# Hybrid Scheduling Methods

# for Paratransit Operations

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## Abstract

We study an extension to the general routing problem, which deals with integrating fixed route service with the general pickup and delivery problem to create a hybrid routing problem. The primary application for such a service is a dial-a-ride system used by transit agencies to transport disabled or elderly individuals. The main aim of the integration is to reduce the vehicle miles of the on-demand vehicles while not significantly reducing the customer service level. Due to the combinatorial nature of the problem, we propose a heuristic algorithm that provides an approximate solution, which is computationally efficient for solving large sized problems. The proposed heuristic is tested using real data from a transit agency.

# 1.0 Introduction

The growth of personalized public transit and demand responsive transit (DRT) began in the late 1970's and early 1980's with large demonstration projects developed in Rochester, NY and Santa Clara County, CA among others. These early systems failed to meet expectations due to low demand requests and deficiency in communication and computer technology to effectively manage such systems (Lave, Teal, and Piras, 1996). However, with the passage of the Americans with Disabilities Act (ADA), which requires that transit agencies provide paratransit or on demand service for the disabled, there has been renewed interest in demand responsive transit. The passage of ADA has increased the obligations of the transit providers to adhere to strict service standards. In addition, the demand of these types of transit services is likely to continue increasing rapidly (Levine, 1997). In Los Angeles County alone more than 5000 vans and 4200 cabs provide service, generating 8 million trips per year.

Besides the increase in demand, ADA also sets strict guidelines for the providers on trip denials and on-time performance (Lewis, Evans, and Koffman, 1998). In essence, transit agencies today are expected to provide better services while experiencing increased usage for demand responsive transit systems. There have been some recent studies that have outlined the potential impact of Advanced Public Transportation Systems (APTS) on service productivity and cost. Technologies part of an APTS include Automatic Vehicle Location (AVL) Systems, Computer Aided Dispatching (CAD) Systems, and Mobil Data Terminals (MDTs). Khattak and Hickman (1998) provide an excellent overview of CAD systems. Stone, Nalevanko, and Gilbert (1994) discuss how computers and advanced algorithms can improve the dispatching and scheduling of paratransit systems. Chira-Chavala and Venter (1997) show the productivity gain of using technology in Santa Clara County. In a survey of paratransit customers in southeastern Michigan, Wallace (1997) concludes that APTS has ample potential to increase customer satisfaction when reserving a trip. Higgins, Laughlin, and Turnbell (2000) state that the implementation of Automatic Vehicle Location (AVL) and advanced scheduling appears to be the primary factor in increasing efficiency by 10.3% for Houston's METROLift Service.

Although the above studies show the potential of APTS in improving the productivity of dial-a-ride programs and alleviating some of the drawbacks of the earlier systems, technology alone cannot solve the problem of operating such systems under increased demand since most dial-a-ride programs for the transport of elderly and disabled persons are heavily subsidized programs. In Los Angeles County, the average cost per trip for such systems is around $17-20 per passenger trip with the average fare being around $2-3 as opposed to fixed route transit which has an average cost per passenger trip of around $2 with average fares of around $0.50. Thus, the increased usage of these curb-to-curb services has put significant budget pressures on most transit agencies. In fact this is such a major concern in Los Angeles County that the agency responsible for paratransit services, Access, is allowing all ADA eligible passengers to ride for free on the fixed route bus lines in order to shift some of the passengers to this mode of transit service. Hence, there is a need to evaluate different service delivery methods that can meet the increased demand of this type of service and satisfy the strict guidelines of ADA. In this paper, we consider an approach that integrates a curb-to-curb system with fixed route bus lines.

The conventional problem of dial-a-ride concerns with routing and scheduling vehicles in order to satisfy transportation requests from customers. Each request has an origin point, destination point, number of riders, and desired pickup or drop-off time. From the perspective of the user, on-demand curb-to-curb service is the most preferred mode of service delivery method. It clearly minimizes the passenger travel time. However, this type of service may not be the most cost-effective method of transporting people.

There is a significant body of research on scheduling curb-to-curb systems. The problem of scheduling curb-to-curb systems is sometimes referred to as the Pickup and Delivery Problem (PDP) in the literature. Savelsbergh and Sol (1995) provide an excellent review of this work. Some exact solution procedures for solving this problem include the work of Psaraftis (1980, 1983), Kalantari et al. (1985), Desrosiers, Dumas, and Soumis (1986), Fischetti and Toth (1989), Dumas, Desrosiers, and Soumis (1991), and Ruland and Rodin (1997). Since the PDP is NP-hard (Savelsbergh and Sol, 1995), most of the research has focused on heuristics. More recent approximation solution procedures include the column management scheme by Savelsbergh and Sol (1998), the insertion heuristic developed by Madsen, Ravn, and Rygaard (1995), the clustering algorithm developed by Ioachim et al. (1995), and the parallel insertion heuristic developed by Toth and Vigo (1997).

Although there has been a significant body of research on scheduling curb-to-curb systems, there has been limited work that attempts to integrate curb-to-curb services with the fixed route bus lines. We refer to a transit system that integrates these two modes of transportation as a "hybrid" service delivery method. Clearly, a curb-to-curb system as opposed to a hybrid system minimizes the travel time for the passenger. However, shifting some of the demand to fixed routes may alleviate some of the demand pressure for the on-demand vehicles caused by ADA requirements. The limited research in this area includes the work of Liaw, White, and Bander (1996), and Hickman and Blume (2000). Both of the above approaches are based on developing insertion heuristics. Hickman and Blume (2000) use the insertion procedure developed by Jaw et al. (1986) in their approach. Liaw, White, and Bander (1996) test their heuristic on a data set from Ann Arbor, Michigan while Hickman and Blum (2000) test their insertion heuristic on a data set from Houston, Texas.

We build on this earlier work by expanding on the insertion heuristic approaches by adding an improvement as well as a Tabu Phase to the solution procedure. Tabu search is considered to be one of the most useful meta-heuristics for solving routing problems. For example, Rochat and Taillard (1995), Badeau et al. (1997), and Taillard et al. (1997) use a tabu search procedure for solving the vehicle routing problem while Nanry and Barnes (2000) use it to solve the pickup and delivery problem.

We test our methodology on an actual paratransit service provider. The selected agency is Antelope Valley Transit Authority (AVTA). This agency is selected since their service area is ideal for a hybrid system. For example, most ADA passengers travel to a central area where the hospitals and shopping malls are located. In addition, the distances traveled by many of the passengers are large enough to justify a transfer to a fixed route bus line.

The rest of this paper is organized as follows. A problem description is given in section 2. The proposed heuristic approach is described in section 3. The experimental results are presented in section 4. The conclusion is provided in section 5.

## 2.0 Problem Description

In a certain day, there are a number of requests of a set of N customers who need to be picked up from origin points and dropped off at destination points. Every request has a desired pickup time and drop-off time. Also, associated with each desired pickup time is a hard time window which specifies the earliest and the latest times the customer can be picked up. We assume that all requests are made in advance. For example, they are made one day before the service day.

The main idea is to integrate two modes of transportation, curb-to-curb service and fixed route transit, to satisfy the above requests. We refer to such a system as a *hybrid* service delivery method. The paratransit curb-to-curb system includes a set of M paratransit vehicles with a known capacity that are used to pick up the passengers from their origins or from the bus stops and drop them off at their final destinations or at the bus stops. All the paratransit vehicles dispatch and return to their depot at the end of the workday.

On the other hand, the fixed bus route system includes a set of R fixed bus routes. Every fixed bus route has a number of buses that travel through it, a set of bus stops and a time schedule. The capacity of the buses is relatively large and is not considered a constraint in this problem. We assume that at most two transfers can be made for any customer in the system and no transfers can be made between the same mode. We refer to passengers that are strictly served by the on-demand vehicles as door-to-door requests while those passengers that transfer to a fixed route bus line are referred to as hybrid requests.

To help illustrate the problem, Figure 1 shows a simple network that has hybrid service. In this example, the triangle with a one is the origin while the triangle with a two is the destination. There are two fixed route bus lines in this example. As the diagram shows, there are a number of different alternatives to go from the origin to the destination using a hybrid service. Obviously, one path is to take the direct distance using only the on-demand vehicle. Another path is to take a vehicle to stop 1C then get on a fixed bus line to either stops TC, 1B, and 1A and transfer to another vehicle. Clearly one of the functions of any solution procedure is to select the entry and the exit bus stops. In this case, stop 1A is the closest to the final destination but requires the passenger to sit in the bus longer. Furthermore, the request that the vehicle serves next may be closer to stop TC. Hence, any solution procedure needs to consider all these factors in selecting the best route/path.

