

REAL-TIME CONTROL OF BUSES FOR SCHEDULE COORDINATION AT A TERMINAL

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ABSTRACT

Recently, bus transit operators have begun to adopt technologies that enable bus locations to be tracked from a central location in real-time. Combined with other technologies, such as automated passenger counting and wireless communication, it is now feasible for these operators to execute a variety of real-time strategies for coordinating the movement of buses along their routes. This paper compares control strategies that depend on technologies for communication, tracking and passenger counting, to those that depend solely on local information (e.g. time that a bus arrived at a stop, and whether other connecting buses have also arrived). We also develop methods to forecast bus arrival times, which are most accurate for lines with long headways, as is usually the case in timed transfer systems. These methods are tested in simulations, which demonstrate that technology is most advantageous when the schedule slack is close to zero, when the headway is large, and when there are many connecting buses.

1. INTRODUCTION

Unlike other industries, transportation companies have traditionally had little opportunity to directly supervise their employees and resources. Drivers have typically spent entire workshifts in the field serving customers, having only infrequent contact with their supervisors. But technology is changing this situation. Global-positioning-systems, and other location technology, can be coupled with wireless communication, enabling vehicles and drivers to be tracked in real-time. Messages can be readily sent back-and-forth between driver and supervisor to convey instructions and status, and communication can be coupled with a variety of sensing and actuation devices to automatically execute control strategies.

The transit industry is no exception from the trend toward increased use of technology, including wireless communication, automated vehicle location, and sensors for counting passengers. These technologies provide a potential to control and coordinate vehicle movements, and to enhance the connectivity between bus lines at transfer points through reductions in waiting time. If transfers between infrequent bus lines can become less time consuming, it may be possible to reconfigure transit networks in a way that provides accessibility for more origin/destination pairs.

This paper compares control strategies that are enabled by communication, tracking and passenger counting technology, to those that depend solely on local information (e.g. time that a bus arrived at a stop, and whether other connecting buses have also arrived). The control consists of deciding when to dispatch buses, and when to hold them to accommodate passengers who will transfer from buses that have not yet arrived. Buses are also controlled at non-transfer stops to ensure that they do not leave earlier than scheduled.

The paper also develops methods for forecasting bus arrival times and passenger loads. These forecasting methods are based on bus tracking and passenger counting information, and therefore constitute a primary difference between local control and technology enabled control. In addition, a simulation model is developed for a wide-area transit network. The model is used to test and compare control strategies for a range of scenarios. An important feature of the simulation is that delay is computed for both the transfer stop (where buses are held) and at downstream stops, which might be affected by any holding strategy at the transfer stop.

The remainder of this paper is divided into five sections. We first review literature on transit modeling. Second, the real-time control strategies are presented. Third, we create models for arrival time and passenger load forecasting. Fourth, the simulation analysis is presented. The final section provides conclusions.

2. RELATED RESEARCH IN PROBLEM AREA

There has been extensive research on controlling transit vehicles traveling along a single line with multiple stops. In routes providing frequent service (headways of 10 minutes or less), the objective in schedule control is largely to ensure consistency in headways (time separation between vehicle arrivals or departures). Customers on short-headway lines typically do not consult schedules before arriving at their stops, and therefore arrival patterns are reasonably stationary relative to the schedule. Second, as demonstrated in Osuna and Newell (1972), average waiting time increases with the square of the coefficient of variation in the headway (ratio of standard deviation to the mean). Completely random Poisson vehicle arrivals generate twice the average wait of deterministic arrivals. In fact, waiting time can be worse than the

Poisson case, as vehicles on frequent lines have a tendency to bunch. Headways on very frequent lines are inherently unstable: when a bus falls slightly behind schedule, it tends to pick up more passengers, causing it to slow further, until it eventually bunches with the trailing bus (Newell, 1974; Barnett, 1974; Turnquist, 1978). This can be controlled, to some degree, by slowing down a trailing bus when it is catching up with the preceding bus. However, the added delay for passengers already on the trailing bus limits the applicability of this (and other) control strategies, except at the very start of lines.

The behavior of infrequent lines differs substantially from frequent lines. Customers generally do consult schedules, making arrival patterns non-stationary. Therefore, waiting time is not defined by the headway, but instead by the random deviations in the bus arrivals at the stop, along with the customer's selected arrival time relative to the schedule. Finally, because late buses generally do not pick up additional passengers, schedules tend to be much more stable. As demonstrated in Dessouky et al. (1999), these attributes, combined with slack time inserted in the schedule, lead to schedule stability. Drivers also have an incentive to catch up to the schedule since most transit agencies penalize them for being excessively late. Thus, the delay in a segment is negatively correlated with the lateness at the start of the segment.

The delay on a bus line segment can either be negatively or positively correlated with the lateness at the start of the segment, depending in large part on the line headway. A positive correlation can occur when passengers do not consult a schedule prior to arrival (as usually occurs on low headway lines), which results in increased boardings when a bus falls behind schedule. A negative correlation can occur when the slack built into the schedule is sufficiently large so that the bus can catch up with the schedule when it is running late.

There has been some work on optimizing holding and departure times at timed transfer centers. Hall (1985) examines transfers to and from a rail line, and develops formulas for optimal "safety margins" (i.e., the expected time between arrival of an inbound bus and an outbound train). Abkowitz et al (1987) simulate a variety of dispatching strategies at a timed transfer hub. Their simulation results on two bus lines show that a no holding strategy is best when the bus lines have unequal headways and a double holding strategy is best when the bus lines have equal headways. Lee and Schonfeld (1991, 1992) simultaneously optimize headways and safety margins at a timed transfer terminal. Knoppers and Muller (1995) find that it is beneficial to coordinate transfers when the variability of the arrival times of connecting buses is low.

Lin et al. (1995) focus on developing bus dispatching criteria at various stops. A holding and stop skipping criterion is analyzed and optimized based on specified cost functions. They also compare various criteria of interest under headway based and scheduled based controls. The work focuses on a single route, not taking into account transfers and transit centers. The study reveals that tight stop skipping control significantly increases the average wait time, while the most critical decision variable is the holding control parameter. Other research in the holding problem where transfers are not considered includes the work by Osuna and Newell (1972), Newell (1974), Barnett (1974, 1978), Koffman (1978), and Abkowitz (1986).

As reported in Benn (1995), bus productivity measures used in the industry focus on the bus line as the fundamental unit, and are not real-time based. He also concludes that intelligent transportation systems will lead to drastic changes in the way bus performance is measured. As an example of the potential use for tracking data, Henderson and Darapeni (1994) discuss how

the New York Transit Authority is using its subway on-time performance data within a multi-variate regression model to assess the causes of delays and develop remedies.

Our work differs from the previous research on transit modeling in several aspects. We analyze strategies that make use of real-time information such as the number of passengers transferring between lines, the bus positions, and the number of passengers waiting at any stop. In addition to taking into account the current status of the transit system, the analyzed strategies *forecast* future states of the system.

An important aspect of modeling bus systems is the probability distribution for travel time along bus line segments. Different probability distributions for the travel time and arrival time random variables have been used in past studies. Most studies used a skewed distribution such as lognormal or gamma because of driver incentives to stay close to, but not ahead of, schedule. Some authors select the probability distribution based on empirical studies (e.g., Turnquist, 1978; Andersson et al., 1979; Talley and Becker, 1987; Guenther and Hamat, 1985; Seneviratne, 1990; Strathman and Hooper, 1993) while others based their selection on model simplification (e.g., Hall, 1985; Bookbinder and Desilets, 1992; Wiransinghe and Liu, 1995).

