to each customer and related analysis.

3.2. *Solving Sub-Problems*

We present the following result that characterizes optimal solutions to sub-problem SUB1.

***Proposition* 1**. There is an optimal solution to SUB1 that only makes deliveries to customer i if the quantity delivered satisfies $r_i > TR_{i,t} / \pi_i$. Also, every optimal solution to SUB1 only makes deliveries if $r_i \geq TR_{i,t} / \pi_i$.

Proof. Assume that in the optimal solution to SUB1, some customer i is delivered r_i that satisfies $r_i \leq TR_{i,t} / \pi_i$, or equivalently $\pi_i(\delta_{i,t} - r_i) + TR_{i,t} \geq \pi_i \delta_{i,t}$. If we consider the modified solution obtained by setting $z_i = r_i = 0$, then the previous inequality shows that the modified solution, which is feasible, is at least as good as the optimal solution. In the case when $r_i < TR_{i,t} / \pi_i$, then the modified solution is strictly better. Thus, the original solution cannot be optimal. □

Based by this result, we construct an efficient feasible solution to SUB1 by guaranteeing that it only makes deliveries when the amount delivered satisfies $r_i > TR_{i,t} / \pi_i$. We develop a greedy algorithm that assigns delivery quantities to potential customers in the order of their π_i values. The algorithm is inspired by the following fractional knapsack problem obtained by considering deliveries to all customers and replacing variables r_i with $w_i = r_i/\delta_{i,t}$ in SUB1:

$$\text{Max } \sum_{i \in ND} \pi_i \delta_{i,t} w_i$$

Subject to:

$$\sum_{i \in ND} \delta_{i,t} w_i \leq \sum_{v=1}^{V} q_v$$

$$w_i \leq 1 \qquad \forall i \in ND$$

The optimal solution for this fractional knapsack problem is obtained by a greedy algorithm with customers sorted by their π_i values. Accordingly, the following is the algorithm that constructs an efficient solution to SUB1:

Procedure SUBALG1
1. Remove customers that have $\delta_{i,t} \leq TR_{i,t} / \pi_i$ from set ND;
2. Let $\Delta Q = \sum_{i \in ND} \delta_{i,t} - \sum_{v=1}^{V} q_v$;
3. Sort customers in set ND in an increasing order of π_i values;
4. For each customer i in the ordered set ND do
   If $\Delta Q \geq \delta_{i,t}$ then let $\Delta Q = \Delta Q - \delta_{i,t}$, $r_i = 0$, remove i from set ND;
   If $\delta_{i,t} > \Delta Q > 0$ then
      If $\delta_{i,t} - \Delta Q > TR_{i,t} / \pi_i$, let $r_i = \delta_{i,t} - \Delta Q$, $\Delta Q = 0$;
      Else let $\Delta Q = 0$, $r_i = 0$, remove i from set ND;
   If $\Delta Q \leq 0$ then let $r_i = \delta_{i,t}$ for all unassigned i in set ND, STOP;
Continue;