Note that in a strictly door-to-door system (e.g., dial-a-ride service) the decision variable is primarily to determine the on-demand vehicle schedule. In a hybrid system, the on-demand vehicle schedule as well as the delivery path for each request needs to be determined as illustrated in the above example.

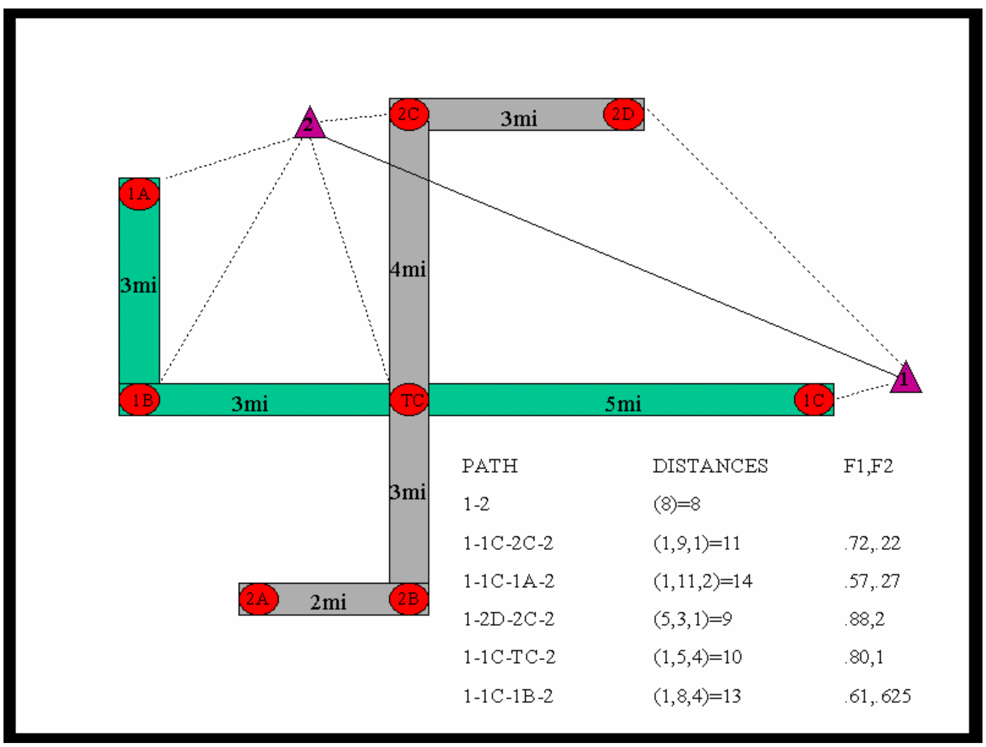


Figure 1. Hybrid Network

Our objective function considers two distinct performance measures. They are:

1. Minimize the total travel distance of the on-demand vehicles, and
2. Minimize the total travel time of the passengers.

Most of the work on scheduling dial-a-ride systems minimizes the first objective (total travel distance of the on-demand vehicles) subject to meeting the time window and vehicle capacity constraints. They typically do not consider the total passenger trip time in the objective function. It is usually incorporated as a constraint expressed either as a maximum ride time constraint or a time window constraint associated with the delivery point. For the extreme case such as taxi service, where there is no ridesharing, the total passenger trip time is insensitive to the schedule. As ridesharing is incorporated to the system the schedule can have some impact on the total passenger trip time. However, for hybrid systems, the selection of the path for a request can greatly impact the total passenger trip time. The passenger trip time can vary significantly depending on the path that is chosen.

In fact, in hybrid systems, it can be easily seen that the two above objectives can conflict. From the viewpoint of the customer, the second objective is the most relevant, and a strict taxi type system minimizes this objective, however, at the expense of a high cost associated with on-demand vehicle travel distance. On the other hand, the service provider prefers to minimize the first objective. Minimizing on-demand vehicle miles minimizes variable costs such as fuel consumption and the number of needed deployed vehicles to satisfy the demand. We do not consider the travel distance of the fixed route buses in the objective since from a day to day operational point of view they are considered a fixed cost.

## 3.0 Proposed Heuristic Approach

Due to the combinatorial nature of the problem, it is not possible to find optimal solutions to practical size problems in a reasonable amount of computation time. Therefore, heuristics are necessary to find a near optimal or good solution to the problem. We first derive an initial solution using an Insertion/Improvement procedure. The Insertion/Improvement procedure consists of three phases. In the first phase, all the candidate routes/paths that meet a certain criterion for each request are identified. In the second phase, an initial solution is determined by identifying a feasible path from the candidates' list that has the shortest on-demand vehicle distance. In the last phase, the initial solution is fed into an Improvement procedure. In this procedure, we try to identify an alternative path that reduces the total passenger trip time for requests that have multiple hybrid paths.

The solution from the Insertion/Improvement procedure is the initial solution in the Tabu Search. The Tabu Search consists of two functions: Re-sequencing and Re-assigning. In the Re-sequencing function, we hold the request to vehicle assignment fixed and consider alternative feasible sequences. In the Re-assigning function, we attempt to exchange requests between vehicles.

Before presenting the methodology of the proposed heuristic algorithms, we present some additional notation.

| | |
|---|---|
| *BB (r, B1, B2)* | *Direct distance from bus stop B1 to bus stop B2 on fixed route r* |
| *DD (n)* | *Direct door to door distance of request n* |
| *PB(r, B1, n)* | *Direct distance from the origin point of request n to bus stop B1 on route r* |
| *DB(r, B2, n)* | *Direct distance from bus stop B2 on route r to the drop-off (destination) point of request n* |

We refer here the direct distance as the Euclidean distance between any two geocoded locations (lat1, long1) and (lat2, long2) using the sphere equation:

distance = (3963*acos(cos (radians (90-lat1))*cos (radians (90-lat2))+sin (radians(90-lat1))*sin (radians (90-lat2))*cos (radians (long1-long2))))

For each bus route, we now have the distance between bus stop B1 to each pick-up point and the distance between bus stop B2 to each drop-off point. This is the total distance of request n that is traveled using the paratransit vehicle for a hybrid system. Let this variable be *DBD (r, B1, B2, n)*.

*DBD (r, B1, B2, n) = PB (r, B1, n) + DB (r, B2, n)*

The hybrid total distance is the total distance that is traveled by the on-demand vehicle and fixed bus line. This distance is always greater than or equal to the direct distance of the same request. Let this variable be *HYB (r, B1, B2, n)*.

$$HYB\ (r, B1, B2, n) = (DBD\ (r, B1, B2, n) + BB\ (r, B1, B2)) \geq DD\ (n)$$

**3.1 Insertion/Improvement Procedure**

Our heuristic procedure consists of three phases. They are:

1. Identify the candidate path set.
2. Identify an initial solution using an Insertion Procedure.
3. Update the solution using an Improvement Procedure.

Phase I of the procedure identifies candidate paths that meet the following three criteria:

- The ratio of the direct distance over the hybrid distance must be greater than or equal to a threshold level F1.
  DD (n) / HYB *(r, B1, B2, n)* ≥ F1
- The ratio of the distance traveled by the on-demand vehicle over the distance on the fixed bus route must be less than or equal to a threshold level F2.
  DBD *(r, B1, B2, n)* / BB *(r, B1, B2)* ≤ F2
- The door-to-door distance of the request must be greater than or equal to a threshold level F3.
  DD (n) ≥ F3

We can look at the first condition as the rider's service level or convenience. The second condition ensures that the door-to-door portion of the hybrid distance is less than the fixed route portion of the trip to justify a transfer. The third condition makes sure that no transfer to a fixed route is made if the distance of the request is short. The above three conditions work together and a path needs to satisfy all of them in order to be considered a candidate path. Figure 1 shows how the candidate paths for the hybrid system can be found using F1=0.7, F2=1, and F3=8. In the Experimental Analysis Section, we show how to compute the best values for the thresholds.