Past studies indicate that there are two categories of passengers who board the bus: those who are aware of the scheduled arrival time and those who are not aware (Barnett, 1974; De Pirey, 1971). Aware passengers time their arrival at a stop according to the bus schedule. Unaware passengers come randomly to a stop, thereby having to wait for a longer duration of time on average. Okrent (1974) found that a headway of 12 to 13 minutes marks a transition period, where a much greater fraction of people is aware of the schedule. Similar results were obtained by Jolliffe and Hutchinson (1975) and Marguier and Ceder (1984). Coslet (1976) used utility theory to predict the arrival time of aware passengers. The study conducted by Bowman

and Turnquist (1981) shows that arrival times of passengers for high headway buses follow a skewed normal distribution, peaking just before the scheduled arrival time and fading steeply beyond it. The study also showed that the variance of the arrival distribution greatly declines as the reliability of the bus service increases. Bowman and Turnquist (1981) also found that for low headway buses, the arrival times of passengers to a stop are uniformly distributed over the duration of the headway. Similar arguments can be found in Turnquist and Bowman (1980, 1981). Hickman and Wilson (1995) study the impact of real time information regarding projected vehicle travel times on passenger route selection. An analytic model is developed that shows how real time information can be used by a passenger in selecting the best route. They demonstrate their model using data from Massachusetts Bay Transportation Authority.

In terms of boarding and alighting times, Kraft (1977) proposed the usage of a 2-Erlang distribution. Andersson et al. (1979) and Andersson and Scalia-Tomba (1980) use a gamma distribution. Turnquist and Bowman (1980) assume a normal distribution. In a simulation model analyzing on-time performance, Seneviratne (1990) referred to a high density stop as one that usually attracts a large number of passengers boarding or alighting. The number of boardings and alightings was simulated using normal and Poisson random variables for high and low density stops, respectively. We build from all the above observations in our simulation model.

3. BUS HOLDING STRATEGIES

We consider a class of bus holding strategies in this section. We assume that buses follow scheduled service, with schedules that are advertised to the public. To minimize the likelihood of leaving a passenger behind at a stop, buses are not permitted to depart before their

scheduled departure time. They may, however, leave later than their scheduled time at transfer stops, with the objective of providing connectivity for incoming buses that arrive late. Buses are not held at non-transfer stops, as no advantage is gained. The rules by which a driver decides to “hold” for a connecting bus, or depart, differentiate the control strategies.

Though executed by individual drivers, the holding strategy may depend on collection of real-time information from other buses, computation of arrival time forecasts, and automatic determination of the proper response. The types of information used in different control strategies include: (1) Schedule for connecting buses at the stop; (2) Whether or not the connecting bus has already arrived at the stop; (3) Forecast arrival time for connecting buses; (4) Forecast number of transferring passengers on connecting buses; and (5) Forecast number of boarding passengers at subsequent stops, and the criteria for deciding whether or not to hold include: (1) Whether or not forecast arrival time for connecting bus is within time window; (2) Whether or not number of connecting passengers exceeds minimum; and (3) Whether or not total waiting time for passengers already on bus and those connecting will decline if bus is held. These criteria are executed in three possible choices: (1) Dispatch immediately; or (2) hold up to a calculated maximum time; or (3) hold until all connecting buses arrive, no matter how late they are. It should be noted that the immediate dispatch choice is irrevocable, but a decision to hold can be changed at a future time, should the information warrant it, by dispatching the bus.

Table 1 outlines seven holding strategies, which are defined by different combinations of the above attributes. The following symbols are used to defined the state of the system at the time a holding strategy is executed.

- i Index of the bus being controlled at the stop
- j Index of a connecting bus approaching the stop (j=1,...,m)

S	Set of subsequent bus stops for bus i
$l(i)$	Index of the next bus arrival after bus i
AD_i	Actual departure time for bus i at current stop
SD_i	Scheduled departure time for bus i at current stop
AA_i	Actual arrival time of bus i at current stop
FA_i	Forecast arrival time of bus i at current stop
TP_{ji}	Expected number of transferring passengers from bus j to bus i
P_i	Expected number of passengers on bus i at current stop
TNOW	Current time
HT	Maximum allowed holding time beyond scheduled departure time
TV	Threshold value for number of connecting passengers
$E(P_s)$	Expected number of boarding passengers at subsequent stop s for bus i
$FA_{i,s AD_i=t}$	Forecast arrival time of bus i at subsequent stop s given the actual departure time at the current stop is t .

Local Strategies

1. No Hold for Connecting Buses

$$AD_i = \text{Max} (AA_i, SD_i) \quad (1)$$

2. Hold for All Connecting Buses

$$AD_i = \text{Max} (AA_1, \dots, AA_m, SD_i) \quad (2)$$

3. Hold for Connecting Buses, but no More than Maximum Holding Time

$$AD_i = \text{Min} (\text{Max}(AA_1, \dots, AA_m, SD_i), \text{Max}(AA_i, (SD_i + HT))) \quad (3)$$

Technology Enabled Strategies

4. Hold if Forecast Arrival Time of Connecting Bus is within a Maximum Holding Time

$$AD_i = \text{Max} (SD_i, AA_i, \text{Max} (t: t < (SD_i + HT) \text{ for } t = FA_1, FA_2, \dots, FA_m)) \quad (4)$$

5. Hold if Forecast Arrival Time of Connecting Bus is within a Maximum Holding Time and the Number of Connecting Passengers is greater than a Threshold Value

$$AD_i = \text{Max} (SD_i, AA_i, \text{Max} (t: t < (SD_i + HT) \text{ and } (\sum_{j|FA_j < t} TP_{ji} > TV) \text{ for } t = FA_1, FA_2, \dots, FA_m)) \quad (5)$$

6. Holding Time is based on Minimizing the Total Passenger Waiting Time at the Current Stop

$$AD_i = (t : \text{Min} (\text{Max}(0, P_i (t - \text{Max}(SD_i, TNOW))) + \sum_{j|FA_j < t} (t - FA_j) TP_{ji} + \sum_{j|FA_j > t} (FA_{l(i)} - FA_j) TP_{ji} \text{ for } t = TNOW, FA_1, FA_2, \dots, FA_m)) \quad (6)$$

7. Holding Time is based on Minimizing the Total Passenger Waiting Time at the Current and Downstream Stops

$$AD_i = (t : \text{Min} (\text{Max}(0, P_i (t - \text{Max}(SD_i, TNOW))) + \sum_{j|FA_j < t} (t - FA_j) TP_{ji} + \sum_{j|FA_j > t} (FA_{l(i)} - FA_j) TP_{ji} + \sum_S E(P_s)(\text{Max} (0, FA_{i,s|AD_i=t} - SD_{i,s})) \text{ for } t = TNOW, FA_1, FA_2, \dots, FA_m)) \quad (7)$$

Strategies 1 (No Hold) and Strategy 2 (All Hold) are two extremes relative to favoring passengers already on holding buses versus passengers on connecting buses. Strategy 3 falls between these extremes by placing an upper bound on holding time. Strategies 4 - 7 require forecast arrival times for connecting buses (hence, they rely on bus tracking). Strategies 5 - 7 additionally require forecast passengers (hence, they rely on passenger counting), and Strategies 6 and 7 additionally account for the net change in waiting time due to holding. That is, the holding time is based on selecting the time that minimizes the total passenger waiting time. Strategy 6 only considers the local impact where the bus is being held, whereas Strategy 7 also accounts for changes in waiting downstream from the stop. In addition, Strategies 6 and 7 do not preset the maximum holding time. Instead, the maximum is determined from minimizing the

total waiting time among passengers currently at the stops and those forecast to arrive before the bus arrives.