The sub-problem SUB2 is a precedence constrained knapsack problem (PCKP) which is known to be NP-hard (Garey and Johnson, 1979). However, Johnson and Niemi (1983) provide a dynamic programming algorithm for the PCKP that can solve the problem in a pseudo-polynomial time, given that the underlying precedence graph is a tree, which is fortunately a property of SUB2. To illustrate this property, consider the sample case for SUB2 illustrated in Fig. 2. The decision variables $u_{i\tau}$ are represented by directed arcs, where the cost saving associated with each arc $S_{i\tau-t} = TR_{i,\tau} - (\tau - t) h_i d_{i\tau}$. A solid vertical line is drawn to represent the time limit TL_i for customer i. Starting from node 0, arcs are to be selected using the order given by their directions, such that the total cost saving is maximized and the given capacity constraint is not violated.
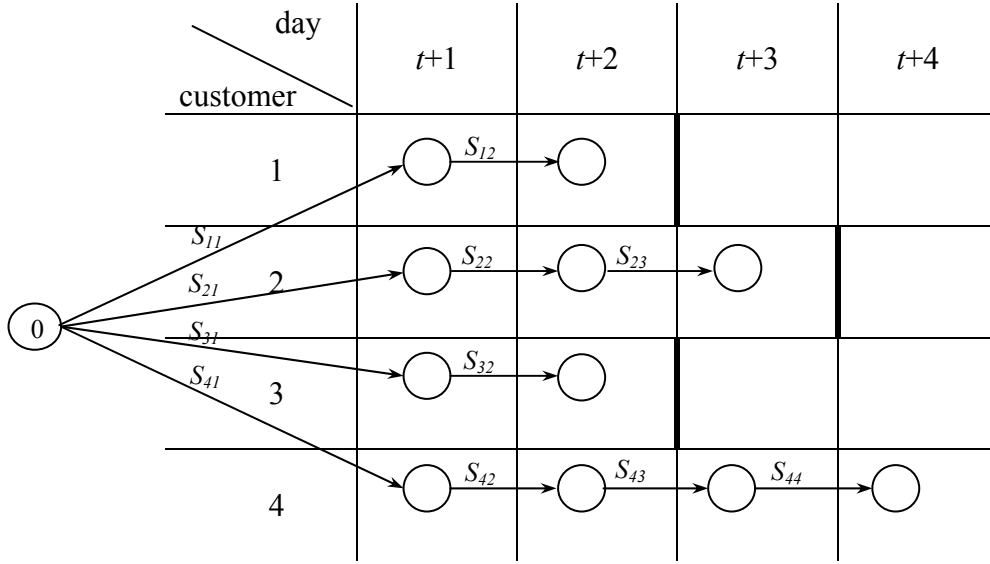
Fig. 2. Tree property of precedence constraints in a sample SUB2 problem

We present here a simpler algorithm based on a greedy search that selects the next possible arc that has the maximum positive saving. This algorithm does not guarantee optimality to the solution of SUB2; however, it can produce relatively good solutions in polynomial time. The following steps describe the algorithm.

Procedure SUBALG2
   1. For every customer i in set ND, Let $\Delta t_i = 1$;
   2. Find customer j in set ND that has the largest positive value of
      $\left(TR_{j,t} - \Delta t_j \, h_j \, d_{j,t+\Delta t_j} \right)$; If none found then STOP;

   3. If $DL \geq d_{j,t+\Delta t_j}$ then

      Let $DL = DL - d_{j,t+\Delta t_j}$;

      Add $d_{j,t+\Delta t_j}$ to customer j's delivery amount;

      Let $\Delta t_j = \Delta t_j + 1$;
      If $\Delta t_j > TL_j$ then remove customer j from set ND;
   End-If
   Else remove customer j from set ND;
   4. If $ND = \emptyset$ then STOP; Else go to step 2.

Obviously, the optimal solutions to the sub-problems depend significanly on the estimated transportation cost values. The following subsection discusses an appropriate method to calculate these estimates.

3.3. *Estimating Individual Customer Transportation Cost*

An appropriate method to estimate the individual customer transportation cost values ($TR_{i,\tau}$) is to calculate the cost reduction that will result when customer i is excluded from the vehicle tour that includes it, given the VRP solutions. The VRP solutions need to be obtained for the current day t and for all future days for which customers' demands can be covered in day t while considering capacity constraints. A suitable heuristic such as the Savings algorithm can be used to generate efficient VRP tours using the appropriate demand values for each day studied. However, if the summation of customers' demands is found to exceed the available vehicle capacity, customers with the lowest unit shortage costs are assigned transportation cost values equal to the transportation costs incurred by a direct shipment from the depot.

The transportation cost estimates must be repeatedly updated during the course of the algorithm. In particular, in SUBALG1 the estimation of transportation costs has to be updated every time a customer is removed in steps 1 and 4. Similarly, in algorithm SUBALG2, after each arc selection made in step 3, the $TR_{i,\tau}$ values for the remaining customers in the corresponding day of the selected arc have to be recalculated for the next iteration's comparison. This recalculation is not expensive in terms of computational time, as it does not necessarily require resolving a VRP.