We note that Liaw, White, and Bander (1996) consider only ratio F1 in their procedure while Hickman and Blume (2000) base their selection criteria on absolute threshold values. We use both ratios F1 and F2 in our procedure since they represent a trade off between the customer service level and overall efficiency of the system. Note that the number of candidate requests and paths increases as the ratio F1 decreases and ratio F2 increases.

Note that for some requests, there may not exist any path that satisfies all the above criteria. These requests will be served strictly by the on-demand vehicles and will be referred to as "Door-to-Door". Alternatively, some candidate requests may have one or more candidate path that satisfies the above conditions. We refer to these requests as "Hybrid". We illustrate this differentiation between candidate request and candidate path using the Antelope Valley Transit Data, AVTA, (see Section 4.0 for a complete description of this data set). A sample of 47 requests (94 points) from the AVTA that is served by one vehicle in one day is taken to test the first phase of the heuristic algorithm using the existing 10 fixed bus routes.

Figure 2 shows the number of the candidate requests that meet the ratio F1 and F2 for various values. Figure 3 shows the number of the candidate paths that meet the ratios F1 and F2 for various values. We emphasis here that a request is considered a candidate if it has at least one candidate path. Thus many requests may have many alternative candidate paths. As it is shown in these two charts, the number of candidate requests and paths increase as the ratio F1 decreases and ratio F2 increases. Using the values of F1=0.7 and F2=1, 25 (53%) requests are found to be candidates for using the hybrid system. These requests have a total of 63 candidate paths. If we let F1=0.4 and maintain F2=1, 43 (91%) requests are found to be candidates for using the hybrid system with a total of 321 candidate paths. The sensitivity of these ratios represents a trade off between the customer service level and overall efficiency of the system.

Our heuristic procedure determines:

1. The on-demand vehicle schedule,
2. The best candidate path for each request.

As previously mentioned, the objective of the heuristic procedure is to minimize both the total distance traveled by the on-demand vehicles, TVD, and the total passenger trip time, TTT. Let $t_n$ be the total trip time of request n including time on-board the on-demand vehicle(s), time on-board the fixed route bus, and waiting time at the enter and exist bus stops, p(i,v) be the direct predecessor of location i in sequence $S_v$ for on-demand vehicle v (the first and last location in the sequence $S_v$ is the depot), and dist(i,j,v) be the direct distance between locations i and j in sequence $S_v$. Given NT total requests, M vehicles, sequence $S_v$ for each vehicle v, and $N_v$ requests in $S_v$, TVD and TTT are computed as follows:

$$TTT = \sum_{n=1}^{NT} t_n$$

$$TVD = \sum_{v=1}^{M} \sum_{i=1}^{2Nv+1} dist(p(i,v),i,v)$$

Note that the depot is represented by locations 0 and $2N_v+1$.

We remark that the on-demand vehicle schedule has a greater impact on the TVD performance measure than that for the TTT performance measure. For example, for taxi systems the schedule has no impact on TTT. However, the selection of the best candidate path for each request greatly impacts the TTT performance measure. Hence, instead of trying to minimize the two above objectives simultaneously, our approach minimizes the appropriate objective based on the purpose of the particular routine of the heuristic. That is, an initial on-demand vehicle schedule is determined using an Insertion procedure that holds the candidate paths fixed. Since the Insertion procedure does not search for the best candidate path, we focus only on TVD in this phase. Then, this initial solution is fed into an Improvement procedure that attempts to find a better solution in terms of the passenger trip time, TTT, that does not reduce, TVD. The Improvement procedure searches the alternative candidate path list in an attempt to reduce TTT.

Figure 4 shows the steps of the Insertion procedure. For each hybrid request, the candidate paths that has the shortest distance traveled by the on-demand vehicles (i.e., the distance from origin to entry bus stop plus the distance from exit bus stop to the final destination) is selected. These candidate paths are held fixed in the Insertion procedure. The requests are then ranked based on the earliest pickup time and stored in set N. The requests are then sequentially inserted into an existing vehicle schedule if it is feasible. Feasibility here refers to no violation of time window or capacity constraints.

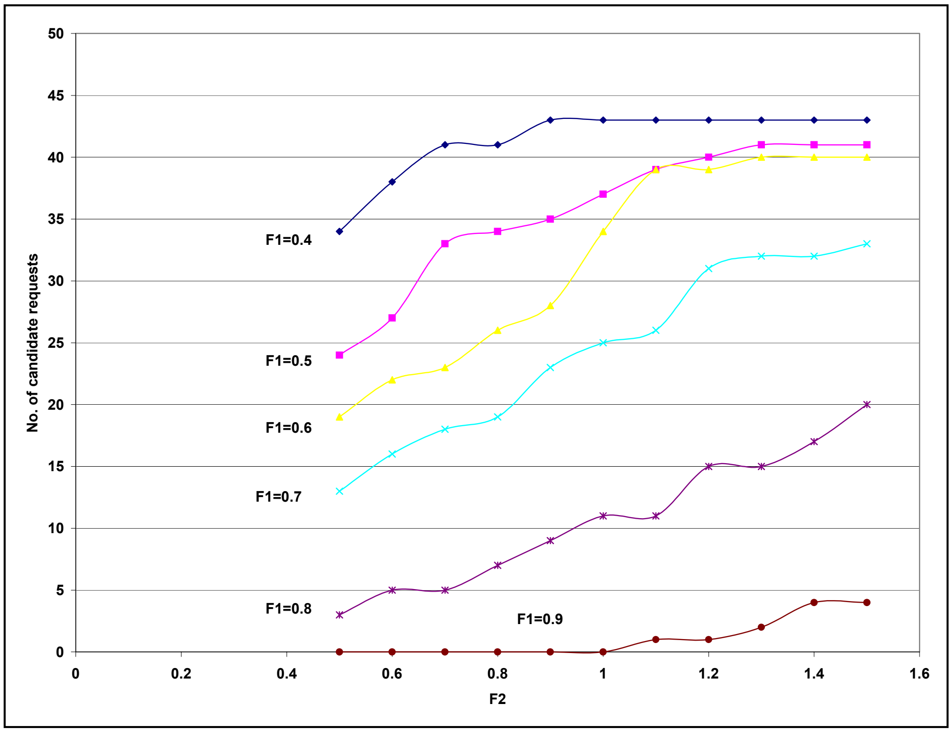


**Figure 2. Number of Candidate Requests**

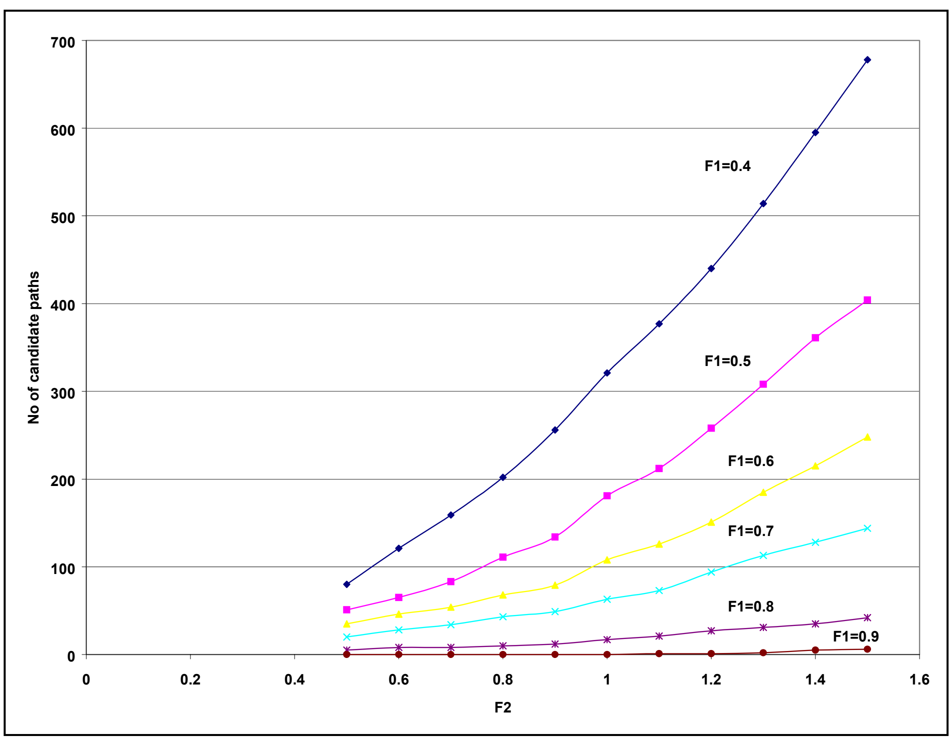


**Figure 3. Number of Candidate Paths**

Otherwise, a new vehicle schedule is created. As the figure shows, the on-demand vehicle that results in the smallest increase in the total distance traveled by the on-demand vehicle, TVD, is selected. The pickup and drop-off points of the current request are tentatively inserted in all feasible combinations after the pickup point of the last request in the vehicle's schedule. Feasible combinations are those in which the pickup point of a request precedes the delivery point of the same request and vehicle capacity is not violated. The request is then permanently inserted in the combination that results in the smallest increase in the total on-demand vehicle distance. Note that if the first leg of a hybrid request is inserted in the schedule, the second leg is placed into set N.

