The Stochastic Vehicle Routing Problem for Minimum Unmet Demand

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Summary. In this paper, we are interested in routing vehicles to minimize unmet demand with uncertain demand and travel time parameters. Such a problem arises in situations with large demand or tight deadlines, so that routes that satisfy all demand points are difficult or impossible to obtain. An important application is the distribution of medical supplies to respond to large-scale emergencies, such as natural disasters or terrorist attacks. We present a chance constrained formulation of the problem that is equivalent to a deterministic problem with modified demand and travel time parameters, under mild assumptions on the distribution of stochastic parameters; and relate it with a robust optimization approach. A tabu heuristic is proposed to solve this MIP and simulations are conducted to evaluate the quality of routes generated from both deterministic and chance constrained formulations. We observe that chance constrained routes can reduce the unmet demand by around 2%-6% for moderately tight deadline and total supply constraints.

1 Introduction

The classical Vehicle Routing Problem (VRP) determines the optimal set of routes used by a fleet of vehicles to serve a given set of customers on a predefined graph; it aims at minimizing the total travel cost (proportional to the travel times or distances) and operational cost (proportional to the number of vehicles used). The Stochastic VRP (SVRP) arises whenever some parameters of the VRP are random. Common examples are uncertain customers and demands, and stochastic travel times or service times. In this work, we address a stochastic routing problem motivated by the problem of distributing medical supplies in large-scale emergency response.

Large-scale emergencies (or major emergencies) are defined as events that overwhelm local emergency responders, which severely impact the operation of normal life, and have the potential to cause substantial casualties and property damage. Examples are natural disasters (earthquake, hurricane, flooding, etc.) and terrorist attacks, like September 11^{th}, 2001.

Careful and systematic pre-planning, as well as efficient and professional execution in responding to a large-scale emergency, can save many lives. Rational policies and procedures applied to emergency response could maximize the effectiveness of the scarce resources available in relation to the overwhelming demands. A key ingredient in an effective response to an emergency is the prompt availability of necessary supplies at emergency sites. Given the challenges of delivering massive supplies in a short time period to dispersed demand areas, operations research models can play an important role in addressing and optimizing the logistical problems in this complex distribution process. Larson et al. [37, 38] conducted a detailed analysis based on well-known and recent large-scale emergencies. They emphasized the need for quantitative, model-oriented methods provided in the operations research field to evaluate and guide the operational strategies and actions in response to major emergencies.

The distinguishing characteristics of large-scale emergencies are high demand for supplies, low frequency of occurrence, and high uncertainty in many aspects (e.g. when and where they happen). In particular, the routing problem for emergency response faces the following unique challenges.

First, the initial pre-positioned supplies will typically not be sufficient to cover all demands in the recommended response times. Unmet demand in an emergency situation can result in loss of life, an impact that outweighs more common VRP objectives such as the travel or operational cost. Therefore, the overriding objective in such problems is to mitigate the effect of the emergency by reducing the amount of unmet demand.

An additional aspect important in meeting the demand is the response time. Since the delivery of supplies to the population within a time-frame makes an appreciable health difference, it is natural to associate a deadline with each demand, to model situations in which a late delivery leads to loss of life.

The highly unpredictable nature of large-scale emergencies leads to significant uncertainty both in demand and in travel time. For instance, at a given demand point (e.g. a neighborhood block), the quantity of required supplies (antidotes, protective equipments, medication, etc.) is often proportional to the size of the population and/or numbers of casualties. The casualties exposures, or demands among "worried well" are hard to accurately predict for an overwhelming emergency occurrence. In addition, the emergency event itself may directly affect road conditions, e.g. possible congestion caused by the emergency event or destruction of the physical road network. Therefore, the travel times between points are stochastic.

We model a stochastic vehicle routing problem that minimizes unmet demand under the uncertainty in demand and travel time, with pre-defined service deadline and limited supply at the depot. Although it may not be possible to service all the nodes in the problem, we seek solutions that visit all the nodes, including nodes after the deadline or with an empty vehicle. Such a planned route provides a starting point for recourse action once demand and travel times are realized and there is the possibility of satisfying additional demand.

There are many ways to address parameter uncertainty in optimization problems, leading to different models and requiring different information on the uncertainty. In particular, we can obtain an answer to a routing problem under uncertainty through the solution of a representative deterministic problem, by using stochastic programming and chance constrained models, by a robust optimization approach, or a markov-decision process model. The type of information available on the uncertain parameters is key in determining which model is most appropriate for a given application. In this work we ignore this application specific concern and assume that any information that is needed is available. The chance constrained programming relies on probabilistic information, supposed to be available, and tries to find a solution which is "optimal" in a probabilistic sense. By adopting this approach, we believe that it is not necessary to look for a solution that is always feasible, since the worst case (if there is one) is very unlikely to occur. However, Robust optimization aims generally to provide the best solution feasible for all the uncertainty considered. In this work, we relate these two approaches by showing that both chance constrained and robust optimization models require the solution of a single deterministic instance of the problem with modified uncertain parameters for our problem at hand. We therefore focus this paper on the solution for these models of uncertainty which do not require specific solution procedures, as is the case in stochastic programming models or Markov-decision models.

The rest of this paper is organized as follows. Section 2 reviews relevant stochastic vehicle routing literature. Section 3 presents our problem formulation and the models to address uncertainty and section 4 presents a tabu heuristic solution approach. We present some numerical experiment results in section 5, and finally, conclude the paper in section 6.