4. Experimentation and Results

Two versions of the ATCH have been programmed. The first one uses a dynamic programming algorithm to solve sub-problem SUB2 optimally, and is referred to as ATCH-DP. The second version uses the provided greedy-search algorithm instead to solve for SUB2, and is referred to as ATCH-G. We benchmark these heuristics against a simple heuristic that does not allow any inventory to be carried from each day. That is, in each period the day's demand is shipped to each customer if there is sufficient capacity. This heuristic, referred to as MPVRP, represents a solution to a multi-period VRP, where backorder decisions are taken only if the summation of the customers' demands in a given period is found to be greater than the available vehicle capacity, and the backorder decisions follow the same approach of the heuristic ATCH. The results of the MPVRP are intended to illustrate the benefit of the inventory decisions made by the ATCH heuristics. These heuristics are benchmarked against the lower and upper bounds obtained by CPLEX 8.1 with a maximum running time of 60 minutes using an Intel Pentium 4 processor running with a clock speed of 2.53 GHz. The *presolve* option of CPLEX is enabled to exploit the initial efficient cuts added by CPLEX to improve the obtained lower bounds.

4.1. *Experimental Design*

We assume that customers are allocated in a square of 20×20 distance units and their coordinates are generated using a uniform distribution within these limits. The depot is located in the middle of the square.

We generate random test problems where backorder decisions are economical, so that the backorder decisions of the heuristic ATCH are assessed. That is, we set the parameter values so it is optimal to carry backorders. Customers' unit holding costs are generated using a normal

distribution with a mean of 0.1 and a standard deviation of 0.02, and each customer has a storage capacity of 120 items. The transportation cost per unit distance is set to 2, the customers' unit shortage costs are generated using a normal distribution with a mean of 3 and a standard deviation of 0.5, and the customers' demands are generated using a uniform distribution from 5 to 50 items per day.

Sixty test problems have been generated by varying the number of customers (N), the number of planning periods (T) and the number of homogenous vehicles (V). We generate three levels of N (5, 10 and 15), two levels of T (5 and 7), and two levels of V (1 and 2). For each combination of N, T, and V, we randomly generate five problems. The total vehicle capacity is selected to be fixed at 150, 300, and 450 for each level of N, respectively.

The naming convention of the test problems starts with the letters 'IIDP', followed by two digits for the number of customers. The third digit represents the length of the planning horizon, and the fourth digit represents the number of vehicles. Finally, the replicate number is given at the last digit, and is separated from the former digits by a hyphen. Thus, the problem IIDP0551-1 represents the first randomly generated test problem with 5 customers, a planning horizon of 5 periods and 1 vehicle.

4.2. Results and Discussion

The results of the experiments are listed in Table 1. The table lists the total cost, the inventory holding, shortage and transportation costs of the three heuristics along with the CPLEX lower and upper bounds. An * next to the lower bound in the tables indicates that CPLEX was able to find the optimal solution within the one hour time limit.

The percentage differences between the total cost obtained by each heuristic and the lower and upper bounds are used as performance indicators. These percentage differences are calculated by taking the ratio of the difference between the heuristic's total cost and the bound to the heuristic's total cost. A comparison against the CPLEX lower bound provides some measure of deviation from optimality. A comparison against the CPLEX upper bound provides some benchmark of the heuristic against solving the problem using a general purpose commercial package. Accordingly, a negative upper bound (UB) percentage difference is an indicator that the heuristic outperforms the general purpose package. These percentage differences and the computational times (in minutes) for the three heuristics are listed in Table 2.

The computational times are the actual CPU times recorded by running the heuristics on an Intel Pentium 4 processor, running at a clock speed of 2.53 GHz. In some cases, the table lists a value of 0.00 for the CPU time which means the time to compute the solution was negligible. The computational times for the ATCH-G and MPVRP heuristics are less than a second in all test problems; whereas for heuristic ATCH-DP the computational time increased with the problem size to a few seconds, due to the pseudo-polynomial part of the algorithm.

Only for the small problem sizes (i.e., for some of the cases with 5 customers) was CPLEX able to find the optimal solution within the one hour time limit. Of the three heuristics (ATCH-DP, ATCH-G, and MPVRP), ATCH-DP provided the lowest costs in most cases with the simple no inventory heuristic performing the worst. The performance of the two ATCH heuristics was close to one another suggesting that the greedy search to solve SUB2 is efficient. A comparison between the CPLEX lower and upper bounds shows that the deviation from the lower bounds for the ATCH heuristics does not significantly increase as the size of the problem increases and that initially for the small problems the CPLEX upper bound outperforms the heuristics. However,

for the larger problems the ATCH heuristics significantly outperform the CPLEX upper bound. This is best illustrated by examining Fig. 3.