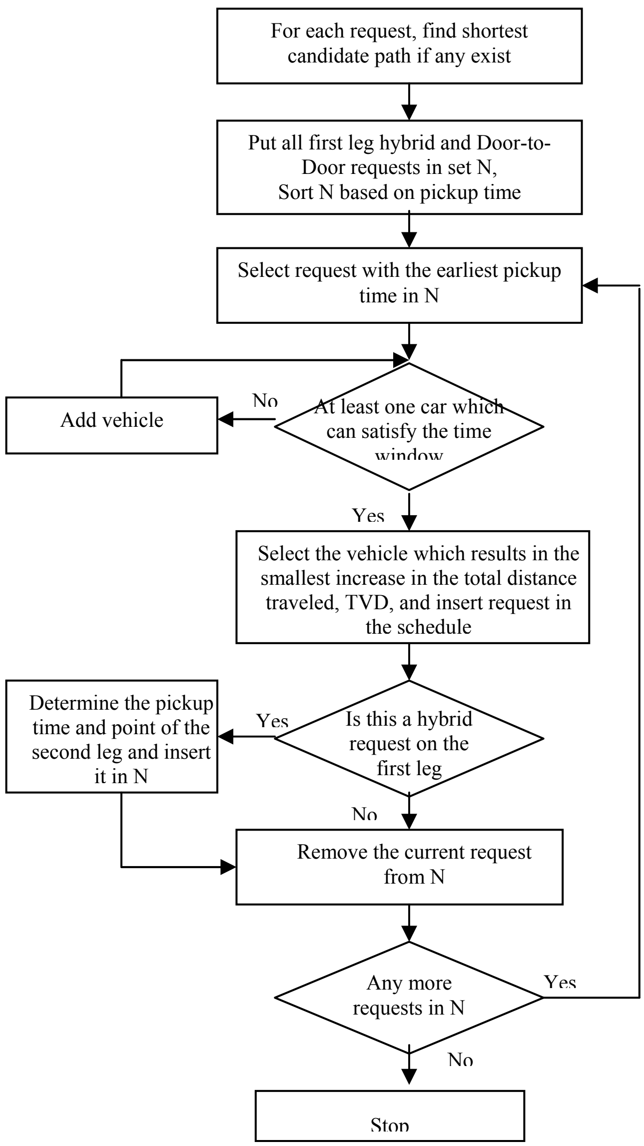


**Figure 4. Insertion Procedure**

The following are two examples to show how we can insert a request in an existing schedule of a vehicle. In the first case, the vehicle picks up the first passenger and then drops the same passenger off before picking up the second passenger (P1D1P2D2). In the second case, the vehicle picks up both passengers first and then drops them of (P1P2D1D2). We now illustrate the possible combinations of inserting a new passenger request. In the first case, three alternatives are evaluated while in the second case nine alternatives are evaluated.

$$\underline{\textbf{\textit{P1 D1 P2 D2}}} \left\{ \begin{array}{l} \text{P1 D1 P2 P3 D3 D2} \\ \text{P1 D1 P2 P3 D2 D3} \\ \text{P1 D1 P2 D2 P3 D3} \end{array} \right.$$

$$\underline{\textbf{\textit{P1 P2 D1 D2}}} \left\{ \begin{array}{l} \text{P1 P2 P3 D3 D1 D2} \\ \text{P1 P2 P3 D1 D3 D2} \\ \text{P1 P2 P3 D1 D2 D3} \\ \text{P1 P2 P3 D3 D2 D1} \\ \text{P1 P2 P3 D2 D3 D1} \\ \text{P1 P2 P3 D2 D1 D3} \\ \text{P1 P2 D1 P3 D3 D2} \\ \text{P1 P2 D1 P3 D2 D3} \\ \text{P1 P2 D1 D2 P3 D3} \end{array} \right.$$

The Improvement Procedure takes the initial solution from the Insertion Method and attempts to find a better solution in terms of the total passenger trip time that does not reduce the TVD. In the Insertion Method, the candidate path with the shortest on-demand vehicle travel distance is selected for the Hybrid requests. This may not be a good rule in terms of minimizing the passenger trip time since the path with the shortest on-demand vehicle travel distance may connect to a fixed route bus line that will require waiting at the bus stop and have a long travel time on the bus. That is, the selection of the candidate path can greatly impact the TTT performance measure. The Improvement Procedure searches for other candidate paths of each hybrid request that can reduce the passenger trip time. Figure 5 shows the steps of the Improvement phase.

Note that the Improvement procedure does not enumerate all possible combinations in determining the solution. That is, once the best candidate path for a particular request is found it is held fixed. Suppose that there are 4 candidate (hybrid) requests that have 8, 7, 6 and 9 candidate paths respectively. The total number of solutions considered will be 8+7+6+9=30. This means that 30 different solutions are considered. However, if we enumerate all the possible combinations of the candidate paths we would need to consider 8*7*6*9=3024.

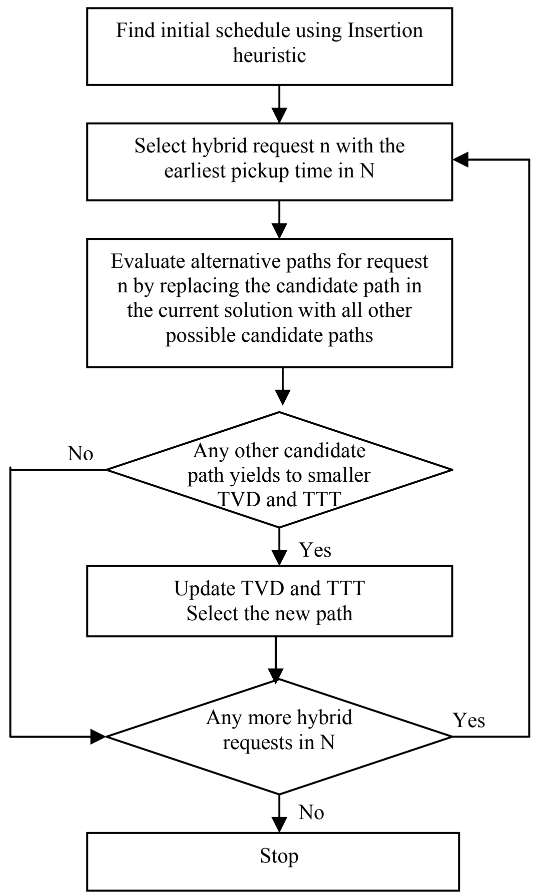


**Figure 5. Improvement Procedure**

### 3.2 Tabu Search

In the Tabu Search procedure we attempt to find an improved vehicle schedule holding the selection of the candidate paths fixed from the Improvement Procedure. Hence, the Tabu Search focuses on finding a better solution in terms of the total travel distance of the on-demand vehicles, TVD, while not degrading the solution quality of the total passenger trip time, TTT. That is, a solution is only updated only if it improves TVD without decreasing TTT.

The main idea behind the Tabu Search is to maintain a list of forbidding moves by which solutions are prevented to be visited again, thus preventing infinite cycles. As we previously mentioned Tabu Search has shown to be effective in solving routing problems.