4. FORECASTING METHODS

With tracking and communication, bus arrival times can be forecast for a transfer terminal based on their current locations. To do this, we first assume (as is customary) that a bus line can include stops that appear in the bus schedule along with intermediate stops that do not appear in the schedule. The forecast is updated each time the bus arrives at, or departs from, a scheduled stop. However, forecasts cannot be made at intermediate stops because no schedule exists. Since arrival and departure times are not identical, the forecast made at departure from a scheduled stop can be different from the one made on arrival.

Consistent with studies by Turnquist (1978) and Strathman and Hopper (1993), we assume a lognormal distribution for actual travel time. A segment is defined as the portion of the line connecting a pair of adjacent scheduled stops (i.e., segment k connects scheduled stop $k-1$ to scheduled stop k). We define the parameter $\gamma_{i,k}$ as the ratio of expected travel time to scheduled travel time on segment k of bus i :

$$E(AT_{i,k}) = \gamma_{i,k} ST_{i,k} \quad (8)$$

where

$AT_{i,k}$: actual travel time from departure at stop $k-1$ to arrival at stop k for bus i

$ST_{i,k}$: scheduled travel time from stop $k-1$ to stop k for bus i

It should be noted our calculation of travel time represents the portion of driving time that is outside of the operator's control. Any additional holding time (for schedule control purposes) is treated separately. Thus, holding time is captured separately from actual travel time.

The value $(1-\gamma_{i,k})$ represents the slack as a proportion of scheduled time in segment k for bus i . Analyzing data from Los Angeles County, Dessouky et al. (1999) found the slack proportion to be on the order of .25. For this reason, buses frequently arrive early at scheduled stops. When this happens, the bus will wait until the scheduled departure time before leaving. On the other hand, if a bus arrives late at a stop it will leave as soon as passengers have boarded. Between scheduled stops, the control strategy is at the driver's discretion. Any effects of intermediate control are assumed to be captured within the distribution for actual travel time, with mean $E(AT_{i,k})$ and variance $\sigma_{i,k}^2$.

A further assumption is that the actual travel time on a segment is independent of the departure time from the stop at the start of the segment. This assumption is clearly most appropriate for infrequent bus lines, for which the number of boarding passengers is not greatly influenced by schedule perturbations. When correlations exist, the model would need to be generalized to account for the dependency. It should be noted, however, that the combined values of actual travel time and holding time on segments are not independent. For instance, if a bus is late on one segment, it is less likely that it will need to be held on the subsequent segment. Thus the model accounts for these negative correlations as observed in Dessouky et al. (1999).

In the analytic model, we assume dwell times are zero. However, we incorporate the dwell times in the simulation. Since in the simulation we model in detail the passenger boarding process, our simulation accounts for the well known bus bunching phenomenon due to increased passenger boardings when a bus is running behind schedule.

Other terms are defined as follows:

- $AA_{i,k}$: actual arrival time at stop k for bus i
 $AD_{i,k}$: actual departure time from stop k for bus i
 $SA_{i,k}$: scheduled arrival time at stop k for bus i
 $SD_{i,k}$: scheduled departure time from stop k for bus i
 $FA_{i,k,k+n}$: forecast arrival time at stop k+n for bus i made at stop k
 $FD_{i,k,k+n}$: forecast departure time from stop k+n for bus i made at stop k
 $f_{i,k}(t)$: probability density function of actual travel time on segment k for bus i

We define the forecast arrival and departure time as the conditional expectations:

$$FA_{i,k,k+n} = E(AA_{i,k+n} | AD_{i,k}) \quad (9)$$

$$FD_{i,k,k+n} = E(AD_{i,k+n} | AD_{i,k}) \quad (10)$$

The forecast arrival time at the next stop is the actual departure time from the previous stop plus the expected travel time over the next segment:

$$FA_{i,k,k+1} = AD_{i,k} + E(AT_{i,k+1}) \quad (11)$$

Since a bus cannot leave earlier than scheduled, the actual departure time ($AD_{i,k+1}$) at a stop is the maximum of the actual arrival time and the scheduled departure time:

$$AD_{i,k+1} = \max(AA_{i,k+1}, SD_{i,k+1}) \quad (12)$$

Thus, the forecast departure time is given by the following equation

$$FD_{i,k,k+1} = E(AD_{i,k+1} | AD_{i,k}) = SD_{i,k+1} \int_{AD_{i,k}}^{SD_{i,k+1}} g_{i,k+1}(t) dt + \int_{SD_{i,k+1}}^{\infty} t g_{i,k+1}(t) dt \quad (13)$$

where $g_{i,k+1}(t)$ is the probability density function of $AA_{i,k+1}|AD_{i,k}$, which is $f_{i,k+1}(t)$ shifted by $AD_{i,k}$.

The variance of the actual departure time can be used to derive the forecast for subsequent stops and is given by the following equation:

$$\begin{aligned} \text{Var}(AD_{i,k+1} | AD_{i,k}) &= E(AD_{i,k+1}^2 | AD_{i,k}) - E^2(AD_{i,k+1} | AD_{i,k}) \\ &= SD_{i,k+1}^2 \int_{AD_{i,k}}^{SD_{i,k+1}} g_{i,k+1}(t) dt + \int_{SD_{i,k+1}}^{\infty} t^2 g_{i,k+1}(t) dt - FD_{i,k,k+1}^2 \end{aligned} \quad (14)$$

To forecast the arrival and departure times for stop k+2, when the bus is at stop k, the following equations can be used.

$$FA_{i,k,k+2} = E(AA_{i,k+2} | AD_{i,k}) = E(AD_{i,k+1} | AD_{i,k}) + E(AT_{i,k+2}) = FD_{i,k,k+1} + E(AT_{i,k+2}) \quad (15)$$

$$FD_{i,k,k+2} = E(AD_{i,k+2} | AD_{i,k}) = SD_{i,k+2} \int_{FD_{i,k,k+1}}^{SD_{i,k+2}} g_{i,k+2}(t) dt + \int_{SD_{i,k+2}}^{\infty} t g_{i,k+2}(t) dt \quad (16)$$

where $g_{i,k+2}(t)$ is the probability density function of $AA_{i,k+2}|AD_{i,k}$. Since the actual departure time from stop k+1 ($AD_{i,k+1}$) is not known at time $AD_{i,k}$, we use the forecast estimate of $AD_{i,k+1}$ ($FD_{i,k,k+1}$) in Eq. 16. The variance can be calculated as in Eq. 14, substituting density function $g_{i,k+2}(t)$ for $g_{i,k+1}(t)$, and increasing other subscripts accordingly. We note that the density $g_{i,k+2}(t)$ is not simply $f_{i,k+2}(t)$ shifted to the right by $FD_{i,k,k+1}$. In fact the density function of $g_{i,k+2}(t)$ is not lognormal.

The arrival time at stop k+2 equals the departure time at stop k+1 plus the travel time from stop k+1 to stop k+2. These two random variables are assumed to be independent, and therefore the variance of $AA_{i,k+2}|AD_{i,k}$ equals $\text{VAR}(AD_{i,k+1}|AD_{i,k}) + \sigma_{i,k+2}^2$. The mean of $AA_{i,k+2}|AD_{i,k}$ is then $FD_{i,k,k+1} + E(AT_{i,k+2})$.