In this figure, the heuristic solution percentage difference with the lower bound for the three heuristics and the CPLEX upper bound are represented graphically. These measures are plotted against the number of binary variables in the IIDP formulation as a representation for the problem size. Each point in the graph represents the average of the bounds percentage differences calculated for the five replications of each problem combination.

As illustrated in Fig. 3, the lower bound percentage differences for both the ATCH-DP and the ATCH-G lie below 20% for small sized problems and stay within this level for larger problems. As the figure illustrates, the ATCH heuristics rapidly outperform the CPLEX upper bound as the problem size increases. The performance of the ATCH-DP is better than the ATCH-G in small sized problems. However, with the increase of the problem size, their results appear to be converging.

Table 1. Detailed costs for the test problems

Problem	CPLEX bounds		ATCH-DP				ATCH-G				MPVRP			
	UB	LB	Hold.	Short.	Transp.	Total	Hold.	Short.	Transp.	Total	Hold.	Short.	Transp.	Total
IIDP0551-1	649.8	649.8*	4.44	148	535	687.83	2.43	197.7	502	702.09	0	304	531	835.25
IIDP0551-2	468	468*	5.46	54.8	477	537.27	5.46	54.81	477	537.27	0	151	525	676.2
IIDP0551-3	400	400*	42.7	16.2	348	406.85	13.37	42.7	374	430.07	0	42.7	425	467.7
IIDP0551-4	475.29	475.29*	5.11	19.8	451	475.95	5.68	19.84	455	480.52	0	39.7	490	529.68
IIDP0551-5	426.01	426.01*	6.75	67.1	408	481.87	8.34	67.12	408	483.46	0	105	540	645.16
IIDP0571-1	522.97	522.97*	19.65	0	621	640.65	31.57	22.8	580	634.37	0	0	854	854
IIDP0571-2	557.89	557.89*	21.49	129	430	580.81	14.59	156.9	439	610.47	0	129	532	661.32
IIDP0571-3	434.86	434.86*	12.86	37.1	461	510.92	12.86	37.06	461	510.92	0	37.1	553	590.06
IIDP0571-4	536.42	536.42*	22.7	56.4	568	647.07	26.32	48.75	571	646.07	0	15.1	770	785.12
IIDP0571-5	498.08	498.08*	12.62	33.6	536	582.22	18.92	33.6	536	588.52	0	15.8	651	666.84
IIDP0552-1	522.82	509	9.13	0	541	550.13	11.53	0	552	563.53	0	0	604	604
IIDP0552-2	940.47	933.76	0	399	593	991.78	0	398.8	593	991.78	0	399	593	991.78
IIDP0552-3	512.44	497.98	16.04	14.5	548	578.56	15.27	39.84	583	638.11	0	0	684	684
IIDP0552-4	537.37	519.91	4.94	26.4	524	555.34	9.01	26.4	546	581.41	0	28.6	560	588.6
IIDP0552-5	553.2	536.52	5.96	0	571	576.96	5.96	0	571	576.96	0	18.4	650	668.41
IIDP0572-1	828.6	789.04	5.81	70.3	904	980.06	5.81	70.25	904	980.06	0	101	933	1034.2
IIDP0572-2	988.31	943.43	2.92	98.9	941	1042.9	2.92	98.94	941	1042.8	0	128	976	1104
IIDP0572-3	864.23	793.38	5.04	55	900	960.04	5.04	55	900	960.04	0	90	965	1055
IIDP0572-4	786.53	738.55	11.47	64.6	860	936.11	13.87	64.64	859	937.51	0	121	933	1054.3
IIDP0572-5	771.35	728.76	20.18	58.2	852	930.36	17.98	58.18	859	935.16	0	41.4	1052	1093.4
IIDP1051-1	528.69	509.59	21.58	19.3	532	572.84	21.58	19.26	532	572.84	0	0	700	700
IIDP1051-2	487.7	423.78	47.26	0	448	495.26	43.41	0	472	515.41	0	2.09	610	612.09
IIDP1051-3	724.13	660.23	3.6	142	553	698.7	3.6	142.1	553	698.7	0	169	565	734.05
IIDP1051-4	456	445.86	21.22	0	442	463.22	18.7	0	456	474.7	0	44.3	530	574.31