The Tabu Search consists of two strategies to improve the vehicle and customer schedule, which are a Re-sequencing strategy and Re-assigning strategy. The Re-sequencing strategy re-sorts the sequence within a vehicle while the Re-assigning strategy searches to remove a request from a particular vehicle to insert it in another vehicle. The two strategies aim to detect better vehicle schedules. Figure 6 displays the proposed overall methodology. Note that we iterate between the Re-sequencing and Re-assigning procedures because we try to find a better sequence when a new request to vehicle assignment is found.

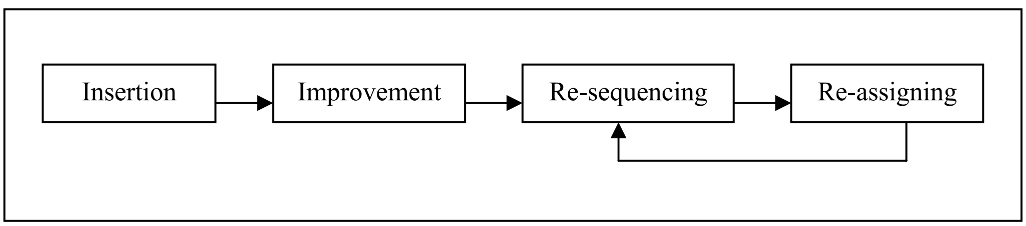


**Figure 6. Proposed Methodology**

Re-sequencing

In this strategy, we use a Tabu Search technique in order to find an improved requests sequence, which leads to shorter vehicle distance in every vehicle while holding the request to vehicle assignment fixed. In the case of a strictly door-to-door delivery method, requests are entirely satisfied at the delivery point. Hence, the schedule of the vehicles is going to be always feasible if we just satisfy the pickup time windows of the requests during Re-sequencing. However, this is not true for the hybrid delivery method where hybrid requests have two delivery points (two legs) serviced by possibly two different vehicles. Also, due to the dependency between the two trips of the hybrid request, it is necessary to set a delivery as well as a pickup time window of every first leg of a hybrid request in each vehicle schedule in order to maintain the feasibility in every vehicle schedule. To clarify this issue, we present the following example where BS1 is the entry bus stop and BS2 is the exist bus stop:

Suppose there is a hybrid request which has the following schedule (7:00 – 7:07) from origin point to BS1 using vehicle 1, (7:10 – 7:25) from BS1 to BS2 in the bus, and (7:27 – 7:32) from BS2 to final destination in vehicle 2. By re-sequencing vehicle 1's schedule without adding a delivery time window, the passenger might end up with a delivery time of 7:11 to BS1. In this case he/she will miss the 7:10 bus and use the 7:30 bus, which arrives at the exit bus stop (BS2) at 7:45. Therefore, ignoring the delivery time of the first leg of a hybrid request in one on-demand vehicle might end up with an infeasible schedule in another on-demand vehicle.

In the Re-sequencing strategy, the schedule is improved by moving individual predecessor (pickup point) and/or successor (delivery point) forward and/or backward in their corresponding route. Three conditions need to be satisfied while moving the pickup and delivery pair in order to have a feasible schedule. The first one is the precedence constraint where the pickup point of any request must be visited before the delivery point of the same request. The second is the on-demand vehicle capacity constraint. The third one is the pickup time window (and the delivery time window for the first leg of the hybrid requests). Since the problem in this technique is single vehicle PDP, the coupling constraint can not be violated. The coupling constraint is that once the vehicle picks up a passenger, the same vehicle must drop him/her off at his/her final destination point.

To minimize the search space and consequently the *Tabu* memory, the search is restricted by what we refer to as min ($S_n$) and max ($E_n$) request positions. These tell how far, the request can be moved backward and forward without causing guaranteed infeasibility (Barnes and Carlton, 1995).

Below we show an illustration of the above concept for two possible cases. Figure 7.a shows a single vehicle schedule where the first and the last nodes represent the dispatching center or the depot. The pickup point and delivery point of request "n", which is intended to be moved, are the fourth and the sixth nodes respectively (e.g., $P_n$ = 4, $D_n$ = 6). After finding the min ($S_n$=2) and max ($E_n$=8) position of the request, we can determine the set of all the possible sequences of request n, which is {(3,4), (3,5), (3,6), (3,7), (4,5), (4,6), (4,7), (5,6), (5,7), (6,7)}.

In figure 7.b, $(P_n,D_n)=(4,5)$ and $(S_n,E_n)=(3,6)$. The set of all the possible sequences equals {(4,5)}. Consequently, determining the values $S_n$ and $E_n$ before starting Re-sequencing is significant in minimizing the search space.

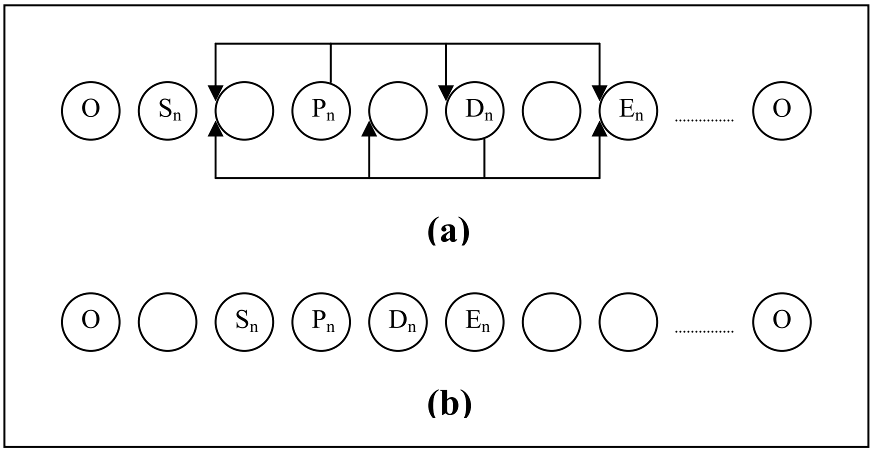


**Figure 7. Illustration of the $S_n$ and $E_n$ concept**

We use a tabu search description similar to the one introduced by Rardin (1998) to illustrate both the Re-sequencing and Re-assigning strategies. The following are the steps of the Re-sequencing strategy:

- Initialization: The initial schedule (SCH (0)) is obtained by the improvement heuristic and the initial sequence is added to the Tabu list. Let the current best schedule (SCH*) equal SCH (0) and let the iteration index (t) equal 0. Let TVD* and TTT* be the resulting total on-demand vehicle distance and total passenger trip time.
- Stopping: If there is no non-tabu sequence leading to a feasible solution neighbor of the current schedule SCH (t) or $t = t_{max}$, then stop. The current SCH* is an approximate optimum schedule.
- Move: Select a non-tabu feasible sequence neighbor to the current schedule SCH(t) and find SCH(t+1).
    - Criterion: In this sequence selection step, all the feasible moves as shown in Figure 7 are evaluated. The sequence that leads to the best value of the objective function, TVD, without increasing TTT, is selected.
- Compare: If the objective function value of SCH(t+1) is superior to that of SCH*, let SCH* = SCH(t+1), set TVD*=TVD and TTT*=TTT.

- Tabu List: Remove from the list of forbidden tabu sequences any that have been on it for a sufficient number of iterations, and add the current sequence of SCH(t+1) to the list.
- Increment: Let t = t + 1, and go back to the stopping step.

Re-assigning

In this strategy, we try to search for a better request to vehicle assignment using a tabu search technique to produce a better solution (shorter vehicle distance). The search space of this method is larger than the one for the Re-sequencing method. Therefore, only few requests with a high potential for improvement are considered to be moved from their current vehicle schedule to another vehicle schedule. The main idea behind the Re-assignment is to move to undiscovered regions and try to search for better solutions there.

In the Re-assigning approach, two questions need to be answered. They are (1) which request should be removed from its current vehicle schedule, and (2) where should it be inserted? To answer the first question, we set the following criteria:

- Remove the request, which has the maximum saving distance if it is removed, which equals the distance of the removed arcs minus the distance of the added arcs.