The calculations certainly become more complex for stops that are further away. As a simplification, we approximate $g_{i,k+2}(t)$ with the lognormal distribution, which we use iteratively to forecast the arrival and departure time at stop $k+n$ when the bus is at stop k :

$$FA_{i,k,k+n} = FD_{i,k,k+n-1} + E(AT_{i,k+n}) \quad (17)$$

where $FD_{i,k,k} = AD_{i,k}$

$$\text{Var}(AA_{i,k+n} | AD_{i,k}) = \text{Var}(AD_{i,k+n-1} | AD_{i,k}) + \sigma_{i,k+n}^2 \quad (18)$$

where $\text{Var}(AD_{i,k}) = 0$

$$FD_{i,k,k+n} = SD_{i,k+n} \int_{FD_{i,k,k+n-1}}^{SD_{i,k+n}} g_{i,k+n}(t) dt + \int_{SD_{i,k+n}}^{\infty} t g_{i,k+n}(t) dt \quad (19)$$

$$\begin{aligned} \text{Var}(AD_{i,k+n} | AD_{i,k}) &= E(AD_{i,k+n}^2 | AD_{i,k}) - E^2(AD_{i,k+n} | AD_{i,k}) \\ &= SD_{i,k+n}^2 \int_{FD_{i,k,k+n-1}}^{SD_{i,k+n}} g_{i,k+n}(t) dt + \int_{SD_{i,k+n}}^{\infty} t^2 g_{i,k+n}(t) dt - FD_{i,k,k+n}^2 \end{aligned} \quad (20)$$

We remark that if the buses are running very late, when the forecast departure time of the previous stop is greater than the scheduled departure time of the successive stop ($FD_{i,k,k+n-1} > SD_{i,k+n}$), Eqs. (19) and (20) are no longer valid since the lower limit of the first integral will be greater than the upper limit. Hence, when this occurs, Eqs. (19) and (20) become

$$FD_{i,k,k+n} = \int_{FD_{i,k,k+n-1}}^{\infty} t g_{i,k+n}(t) dt \quad (21)$$

$$\text{Var}(AD_{i,k+n} | AD_{i,k}) = \int_{FD_{i,k,k+n-1}}^{\infty} t^2 g_{i,k+n}(t) dt - FD_{i,k,k+n}^2 \quad (22)$$

We now test the error of the forecast arrival and departure times using the approximation technique. The error should be greatest for values of γ in the vicinity of 1. As γ approaches zero (slack proportion approaches 1), the probability of a late arrival approaches 0. Hence the actual departure time is easily forecast as the scheduled departure time. When γ is large (negative slack), most buses will arrive late. Hence, the actual arrival time and actual departure time are almost identical. Furthermore, for these two cases, the forecasts do not depend on the shape of the density function $g_{i,k}(t)$. The least intuitive and difficult forecasts to obtain are the cases when slack is approximately zero, at which point the truncation in Eq. 19 has the greatest effect on the results.

The approximation is tested for $\gamma = 1$. We assume ten stops in our simulations. The actual travel times on each segment were sampled from a lognormal distribution with expected travel time of 2.5 minutes. Tables 2, 3, and 4 show the results for different ranges of the standard deviation of the travel time on each segment ($\sigma = 0.5, 1.5, \text{ and } 2.5$). In each table, two sets of results are provided. The first set of results show the mean absolute difference between the forecast and actual arrival times averaged over 500 runs as the bus moves along its route. For example, the entries in the first row represent the forecasts made at the first stop for subsequent stops. The second set of results show the mean absolute difference between the forecast departure time and the actual departure times. As the tables show, the forecasts made by the above approximation are relatively close to the actual values, especially for cases when the standard deviation is small and the buses are closer to the stop. As expected, in all cases the forecasts are more accurate (i.e., closer to actual values) as the bus gets closer to the stop.

Passenger Forecast

We also forecast passenger loads at downstream stops, with one forecast sent on arrival and another at departure. The expected load size depends on the number of originating passengers, number of passengers continuing on the bus, and the number of passengers transferring to the bus. We describe the approach used in the model next.

$$FP_{i,k+1} = P_{i,k} * C_{i,k} + O_{i,k} + TP_{i,k} \quad (22)$$

where:

$FP_{i,k}$	The forecast number of passengers on bus i when arriving to stop k
$P_{i,k}$	The number of passengers on board bus i at stop k
$C_{i,k}$	The fraction of continuing passengers on bus i at stop k
$O_{i,k}$	The number of originating passengers at stop k for bus i
$TP_{i,k}$	The number of transferring passengers at stop k to bus i

The number of continuing passengers at stop k is dependent on the fraction of passengers that remain on board the bus at stop k. The number of transferring passengers, $TP_{i,k}$, to bus i at stop k depends on the number of connecting buses. For each connecting bus, we need to forecast its arrival time and the number of passengers on board the bus. A transferring passenger is assumed to be able to make the connection to bus i at stop k if the forecast arrival time of the connecting bus is less than the forecast departure time of bus i. Our estimate for $TP_{i,k}$ is as follows.

$$TP_{i,k} = \sum_j FP_{j,k} * C_{j,i,k} * I_{j,i} \quad (23)$$

where:

- $FP_{j,k}$ The forecast number of passengers on bus j at stop k
- $C_{j,i,k}$ The fraction of passengers transferring from bus j to bus i at stop k
- $I_{j,i,k}$ Equals 1 if the forecast arrival time of bus j is before the forecast departure of bus i at stop k ($FA_{jk} < FD_{ik}$)

5. SIMULATION ANALYSIS

A simulation model of a wide-area transit network was developed to evaluate the control strategies. The model was developed using a general-purpose simulation language, AweSim (Pritsker, 1997). The advantage of using a process-oriented language to model bus operations is that a small network model, which has the flexibility to test many different control strategies, can be used to represent detailed bus movement. The simulation model is generic and independent of any dedicated transit network. The model has high flexibility and can be used to simulate different kinds of transit networks with varying number of bus lines and different travel patterns. The user has the flexibility to input the appropriate control strategy at each stop. With this approach a replica of an actual system can be simulated.

The simulation models in detail the actual bus movement process and the passenger boarding and alighting process. The passenger boarding and alighting times are modeled as Gamma random variables. Alighting and boarding take place simultaneously at a stop. The delay to the system due to passengers' boarding is not simply the sum of the individual passenger boarding times. It depends on numerous factors including passenger arrival times, number of passenger boarding or alighting, bus arrival time, and holding time. There are many scenarios in

which the total passenger boarding time is not simply the sum of the individual boarding times. We present several of these scenarios next.

- If all the originating passengers have arrived while the bus is waiting for the scheduled departure time, then there is no additional delay to the bus departure time due to passenger's boarding. If an originating passenger couldn't complete their boarding by the scheduled departure time, then the incremental time difference is added to the actual departure time.
- If the bus is holding for another bus, then the actual departure is delayed by the boarding process and will include the boarding time of the transfer passengers and any originating passengers who couldn't have completed their boarding process before the arrival of the transfer passengers.
- For the alighting process, the actual departure time is only delayed if the passengers could not get off the bus by either the scheduled departure time or the holding time. When this is not the case, again only the incremental time difference is added to the actual bus departure time.

Because of all these complicating factors, it is difficult to explicitly account for the boarding and alighting times in the forecasting model. One method to account for them in the forecasting model is to slightly increase the travel time to represent this additional delay. We chose to explicitly represent the passenger boarding and alighting process in the simulation model to test the forecasting model and control strategies under realistic and as close as possible to actual situations.