IIDP1051-5	591.03	546.62	33.61	27.8	546	607.43	19.26	27.82	582	629.08	0	51.4	675	726.36
IIDP1071-1	784.36	728.48	35.77	47.8	731	814.52	35.33	47.75	731	814.08	0	33	861	894
IIDP1071-2	842.4	730.1	35.37	37.7	758	831.05	37.67	37.68	753	828.35	0	37.7	875	912.68
IIDP1071-3	748.65	668.8	72.15	0	697	769.15	54.83	0	753	807.83	0	77.7	959	1036.7
IIDP1071-4	897.24	799.72	65.85	27.7	795	888.57	57.8	13.86	865	936.66	0	27.7	1155	1182.7
IIDP1071-5	763.69	712.32	36.19	19.6	724	779.75	27.63	19.56	778	825.19	0	32.6	889	921.6
IIDP1052-1	829.24	758.39	10.77	105	715	831.04	10.77	105.3	715	831.04	0	95.4	797	892.39
IIDP1052-2	676.22	566.94	35.34	0	662	697.34	52.66	0	694	746.66	0	158	787	945.18
IIDP1052-3	759.4	659.22	19.28	26.4	724	769.63	20.58	26.35	728	774.93	0	22.9	830	852.9
IIDP1052-4	630.89	509.46	26.17	0	669	695.17	26.8	0	667	693.8	0	5.46	699	704.46
IIDP1052-5	799.24	718.07	8.5	46.9	742	797.42	6.41	46.92	761	814.33	0	46.9	802	848.92
IIDP1072-1	955.63	808.46	37.19	43.9	932	1013.1	40.69	69.8	893	1003.4	0	126	1201	1326.6
IIDP1072-2	1266.9	1029.21	31.53	99.8	1144	1275.3	31.53	99.81	1144	1275.3	0	169	1339	1508.4
IIDP1072-3	1037.6	857.06	44.7	48.1	1026	1118.8	55.12	99.63	975	1129.7	0	0	1216	1216
IIDP1072-4	1135.9	896.78	39.79	45.2	1088	1173	55.41	45.18	1077	1177.5	0	155	1170	1325.3
IIDP1072-5	938.11	750.97	27.41	35	882	944.37	29.88	34.96	887	951.84	0	261	943	1203.9
IIDP1551-1	823.1	736.42	22.01	45.2	739	806.16	20.97	45.15	757	823.12	0	45.2	830	875.15
IIDP1551-2	781.1	725.42	6.8	0	779	785.8	12.67	0	763	775.67	0	36.9	835	871.86
IIDP1551-3	800.63	666.94	20.23	77.2	643	740.41	20.79	77.18	644	741.97	0	169	765	934.47
IIDP1551-4	739.67	608.49	39.51	0	675	714.51	38.72	0	691	729.72	0	0	785	785
IIDP1551-5	1012.9	971.66	3.8	195	834	1033.3	3.78	182.7	852	1038.4	0	366	851	1217
IIDP1571-1	1095.1	747.3	80.96	48.2	813	942.16	91.35	0	857	948.35	0	219	1109	1328.1
IIDP1571-2	1097.7	660.8	85.18	0	752	837.18	63.73	0	825	888.73	0	338	1072	1409.8
IIDP1571-3	1217.2	800.45	94.91	0	929	1023.9	106.83	0	931	1037.8	0	0	1281	1281
IIDP1571-4	1095.3	803.99	86.96	0	917	1004	83.15	20.1	950	1053.2	0	233	1274	1506.8
IIDP1571-5	1383.8	1130.8	27.53	211	999	1237.4	38.34	210.9	998	1247.2	0	207	1169	1376.2
IIDP1552-1	924.1	620.96	39.04	14.6	795	848.64	56.67	14.6	761	832.27	0	185	914	1098.8
IIDP1552-2	818.36	595.9	39.02	0	707	746.02	50.05	25.28	682	757.33	0	40.3	855	895.32
IIDP1552-3	1103.7	923.58	55.26	22.5	888	965.73	56.45	0	912	968.45	0	10.1	1081	1091.1
IIDP1552-4	1086.4	923.82	14.05	101	937	1052	14.05	101	937	1052.0	0	129	985	1113.8
IIDP1552-5	1125.5	729.65	30.91	0	902	932.91	31.11	0	909	940.11	0	33.9	1009	1042.9
IIDP1572-1	1375.1	881.63	52.65	19.1	1118	1189.7	87.65	0	1108	1195.6	0	92	1277	1369
IIDP1572-2	1415.2	972.09	22.55	20.6	1146	1189.1	25.87	23.14	1152	1201.0	0	113	1220	1333.1
IIDP1572-3	1768.9	1042.43	67.25	21.1	1221	1309.4	60.24	21.12	1275	1356.3	0	111	1526	1636.9
IIDP1572-4	1328.7	920.04	83.92	0	1150	1233.9	106.63	0	1157	1263.6	0	0	1566	1566
IIDP1572-5	1575.2	1117.42	20.6	116	1199	1335.8	21.16	116.2	1207	1344.3	0	194	1257	1450.6