Figure 8 illustrates the above criteria. In Figure 8.a, the request that will be removed from the current on-demand vehicle schedule is request “n” which has adjacent pickup and delivery points. The eliminated arcs are $X_{\alpha,Pn}$, $X_{Dn,\beta}$ and $X_{Pn,Dn}$. The arc $X_{\alpha,Pn}$ represents the distance required to reach the pickup point of the request from the previous adjacent point ”$\alpha$”. The arc $X_{Dn,\beta}$ represents the distance from the request delivery point to the next adjacent point ”$\beta$”. The arc $X_{Pn,Dn}$ represents the distance from the request pickup point to the delivery point. The new arc that will be added after removing the request ”n” is $X_{\alpha,\beta}$ which represents the distance from $\alpha$ to $\beta$. The double lines in Figure 8.a are the eliminated arcs while the dotted lines are the new added arcs.

$$\text{Saving} = X_{\alpha,Pn} + X_{Dn,\beta} + X_{Pn,Dn} - X_{\alpha,\beta}$$

In Figure 8.b, the request that will be removed "n" does not have adjacent pickup and delivery points. Following the same notation concept, the saving distance will be:

$$\text{Saving} = X_{\alpha,Pn} + X_{Dn,\beta} + X_{Pn,\gamma} + X_{\gamma,Dn} - X_{\alpha,\gamma} - X_{\gamma,\beta}$$

We use the saving function as the criterion for selecting the node removal. Also, the same concept of removing is followed for insertion. We will evaluate the insertion of this request on all other on-demand vehicles. The feasible schedule that has the minimum increment of distance will be chosen. The time window and capacity constraints need to be satisfied in order to call the schedule feasible.

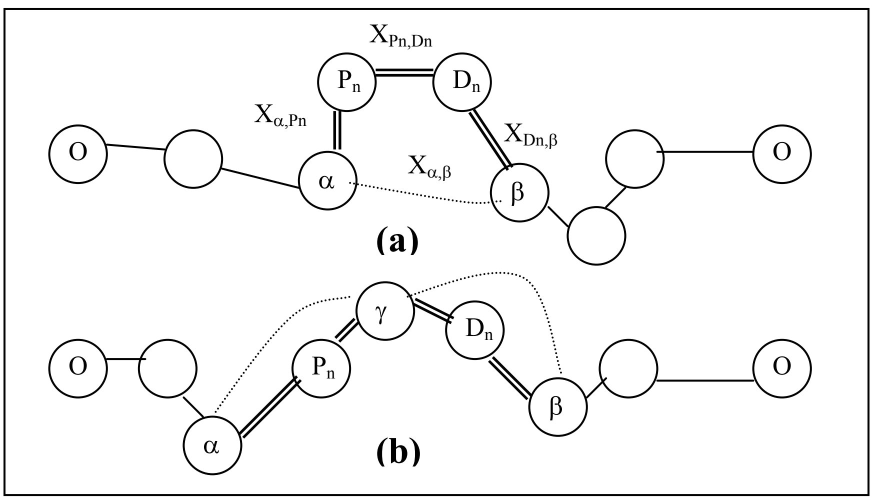


**Figure 8. Illustration of the saving concept**

The following are the steps of the Re-assigning strategy:

- Initialization: The initial schedule (SCH (0)) is obtained by the final solution of the Tabu sequencing procedure and the Tabu List is empty. Let the current best schedule (SCH*) equal SCH (0) and let the iteration index (t) equal 0. Let TVD* and TTT* be the resulting total on-demand vehicle distance and total passenger trip time.

- Stopping: If there is no non-tabu move leading to a feasible solution neighbor of the current schedule SCH (t) or t = $t_{max}$, then stop. The current SCH* is an approximate optimum schedule.
- Move: Select a non-tabu feasible move neighbor to the current schedule SCH(t) and find SCH(t+1).
  - Removing Criterion: Remove the request from vehicle (V1) that results in the largest decrease in the total distance traveled.
  - Insertion Criterion: Insert the request in vehicle (V2) that results in the smallest increase in the total distance traveled.
  - Re-sequence V1 and V2 using the Tabu re-sequencing procedure.
- Compare: If the objective function value of SCH(t+1) is superior to that of SCH*, let SCH* = SCH(t+1), set TVD*=TVD and TTT*=TTT
- Tabu List: Remove from the list of forbidden tabu requests any that have been on it for a sufficient number of iterations, and add the current request of SCH(t+1) to the list.
- Increment: Let t = t + 1, and go back to the stopping step.

## 4.0 Experimental Analysis

This section presents the computational experiments that have been performed to test the heuristics using Antelope Valley Transit Authority (AVTA). This agency was selected because the travel distances in Antelope Valley are large enough to justify a transfer point between the two different types of transit services. Furthermore, most of the disabled and elderly passengers travel to a central location where most of the hospitals are located. This is shown in Figure 9. The area around Palmdale Street (Lower circle) is mostly a residential area while the area around Avenue J-8 (Upper circle) is mostly hospitals and a commercial district. Furthermore, there are fixed bus routes that connect these two areas, which are also shown in the figure by dark lines.

Antelope Valley Transit Authority provides both fixed route and dial-a-ride services in Lancaster County, California. The fixed route portion of their service consists of ten lines with each line consisting of many bus stops. In terms of their on-demand service, we received data for the period of two weeks, February 14, 2000 to February 25, 2000 for a total consisting of nine weekdays. AVTA operates from 6:00 AM to 7:30 PM during the weekdays. The total number of requests served during these days was 1029. The number of drivers per day varied from 4 to 12 during this period (Average=9.22). The average number of requests served per day per driver was 15.79 with the maximum number being 30.33 on any given day.

Figure 10 shows a histogram of the distance traveled in miles. As the figure shows, there are a significant number of passengers traveling greater than 10 miles which may be a significant enough of a distance to justify a transfer to an accessible fixed route bus.