The scheduled departure times at each major stop for each bus line are input to the model. The scheduled travel time between major stops defines a particular segment along a bus line. The model simulates the movement of buses on each segment of its line until it finishes visiting

all scheduled stops. The actual travel time on each segment is sampled from a lognormal distribution with mean $E(AT_{i,k})$. Consistent with the simulation model of Seneviratne (1990), we assume that the number of passengers that arrive between scheduled bus departures is Poisson distributed. Based on the observations of Bowman and Turnquist (1981), the arriving passengers are randomly categorized into aware and unaware passengers. The arrival time for each unaware passenger is simulated as a uniform random variable over the interval between scheduled departures. The arrival time for aware passengers is simulated as a normal random variable with a mean of one minute before the bus is scheduled to arrive at the stop. It should be noted that within the simulation, passengers can board a different bus than intended, depending on the actual bus arrival times (which can differ from schedule).

The primary evaluation criterion is the average passenger trip time, where trip time is defined to be the arrival time at the final bus stop minus the *scheduled* departure time at the initial bus stop. We assume that passenger arrival times at their initial stop can depend on the scheduled time (i.e., as it appears in a printed schedule), and deviations between their arrival time at the stop and the printed time are constant across all control strategies. Hence, differences in travel time among control strategies are captured by the arrival time at final stop minus scheduled departure time at the initial stop. Note that included in the trip time is the waiting time (and transfer time) of the passenger at all intermediate stops.

Another evaluation criterion is the total passenger *delay*. The definition of delay depends on the phase of the trip that the passenger is currently in. The three phases of a passenger trip are: at stop (originating), on board bus, and transferring to another line. Note that all passengers will experience the first two phases and some may not go through the transfer phase. For passengers already on board, passenger delay is the difference between actual bus departure time

and the scheduled departure time, minus the bus lateness at the previous stop. The lateness is subtracted to insure the delay is not double counted for multiple stops. The passenger delay is then the maximum of this value and zero. For originating and transfer passengers, the delay is defined to be the maximum of zero and the difference between the actual departure time and the scheduled departure time. For originating passengers, we use the passenger's scheduled departure time if they arrive either early or on-time. If the passenger arrives late, we use the scheduled departure time of the next bus to arrive. This ensures that we do not penalize the system for passengers missing their bus because of passenger late arrival. For transferring passengers, the scheduled departure time is for the bus line the passenger is coordinated to meet irrespective of the passenger's actual arrival time at the stop. Thus, if the passenger arrives to the stop late and misses the connection, the delay penalty will be large since they will have to wait for the next bus arrival.

Experimental Design

Bus networks are defined by numerous factors, including the rate of passenger boardings, distribution of arrival times, amount of slack in the schedule, number of transfer passengers, headway of buses, etc. To treat all of these parameters as exogenous controllable variables would create so many alternatives that meaningful comparisons of the performance measures would be virtually impossible. The focus of this experimental design is to study the impact of the various control strategies on the average passenger trip time as a function of the parameters γ (assumed constant for all segments), H (bus headway), and N (number of connecting bus lines at a transfer station). Hence, these experiments are designed to provide a guideline for selecting appropriate holding strategies for specific system parameters.

The base case of the sensitivity study is $\gamma = 1$, $H = 60$, and $N = 5$. When one of the three factors is studied, the other two factors are set at their base case values. We next describe the other characteristics of the bus network that are held fixed for the purpose of this analysis.

The actual travel times on each segment were sampled from a lognormal distribution with expected travel time of 2.5 minutes and standard deviation of 1.5 minutes. We assume that 2 passengers, on average, arrive during a headway at a stop, and that 50% are “aware” of schedules. The mean boarding and alighting times per passenger are set to 4.2 and 2.1 seconds, as proposed by Koffman (1978). Because of the presence of unaware passengers, the simulation can produce negative correlations in travel time between sequential bus runs. This possibility was included to provide for additional realism that would be difficult to capture in the forecasting model, and to provide a more realistic test for the forecasts.

Each bus line has 12 stops in total, and stop 6 acts as a timed transfer station where all N lines coverage. A center location is selected for timed transfer so that the effect of holding on passenger waiting times at subsequent stops can be evaluated. All stops except for the timed transfer stop (stop 6) use a no hold strategy since these stops have no timed passenger transfers. At stop 6, each passenger has a .50 probability of continuing on the bus and .50 probability of transferring with equal probability allocated to each connecting bus. Here, we assume that there is equal probability that passengers will continue their trip along the original bus line or transfer to other lines. Also, transfer passengers are assumed to transfer between busses with equal probability.

Different holding strategies are evaluated at the timed transfer stop only. Preliminary experimentation was used to determine the best holding parameters for strategies 3-5. The experiments showed that in most scenarios a maximum holding time of 3 minutes minimized the

average trip time. In comparing the different holding strategies for a given scenario, we set the input control parameters for strategies 3-5 at the values that minimize the average passenger trip time.

In our simulations the standard errors of the average passenger trip times were on the order of 0.05 minutes. This small standard error is due to the fact that we ran the simulation for a long time (100000 minutes) with multiple runs. Thus, any differences in the means are in most cases statistically significant.

Experimental Results

Figure 1 shows the average passenger trip time as a function of γ , holding the scheduled travel time constant, with $H = 60$ and $N = 5$. As expected, the passenger trip time decreases with a lower γ since system performance should improve as the expected travel time declines. In general, the technology enabled strategies tend to outperform the local strategies. Therefore, the results show the benefit of being able to forecast arrival times in order to determine the best holding times. In fact, for all values of γ shown, the most technology enabled strategy (Strategy 7) performs best, especially for γ in the vicinity of 1. Among the local control strategies, an all hold strategy is better than a no hold strategy. When γ is small, there is no significant difference among the strategies when $H = 60$ and $N = 5$, as the schedule slack greatly eliminates the need for holding. As the figures show, as γ increases the actual arrivals will start becoming much greater than the scheduled times. We note that this model will no longer be valid as γ approaches infinity because the average passenger trip time will be infinity in this case.

In the experiments the only difference in the control logic is applied at the transfer station (Stop 6). Hence, the primary differences in the trip time are the delay incurred by the passengers at stop 6 and the subsequent delay at downstream stops due to holding the bus at Stop 6.

Figure 2 plots the total passenger delay at the transfer station (Stop 6) and Figure 3 plots the total passenger delay at the subsequent stops (Stops 7-12). From these plots, we can see that the all hold strategy provides shorter delay at stop 6 by sacrificing performance from stop 7 to 12, while the no hold strategy minimizes the delay for the subsequent stops. The overall gain or loss in system performance is determined by a tradeoff between these two delay components: time saving for transfer passengers because of holding and delay for on-board or originating passengers at subsequent stops. By forecasting the bus arrival times and the number passengers on-board the bus, Strategy 7 accounts for these two parts of delay and attempts to achieve a global optimum.

We next study the effect of headway on system performance under the different holding strategies (Figure 4). As expected, when headway is small there is no significant difference among the strategies. Otherwise, the global optimized strategy 7 is again the best performing strategy. The benefit is most significant when the headway is large. Another interesting observation is that for all strategies except strategy 2 (all hold), system performance gets worse when headway increases. It can be intuitively explained that for bus lines with high headway, a delay related to missing a connection is expected to be longer.