* Optimal solution found

Table 2. Lower and upper bounds percentage differences and computational time results

| Problem	ATCH-DP			ATCH-G			MPVRP		
	LB diff. %	UB diff. %	Time (min.)	LB diff. %	UB diff. %	Time (min.)	LB diff. %	UB diff. %	Time (min.)
IIDP0551-1	5.5	5.5	0.00	7.4	7.4	0.00	22.20	22.20	0.00
IIDP0551-2	12.9	12.9	0.00	12.9	12.9	0.00	30.79	30.79	0.00
IIDP0551-3	1.7	1.7	0.00	7.0	7.0	0.00	14.48	14.48	0.00
IIDP0551-4	0.1	0.1	0.00	1.1	1.1	0.00	10.27	10.27	0.00
IIDP0551-5	11.6	11.6	0.00	11.9	11.9	0.00	33.97	33.97	0.00
IIDP0571-1	18.4	18.4	0.00	17.6	17.6	0.00	38.76	38.76	0.00
IIDP0571-2	3.9	3.9	0.00	8.6	8.6	0.00	15.64	15.64	0.00
IIDP0571-3	14.9	14.9	0.00	14.9	14.9	0.00	26.30	26.30	0.00
IIDP0571-4	17.1	17.1	0.00	17.0	17.0	0.00	31.68	31.68	0.00
IIDP0571-5	14.5	14.5	0.00	15.4	15.4	0.00	25.31	25.31	0.00
IIDP0552-1	7.5	5.0	0.00	9.7	7.2	0.00	15.73	13.44	0.00
IIDP0552-2	5.9	5.2	0.00	5.9	5.2	0.00	5.85	5.17	0.00
IIDP0552-3	13.9	11.4	0.00	22.0	19.7	0.00	27.20	25.08	0.00
IIDP0552-4	6.4	3.2	0.00	10.6	7.6	0.00	11.67	8.70	0.00
IIDP0552-5	7.0	4.1	0.00	7.0	4.1	0.00	19.73	17.24	0.00
IIDP0572-1	19.5	15.5	0.00	19.5	15.5	0.00	23.70	23.7	0.00
IIDP0572-2	9.5	5.2	0.00	9.5	5.2	0.00	14.55	10.48	0.00
IIDP0572-3	17.4	10.0	0.00	17.4	10.0	0.00	24.80	18.08	0.00
IIDP0572-4	21.1	16.0	0.00	21.2	16.1	0.00	29.95	25.40	0.00
IIDP0572-5	21.7	17.1	0.00	22.1	17.5	0.00	33.35	29.45	0.00
IIDP1051-1	11.0	7.7	0.00	11.0	7.7	0.00	27.20	24.47	0.00
IIDP1051-2	14.4	1.5	0.00	17.8	5.4	0.00	30.77	20.32	0.00
IIDP1051-3	5.5	-3.6	0.00	5.5	-3.6	0.00	10.06	1.35	0.00
IIDP1051-4	3.7	1.6	0.00	6.1	3.9	0.00	22.37	20.60	0.00
IIDP1051-5	10.0	2.7	0.00	13.1	6.0	0.00	24.75	18.63	0.00
IIDP1071-1	10.6	3.7	0.00	10.5	3.7	0.00	18.51	12.26	0.00
IIDP1071-2	12.1	-1.4	0.00	11.9	-1.7	0.00	20.00	7.70	0.00
IIDP1071-3	13.0	2.7	0.00	17.2	7.3	0.00	35.49	27.79	0.00
IIDP1071-4	10.0	-1.0	0.00	14.6	4.2	0.00	32.38	24.14	0.00