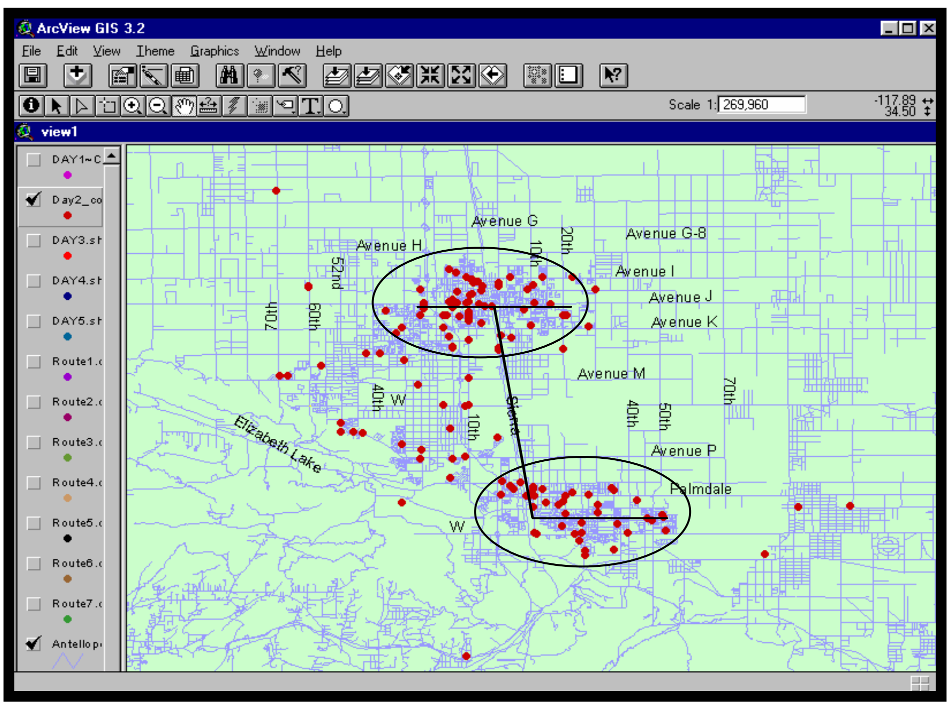


**Figure 9. Snapshot of Arcview for the AVTA data**

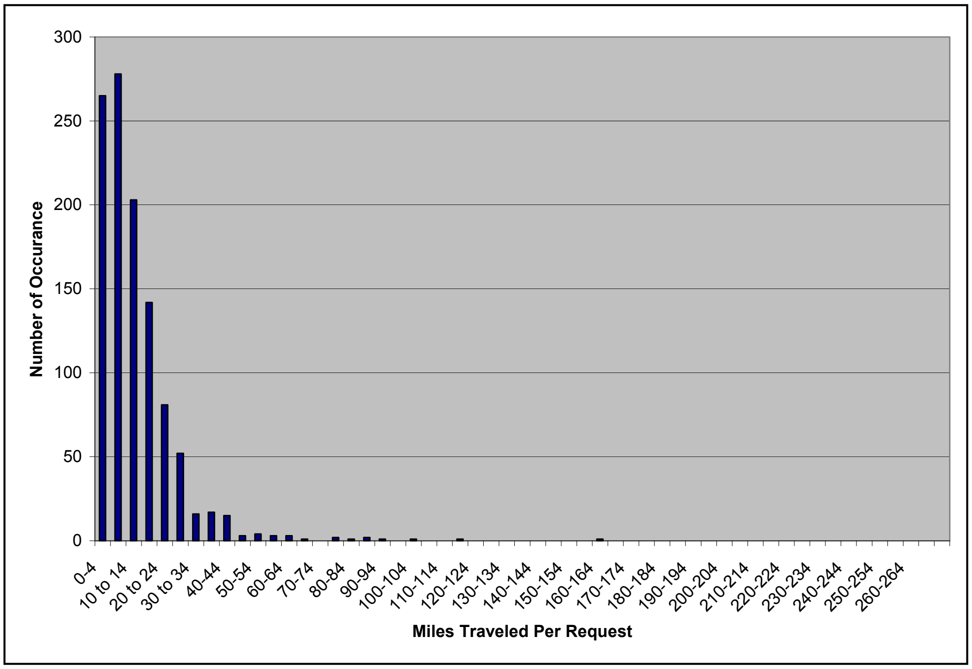


**Figure 10. Miles Traveled Per Request**

The various heuristics methods are compared using the AVTA data in terms of total on-demand vehicle distance and total customer trip time. We benchmark the results against AVTA's manual schedule. In these comparisons, we use the same number of on-demand vehicles as used by AVTA. The heuristic methods are compared to each other in terms of solution quality and computational time. We implemented the algorithms using C++ on a SUN Enterprise E4500/E5500 System.

Table 1 summarizes the AVTA data. For each day, we list the number of on-demand vehicles, the number of transportation requests, and the number of requests that are candidates for the hybrid system. We also list the F1, F2, and F3 values used for that day. In general, the selected ratios give 18.6% of the requests as candidates for the hybrid system.

We next demonstrate the analysis that we used for selecting the appropriate ratios F1 and F2. Figures 11 and 12 show the sensitivity of the vehicle distance and customer trip time to various values of the ratios F1 and F2 for one of the days using the Insertion Procedure. As the figures show, these two measures are sensitive to the ratios F1 and F2, and there is a trade off between balancing the vehicle distance and customer trip time.

For example, the combination (F1 = 0.6, F2 = 0.95) that gives the smallest vehicle distance results in a high customer trip time. Thus, for this day we use a combination of (F1 = 0.6, F2 = 1.1).

**Table 1. Summary Data**

| Day | Number of Vehicles | Requests | Hybrid Requests | F1,F2, F3 |
|---|---|---|---|---|
| 1 | 6 | 76 | 15 | .6,1.1, 9 |
| 2 | 12 | 155 | 30 | .65,1.2, 9 |
| 3 | 10 | 150 | 29 | .65,.95, 9 |
| 4 | 10 | 135 | 18 | .75,.9, 9 |
| 5 | 10 | 138 | 17 | .75,1, 9 |
| 6 | 10 | 103 | 19 | .65,.85, 9 |
| 7 | 12 | 139 | 22 | .6,.8, 9 |
| 8 | 4 | 42 | 14 | .6,.85, 9 |
| 9 | 9 | 91 | 27 | .65,.9, 9 |
| Total | 83 | 1029 | 191 | |
| Percent | | | 18.6% | |

Table 2 presents a general comparison between all the rules used in this paper in terms of total vehicle distance and total customer time. The Re-sequencing strategy is called TABU-S and the Re-assigning strategy is called TABU-A in this section. Recall, the initial solution for the TABU-S heuristic is the solution from the Improvement procedure, and the TABU-A procedure reiterates between the Re-sequencing and Re-assigning strategies. As the table shows, the hybrid scheduling delivery method (TABU-S) decreases both the total vehicle distance and total customer time by 16.6% and 8.7% respectively compared to the manual schedule that uses strictly the curb-to-curb delivery method. Table 3 shows the on-demand vehicle distance and customer trip time for each day for each routing heuristic. For the nine days, the distance traveled by the on-demand vehicles decreases as the CPU time needed to implement the heuristic increases, which shows the common trade off between the solution quality and the computational time. Also, it is shown that by applying TABU-A after TABU-S, the solutions are not significantly improved at the expense of a relatively large CPU time.

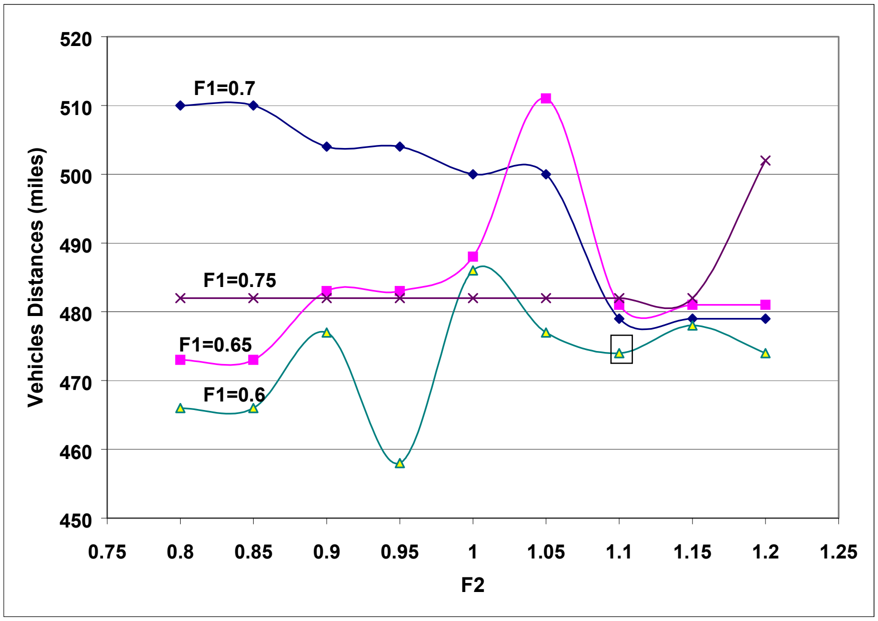


**Figure 11. Sensitivity analysis (vehicle distance)**

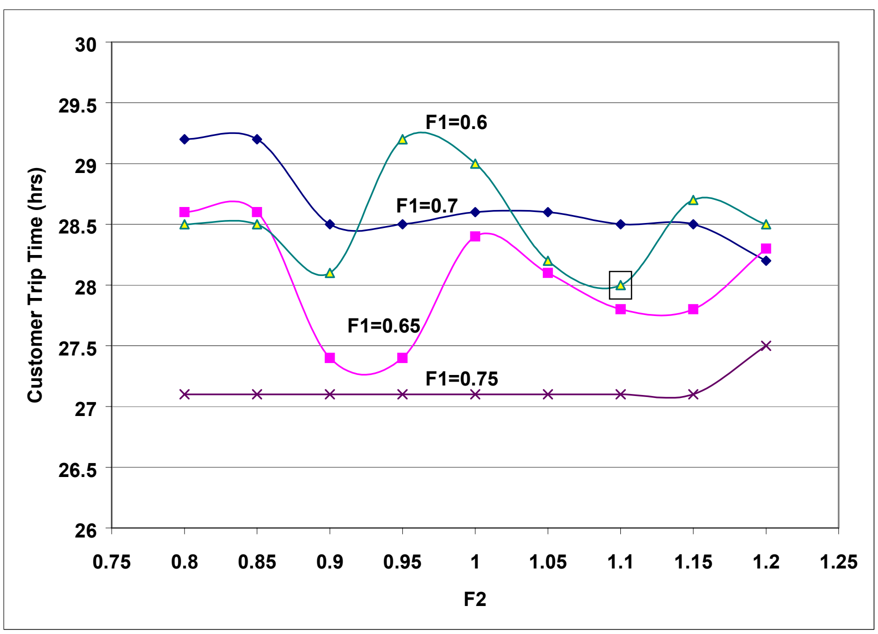


**Figure 12. Sensitivity analysis (customer time)**

**Table 2. Total Vehicle Distance and Customer Time**

| Rule | Vehicle Distance (miles) | Customer Time (hours) | (+/-) % of Manual Distance | (+/-) % of Manual Time | CPU Time (seconds) |
|---|---|---|---|---|---|
| Insertion | 6737 | 442.5 | -11.8% | -1.6% | 9 |
| Improvement | 6593 | 422.7 | -13.6% | -6.0% | 231 |
| TABU-S | 6365 | 410.6 | -16.6% | -8.7% | 623 |
| TABU-A | 6338 | 411.7 | -17.0% | -8.5% | 1357 |
| Manual | 7635 | 449.7 | 0.0% | 0.0% | NA |