Figure 5 shows the average trip time as a function of the number of connecting buses, N . When N increases, an all hold strategy becomes the worst performing strategy since it does not make sense to hold all the connecting bus lines for just one late bus. For a small N , a no hold strategy is the worst performing strategy since in this case it is beneficial to wait for all of the

buses to arrive. Again, Strategy 7 attempts to balance the tradeoff between these two rules in determining the dispatching time.

6. CONCLUSIONS

A simulation model of a wide-area transit network was developed to evaluate localized, and technology enabled (e.g., bus tracking and passenger counting), strategies for controlling departure times from stops. For the technology-enabled strategies we developed methods to forecast bus arrival times and passenger loads.

In terms of minimizing the average passenger trip time, technology provided benefits. The most technology enabled strategy (Strategy 7), which determines the holding time by minimizing the total waiting time among passengers currently at the stops and those forecast to arrive before the bus arrives, performed best. This strategy tends to balance the time saved for late-arriving transfer passengers, due to holding, against the delay for passengers who are already on-board, or will board at subsequent stops. This strategy is most advantageous when the schedule slack is close to zero, when the headway is large, and when there are many connecting buses.

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Table 1. Holding Strategies

Strategy		Information Used					Holding Criteria			Implementation		
		Schedule	Connecting Bus Arrived/Not Arrived	Forecasted Arrival Times	Number of Transferring Passengers	Number of Passengers at Subsequent Stops	Forecast Arrival Within Time Window	Transferring Passengers > Minimum	Net Reduction in Waiting Time	Maximum Holding Time	Hold Indefinitely	No Hold
1	No Hold											X
2	All Hold		X							X		
3	Maximum Scheduled	X	X							X		
4	Forecast Time			X			X			X		
5	Forecast Time/Pass			X	X		X	X		X		
6	Net Wait at Stop			X	X		X	X	X	X		
7	Net System Wait			X	X	X	X	X	X	X		

Table 2. Forecast Arrival/Departure Time when $ST=2.5$, $\sigma = 0.5$, $\gamma = 1.0$

Mean Absolute Difference Between Forecast and Actual Arrival Time (min)

	Stop 2	Stop 3	Stop 4	Stop 5	Stop 6	Stop 7	Stop 8	Stop 9	Stop 10
Stop 1	0.427	0.453	0.563	0.631	0.656	0.697	0.820	0.894	1.062
Stop 2	0.000	0.393	0.500	0.573	0.615	0.665	0.770	0.833	0.984
Stop 3	0.000	0.000	0.410	0.531	0.576	0.614	0.711	0.776	0.907
Stop 4	0.000	0.000	0.000	0.434	0.510	0.565	0.688	0.735	0.843
Stop 5	0.000	0.000	0.000	0.000	0.383	0.504	0.647	0.668	0.743
Stop 6	0.000	0.000	0.000	0.000	0.000	0.376	0.569	0.606	0.673
Stop 7	0.000	0.000	0.000	0.000	0.000	0.000	0.425	0.517	0.607
Stop 8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.361	0.507
Stop 9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.412

Mean Absolute Difference Between Forecast and Actual Departure time (min)

	Stop 2	Stop 3	Stop 4	Stop 5	Stop 6	Stop 7	Stop 8	Stop 9	Stop 10
Stop 1	0.249	0.340	0.451	0.564	0.619	0.689	0.831	0.956	1.169
Stop 2	0.000	0.285	0.388	0.499	0.573	0.643	0.767	0.873	1.055
Stop 3	0.000	0.000	0.308	0.460	0.528	0.575	0.683	0.806	0.946
Stop 4	0.000	0.000	0.000	0.360	0.459	0.524	0.645	0.747	0.848
Stop 5	0.000	0.000	0.000	0.000	0.335	0.453	0.585	0.660	0.731
Stop 6	0.000	0.000	0.000	0.000	0.000	0.323	0.498	0.576	0.634
Stop 7	0.000	0.000	0.000	0.000	0.000	0.000	0.356	0.470	0.546
Stop 8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.315	0.443
Stop 9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.336

Table 3. Forecast Arrival/Departure Time when $ST=2.5$, $\sigma = 1.5$, $\gamma = 1.0$

Mean Absolute Difference Between Forecast and Actual Arrival Time (min)

	Stop 2	Stop 3	Stop 4	Stop 5	Stop 6	Stop 7	Stop 8	Stop 9	Stop 10
Stop 1	1.062	1.383	2.065	3.195	3.518	3.754	3.906	3.934	3.972
Stop 2	0.000	1.126	1.504	2.170	3.129	3.405	3.814	3.912	3.967
Stop 3	0.000	0.000	1.149	1.556	2.305	3.013	3.466	3.816	3.944
Stop 4	0.000	0.000	0.000	1.097	1.551	2.339	3.073	3.458	3.843
Stop 5	0.000	0.000	0.000	0.000	1.153	1.521	2.507	3.061	3.479
Stop 6	0.000	0.000	0.000	0.000	0.000	1.062	1.522	2.649	3.067
Stop 7	0.000	0.000	0.000	0.000	0.000	0.000	1.079	1.511	2.691
Stop 8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.117	1.511
Stop 9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.043

Mean Absolute Difference Between Forecast and Actual Departure Time (min)

	Stop 2	Stop 3	Stop 4	Stop 5	Stop 6	Stop 7	Stop 8	Stop 9	Stop 10
Stop 1	0.709	1.619	2.913	3.361	3.653	3.695	3.766	3.809	3.840
Stop 2	0.000	0.860	1.771	2.916	3.289	3.596	3.743	3.803	3.839
Stop 3	0.000	0.000	0.934	1.955	2.866	3.223	3.643	3.779	3.833
Stop 4	0.000	0.000	0.000	0.932	2.100	2.803	3.273	3.675	3.808
Stop 5	0.000	0.000	0.000	0.000	0.990	2.151	2.856	3.298	3.700
Stop 6	0.000	0.000	0.000	0.000	0.000	0.919	2.339	2.874	3.316
Stop 7	0.000	0.000	0.000	0.000	0.000	0.000	0.937	2.501	2.884
Stop 8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.983	2.543
Stop 9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.901

Table 4. Forecast Arrival/Departure Time when $ST=2.5$, $\sigma = 2.5$, $\gamma = 1.0$

Mean Absolute Difference Between Forecast and Actual Arrival Time (min)

	Stop 2	Stop 3	Stop 4	Stop 5	Stop 6	Stop 7	Stop 8	Stop 9	Stop 10
Stop 1	1.567	2.187	3.023	3.847	3.961	4.114	4.342	4.521	4.533
Stop 2	0.000	1.679	2.375	3.325	3.969	3.971	4.299	4.509	4.529
Stop 3	0.000	0.000	1.742	2.473	3.627	3.967	4.176	4.468	4.516
Stop 4	0.000	0.000	0.000	1.655	2.437	3.758	4.164	4.355	4.473
Stop 5	0.000	0.000	0.000	0.000	1.723	2.360	4.066	4.339	4.357
Stop 6	0.000	0.000	0.000	0.000	0.000	1.571	2.328	4.358	4.341
Stop 7	0.000	0.000	0.000	0.000	0.000	0.000	1.588	2.365	4.430
Stop 8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.686	2.335
Stop 9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.510

Mean Absolute Difference Between Forecast and Actual Departure Time (min)