IIDP1071-5	8.6	2.1	0.00	13.7	7.5	0.00	22.71	17.13	0.00
IIDP1052-1	8.7	0.2	0.00	8.7	0.2	0.00	15.02	7.08	0.00
IIDP1052-2	18.7	3.0	0.00	24.1	9.4	0.00	40.02	28.46	0.00
IIDP1052-3	14.3	1.3	0.00	14.9	2.0	0.00	22.71	10.96	0.00
IIDP1052-4	26.7	9.2	0.00	26.6	9.1	0.00	27.68	10.44	0.00
IIDP1052-5	10.0	-0.2	0.00	11.8	1.9	0.00	15.41	5.85	0.00
IIDP1072-1	20.2	5.7	0.00	19.4	4.8	0.00	39.06	27.96	0.00
IIDP1072-2	19.3	0.7	0.00	19.3	0.7	0.00	31.77	16.00	0.00
IIDP1072-3	23.4	7.3	0.00	24.1	8.2	0.00	29.52	14.67	0.00
IIDP1072-4	23.5	3.2	0.00	23.8	3.5	0.00	32.33	14.29	0.00
IIDP1072-5	20.5	0.7	0.00	21.1	1.4	0.00	37.62	22.07	0.00
IIDP1551-1	8.7	-2.1	0.00	10.5	0.0	0.00	15.85	5.95	0.00
IIDP1551-2	7.7	0.6	0.00	6.5	-0.7	0.00	16.80	10.41	0.00
IIDP1551-3	9.9	-8.1	0.00	10.1	-7.9	0.00	28.63	14.32	0.00
IIDP1551-4	14.8	-3.5	0.00	16.6	-1.4	0.00	22.49	5.77	0.00
IIDP1551-5	6.0	2.0	0.00	6.4	2.5	0.00	20.16	16.78	0.00
IIDP1571-1	20.7	-16.2	0.00	21.2	-15.5	0.00	43.73	17.55	0.00
IIDP1571-2	21.1	-31.1	0.02	25.6	-23.5	0.00	53.13	22.14	0.00
IIDP1571-3	21.8	-18.9	0.01	22.9	-17.3	0.00	37.51	4.98	0.00
IIDP1571-4	19.9	-9.1	0.01	23.7	-4.0	0.00	46.64	27.31	0.00
IIDP1571-5	8.6	-11.8	0.00	9.3	-10.9	0.00	17.83	-0.55	0.00
IIDP1552-1	26.8	-8.9	0.01	25.4	-11.0	0.00	43.49	15.90	0.00
IIDP1552-2	20.1	-9.7	0.01	21.3	-8.1	0.00	33.44	8.60	0.00
IIDP1552-3	4.4	-14.3	0.02	4.6	-14.0	0.00	15.35	-1.15	0.00
IIDP1552-4	12.2	-3.3	0.00	12.2	-3.3	0.00	17.06	2.46	0.00
IIDP1552-5	21.8	-20.6	0.00	22.4	-19.7	0.00	30.04	-7.92	0.00
IIDP1572-1	25.9	-15.6	0.03	26.3	-15.0	0.00	35.60	-0.44	0.00
IIDP1572-2	18.3	-19.0	0.00	19.1	-17.8	0.00	27.08	-6.16	0.00
IIDP1572-3	20.4	-35.1	0.00	23.1	-30.4	0.00	36.32	-8.07	0.00
IIDP1572-4	25.4	-7.7	0.05	27.2	-5.1	0.00	41.25	15.15	0.00
IIDP1572-5	16.3	-17.9	0.00	16.9	-17.2	0.00	22.97	-8.59	0.00
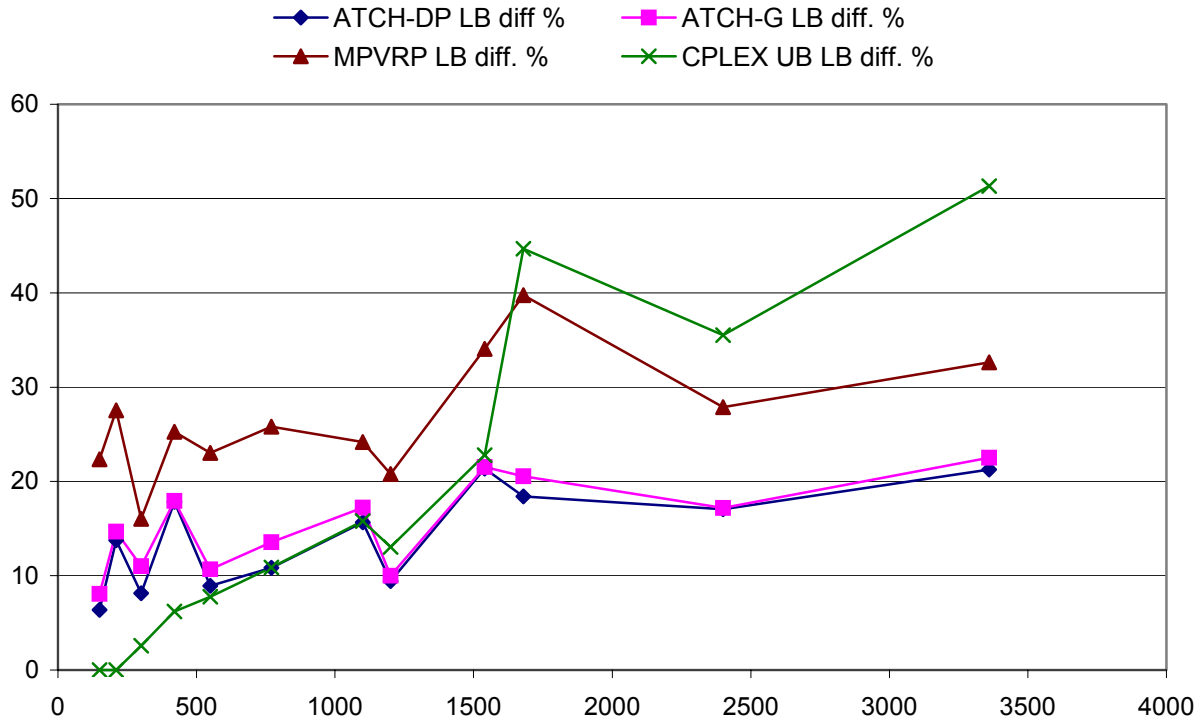
Fig. 3. % percentage difference against lower bounds vs. number of binary variables