**Table 3. Daily Vehicle Distance and Customer Time**

| | Insertion | | Improvement | | TABU-S | | TABU-A | |
|---|---|---|---|---|---|---|---|---|
| Day | Vehicle Distance (miles) | Customer Time (hours) | Vehicle Distance (miles) | Customer Time (hours) | Vehicle Distance (miles) | Customer Time (hours) | Vehicle Distance (miles) | Customer Time (hours) |
| 1 | 487 | 29.0 | 474 | 28.0 | 459 | 26.6 | 453 | 25.9 |
| 2 | 1001 | 67.9 | 973 | 62.6 | 949 | 61.5 | 945 | 61.4 |
| 3 | 918 | 61.3 | 915 | 60.5 | 882 | 59.3 | 881 | 59.3 |
| 4 | 809 | 47.9 | 804 | 47.8 | 787 | 46.5 | 779 | 47.1 |
| 5 | 947 | 58.1 | 915 | 55.0 | 897 | 53.4 | 897 | 53.4 |
| 6 | 694 | 49.7 | 657 | 43.6 | 640 | 42.8 | 640 | 42.8 |
| 7 | 1024 | 74.6 | 1007 | 72.5 | 946 | 69.4 | 943 | 69.4 |
| 8 | 297 | 16.5 | 295 | 16.4 | 273 | 15.7 | 273 | 15.7 |
| 9 | 560 | 37.5 | 553 | 36.3 | 532 | 35.4 | 527 | 36.7 |

We next examine the total customer trip time in more detail for the hybrid requests. Note that the previous reported trip times included all requests (i.e., hybrid and strictly door-to-door). For hybrid requests, the total trip time includes the time on the on-demand vehicles, time waiting at the entry and exit bus stops, and time on the fixed bus routes. Table 4 shows the breakdown of the total trip time for the schedules generated by the TABU-S heuristic. For example, in Day 2, of the 61.5 total hours of customer trip time 37.0 is for the hybrid requests. The last four columns show the breakdown into the various components. The entry bus stop is stop 1 and the exit bus stop is stop 2. In the table, we also list the percentage above the manual schedule for the hybrid customer trip times. When considering all the customers, we previously showed that the TABU-S heuristic generated schedules with lower total customer trip times than the manual schedule. However, when only counting the hybrid requests, Table 4 shows that for these passengers their trip times will increase moderately with an overall 5.4% increase over the manual schedule. This is to be expected since these passengers require two modes of transportation to satisfy their requests.

Another important observation from this table is that the customer waiting time at the exit bus stop is significantly less than the customer waiting time at the entry bus stop. This is due to the fact that there are time windows associated with the exit bus stop since it is considered a pickup point. Thus these windows place a restriction on the waiting time at the exit bus stop. Since the entry bus stop is considered a destination point, there are no time windows associated with these locations. We attempt to minimize the waiting time at the entry bus stops by inserting vehicle idle time if possible in order to have the passenger arrive as close as possible to the departure time of the fixed bus line.

**Table 4. Components of Customer Trip Time for TABU-S**

| Day | All Requests Total Time | Door-to-Door Requests Total Time | Hybrid Requests Total Time | % Above Manual | Hybrid Requests Time in Vehicle | Hybrid Requests Time in Bus | Hybrid Requests Time at Bus Stop1 | Hybrid Requests Time at Bus Stop2 |
|---|---|---|---|---|---|---|---|---|
| 1 | 26.6 | 14.65 | 11.95 | -7.4% | 4.1 | 6.6 | 1.2 | 0.05 |
| 2 | 61.5 | 37.0 | 24.5 | +1.3% | 8.9 | 11.2 | 3.4 | 1.0 |
| 3 | 59.3 | 33.9 | 25.4 | +1.5% | 6.7 | 11.1 | 6.6 | 1.0 |
| 4 | 46.5 | 34.1 | 12.4 | +1.8% | 4.5 | 5.3 | 1.8 | 0.8 |
| 5 | 53.4 | 41.0 | 12.4 | +6.5% | 4.5 | 5.3 | 1.3 | 1.3 |
| 6 | 42.8 | 27.1 | 15.7 | +36.5% | 7.3 | 5.8 | 1.9 | 0.7 |
| 7 | 69.4 | 51.4 | 18.0 | +17.9% | 5.1 | 8.5 | 3.6 | 0.8 |
| 8 | 15.7 | 7.2 | 8.5 | -11.6% | 2.4 | 4.0 | 1.8 | 0.3 |
| 9 | 35.4 | 15.7 | 19.7 | +5.9% | 6.3 | 9.6 | 3.0 | 0.8 |
| Total | 410.6 | 262.05 | 148.55 | +5.4% | 49.8 | 67.4 | 24.6 | 6.75 |

Although the above analysis shows that the hybrid approach outperforms the manual scheduling method used at AVTA, we next compare it against using an insertion algorithm on the data set assuming all requests are served as strictly door-to-door. Table 5 displays the results for this case. Comparing it against the Improvement Method when some of the requests are hybrid (results in Table 2), the total vehicle miles increases by 11.04% when there are no hybrid requests, but the total customer trip time decreases by 1.23%. This illustrates the benefit of using a hybrid approach in reducing the miles traveled by the on-demand vehicles. However, this may come at the expense of increased passenger trip times. Our proposed approach attempts to balance these two components in determining a schedule.

**Table 5. Comparison with Strictly Door-to-Door Service**

| | Door-to-Door | |
|---|---|---|
| Day | Vehicle Distance (miles) | Customer Time (hours) |
| 1 | 507 | 28.2 |
| 2 | 1125 | 63.2 |
| 3 | 1022 | 56.3 |
| 4 | 876 | 49.5 |
| 5 | 1012 | 55.6 |
| 6 | 704 | 41.5 |
| 7 | 1107 | 70.6 |
| 8 | 323 | 16.3 |
| 9 | 645 | 36.3 |
| Total | 7321 | 417.5 |

## 5.0 Conclusions

Since most dial-a-ride programs for the transport of elderly and disabled persons are heavily subsidized programs, the increased usage of these curb-to-curb services has put significant budget pressures on most transit agencies. In this paper, we have investigated a hybrid system consisting of both on-demand vehicles and fixed route lines for servicing this type of request. We developed a heuristic solution approach for scheduling a hybrid system.

The first heuristic procedure consists of three phases. In the first phase, all the candidate routes/paths that meet a certain criterion for each request are identified. In the second phase, a feasible path from the candidates' list that has the shortest on-demand vehicle distance is selected and inserted into the vehicle schedule. The solution of the Insertion procedure is fed into the Improvement procedure. In this procedure, we try to identify an alternative path for requests that have multiple hybrid paths that can satisfy the demand. The second type of heuristic is based on Tabu Search which uses the solution from the Improvement procedure as the initial solution.

We tested our heuristics on actual data from AVTA. Overall, this analysis showed that shifting some of the demand to a hybrid service route (18.6% of the requests) reduces the on-demand vehicle distance by 16.6% and the overall customer trip time by 8.7% over the manual schedule using the TABU-S heuristic. However, for these customers who take the hybrid delivery method (18.6% of the requests), their trip time will increase on average by 5.4%.

In terms of computation time, the Tabu methods (TABU-S and TABU-A) took considerable more CPU time in finding a solution. The Improvement heuristic was computationally efficient with a solution quality close to the Tabu methods. This characteristic is especially important when applying the proposed approach to a real-time environment. The Tabu methods may take too long computationally to apply in real-time. The next phase of our research will be targeted at adapting the Improvement heuristic to a real-time environment.

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