	Stop 2	Stop 3	Stop 4	Stop 5	Stop 6	Stop 7	Stop 8	Stop 9	Stop 10
Stop 1	1.122	2.119	3.216	3.424	3.735	3.850	4.026	4.236	4.253
Stop 2	0.000	1.324	2.575	3.432	3.576	3.797	4.012	4.232	4.252
Stop 3	0.000	0.000	1.459	2.977	3.568	3.648	3.965	4.218	4.248
Stop 4	0.000	0.000	0.000	1.427	3.300	3.634	3.834	4.174	4.234
Stop 5	0.000	0.000	0.000	0.000	1.493	3.471	3.819	4.054	4.188
Stop 6	0.000	0.000	0.000	0.000	0.000	1.366	3.774	4.040	4.066
Stop 7	0.000	0.000	0.000	0.000	0.000	0.000	1.389	4.100	4.043
Stop 8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.505	4.169
Stop 9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.323

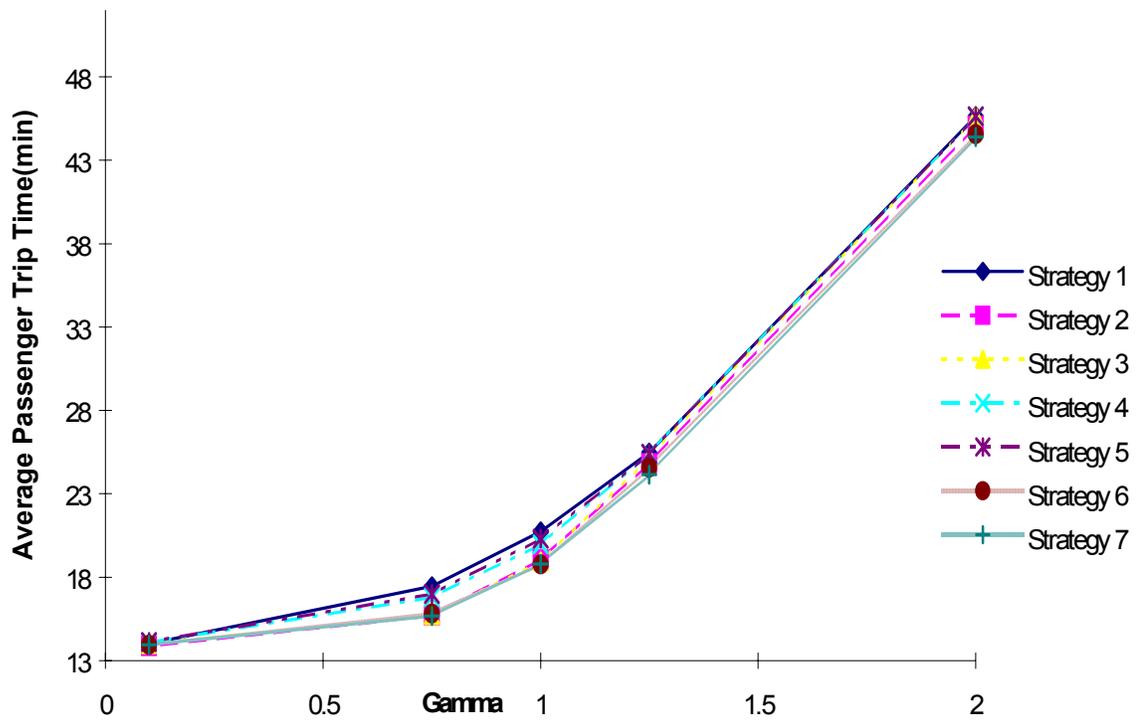


Figure 1. Average Passenger Trip Time (minutes) as a Function of γ

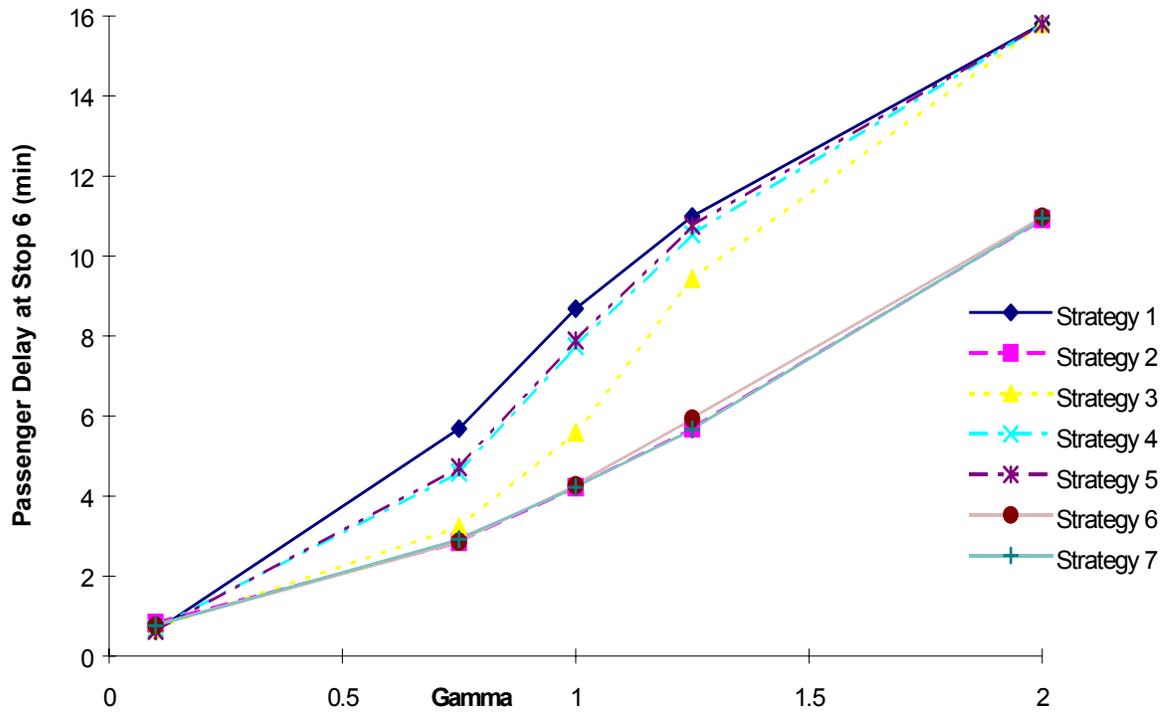


Figure 2. Passenger Delay (minutes) at Stop 6 as a Function of γ

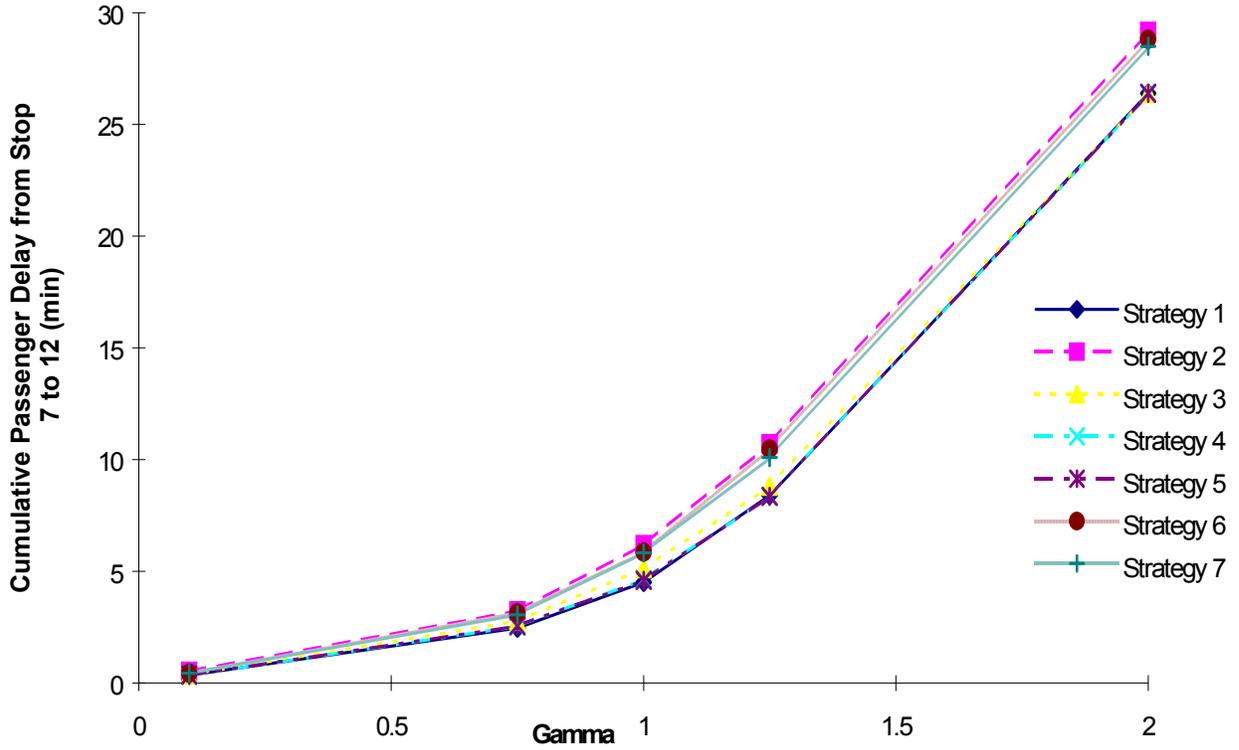


Figure 3. Total Passenger Delay after Stop 6 (minutes) as a Function of γ

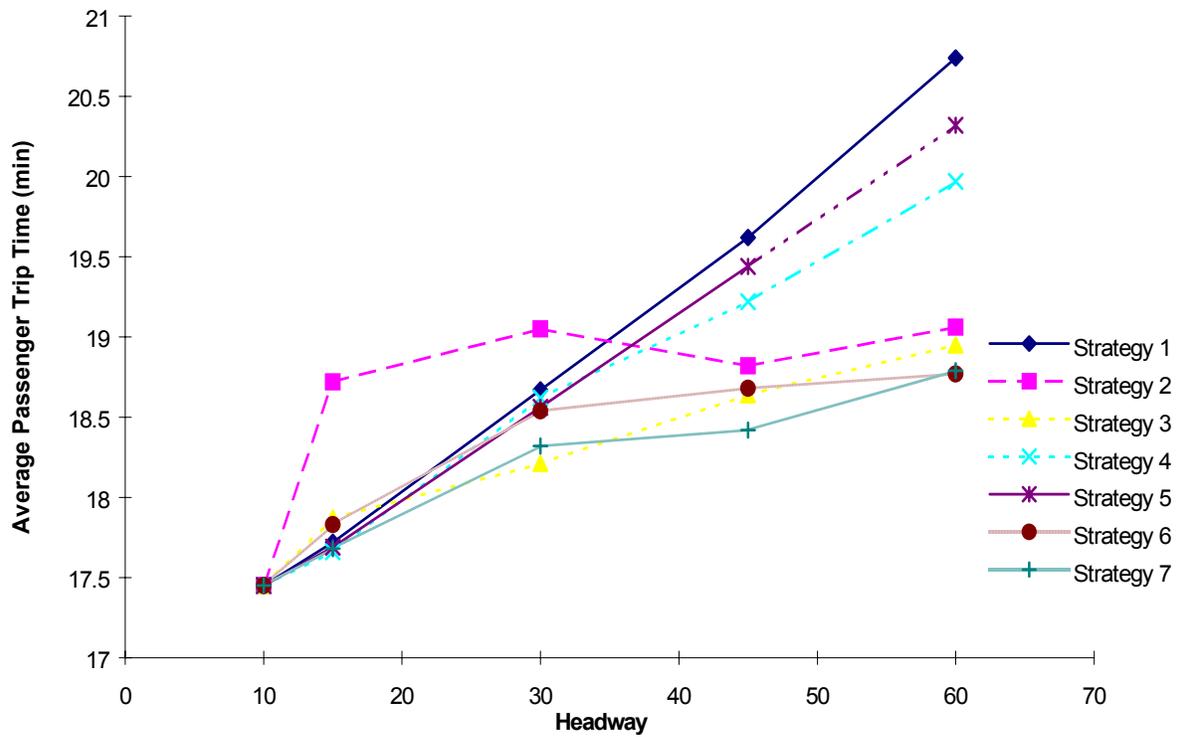


Figure 4. Average Passenger Trip Time (minutes) as a Function of the Headway

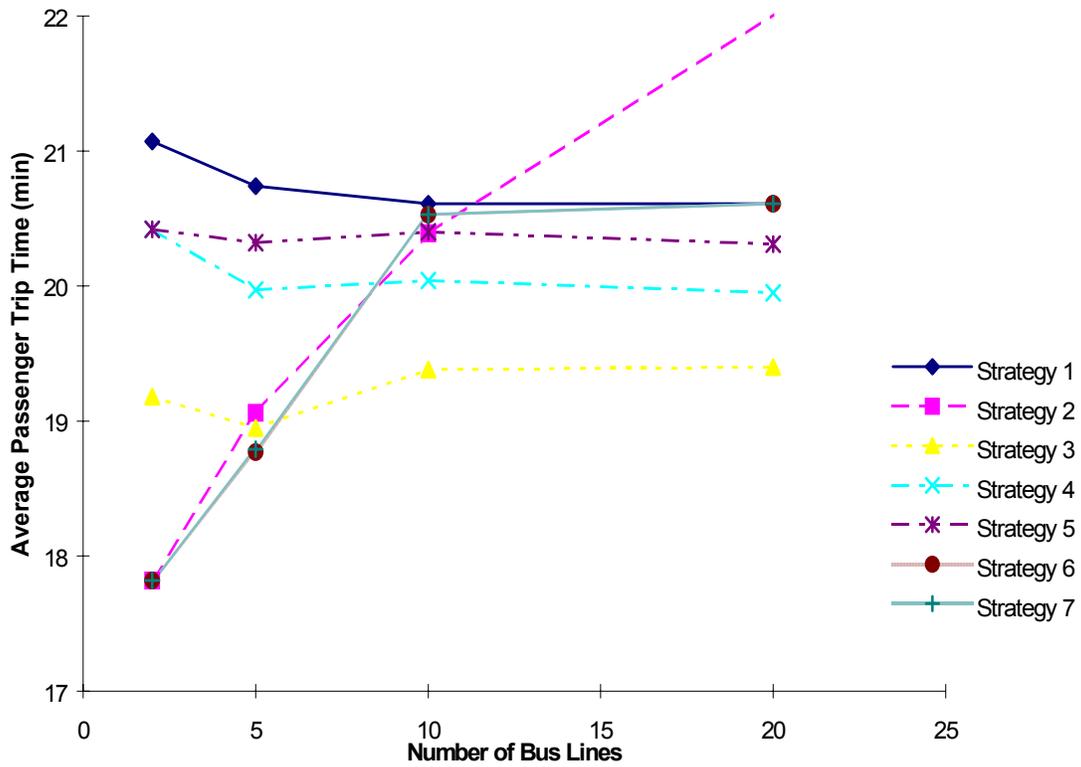


Figure 5. Average Passenger Trip Time (minutes) as a Function of N