5. Conclusion and Future Work

This article addressed the integrated inventory distribution problem in which vehicle routing and inventory holding and backorder decisions for a set of customers are to be made for a specific planning horizon. We considered an environment in which the demand of each customer is relatively small compared to the vehicle capacity, and the customers are closely located such that a consolidated shipping strategy is appropriate. We presented a heuristic approach based on the idea of allocating single transportation cost estimates for each customer. Two sub-problems, comparing inventory holding and backorder decisions with these transportation cost estimates, are formulated and their solution methods are incorporated in the developed heuristic. A mixed

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integer programming formulation is provided and used to obtain lower and upper bounds using CPLEX to assess the performance of the heuristic. The benchmarking results show that the developed heuristic can obtain solutions that are within 20% from optimal for this NP-hard problem in a reasonable amount of computation time.

The solution heuristic is based on guiding principles from the uncapacitated lot-sizing problems, which assume inventory replenishment decisions only when the customer's inventory reaches zero. Using these principles, the heuristic generates solutions in which delivery schedules cover customers' exact demand requirements in future days. That is, partial fulfillment of a customer's demand in a future day is not considered by the heuristic. This approximation is reasonable when each individual customer order quantity is significantly less than the vehicle capacity since neglecting these partial demand fulfillments facilitates the decisions involved and results in significant reductions in transportation costs. However, this strategy is clearly not always optimal. Thus, further research can focus on developing approximate solution approaches that allow for partial deliveries.

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