2 Literature Review

In this section, we review well-known problems in the literature which are relevant to our problem at hand. This section is organized as follows: the general SVRP problem is briefly introduced and classified according to type of uncertainty, modeling methods and solution techniques. Two specific topics, SVRP with stochastic customers and demands and SVRP with stochastic travel time and service time, follow the general discussion.

2.1 Stochastic VRPs

The Vehicle Routing Problem is defined on a given graph $G = (V, \mathcal{A})$, where $V = \{v_1, v_2, \ldots, v_n\}$ is a set of vertices and $\mathcal{A} \subseteq \{(v_i, v_j) : i \neq j, v_i, v_j \in V\}$ is the arc set. An optimal set of routes, composed of a cyclic linkage of arcs starting and ending at the depot, is selected to serve a given set of customers at vertices. This problem was first introduced by Dantzig and Ramser in 1959 to solve a real-world application concerning the delivery of gasoline to service stations. A comprehensive overview of the Vehicle Routing Problem can be found in [48] which discusses problem formulations, solution techniques, important variants and applications. Other general surveys on the deterministic VRP also can be found in [34]. The Stochastic VRP (SVRP) introduces some element of uncertainty in the parameters within the system in question. The SVRP differs from its deterministic counterpart: several fundamental properties of the deterministic VRPs no longer hold in the stochastic case, and solution methodologies are considerably more complicated. A general review on the SVRP appeared in [23]. The Stochastic Vehicle Routing Problem can be broadly classified based on the following criteria:

- **Uncertainty in the problem:** The uncertainty can be present in different parts of the Vehicle Routing Problem. It can be divided into VRP with stochastic customers (VRPSC), VRP with stochastic demands (VRPSD), VRP with stochastic travel time (VRPSTT) and VRP with stochastic service time (VRPSST).
- **Modeling method:** The modeling method can also be the criterion to classify SVRPs. Stochastic VRPs can be cast into a stochastic programming framework, which is further divided into chance constrained program (CCP) and stochastic program with recourse (SPR). Another approach to model the SVRP is to view it as a Markov decision process. Some more recent modeling approaches include neurodynamic programming/reinforcement learning methodology.
- **Solution techniques:** Different solution techniques are the direct result of different modeling methods and the nature of the model. They usually broadly fall into two categories: exact methods and heuristic methods. The exact solution methods successfully applied to the SVRP include branch and bound, branch and cut, integer L-shape method and generalized dynamic programming. Under some mild assumptions, several classes of chance-constrained SVRPs can be transformed into equivalent deterministic VRPs [42, 35]. Numerous heuristics have been used to solve the SVRP, e.g. the savings algorithm, tabu search, etc. The heuristic solution techniques usually can be further divided as constructive heuristics, improvement heuristics and meta-heuristics.

2.2 SVRPs with Stochastic Customers and Demands

The VRP has stochastic demands (VRPSD) when the demands at the individual delivery (pickup) locations behave as random variables. The first proposed algorithm for the VRPSD by Tillman [46] was based on Clarke and Wright savings algorithm [15]. Another early major contribution on the

VRPSD comes from Stewart and Golden [42] who applied the chance constrained programming and recourse methods to model the problem. Later on, Dror [18] illustrated the impact that the direction of a designed route can have on the expected cost. A major contribution to the study of the VRPSD comes from Bertsimas [4, 5]. This work illustrated the a priori method with different recourse policies (re-optimization is allowed) to solve the VRPSD and derived several bounds, asymptotic results and other theoretical properties. Bertsimas and Simchi-Levi [9] surveyed the development in the VRPSD with an emphasis on the insights gained and on the algorithms proposed. Besides the conventional stochastic programming framework, a Markov decision process for single stage and multistage stochastic models were introduced to investigate the VRPSD in [17, 16]. More recently, a re-optimization type routing policy for the VRPSD was introduced by [41].

The VRP with stochastic customers (VRPSC), in which customers with deterministic demands and a probability p_i of being present, and the VRP with stochastic customers and demands (VRPSCD), which combines the VRPSC and VRPSD, first appeared in the literature of [29, 27, 28]. Bertsimas [4] gave a more systematic analysis and presented several properties, bounds and heuristics. Gendreau et al. [22, 24] proposed the first exact solution, L-shaped method, and a meta-heuristic, tabu search, for the VRPSCD.

Another research direction of the VRP with stochastic demands or customers incorporates a dynamically changing environment, where the demands vary over time. Bertsimas and Van Ryzin [6, 7] pioneered this work and named it dynamic traveling repairman problem (DTRP). They applied queuing theory to solve it and analyzed several dispatching policies for both light and heavy traffic. Based on their results, Papastavrou et al. [40, 44] defined and analyzed a new routing policy using the branching process. The multiple scenario approach (MSA) [3] and waiting strategies [11] have also been introduced for this problem recently.

2.3 SVRPs with Stochastic Travel Time and Service Time

Compared with stochastic customers and demands, the research on the stochastic travel time and service time problem of the TSP and VRP have received less attention. The VRP with stochastic travel time (VRPSTT) describes the uncertain environment of the road traffic condition. Kao [31] first proposed heuristics based on dynamic programming and implicit enumeration for the TSP with stochastic travel time (TSPSTT). Carraway et al. [13] used a generalized dynamic programming methodology to solve the TSPSTT. Laporte, Louveaux and Mercure [36] performed systematic research on the VRP with stochastic service and travel time (VRPSSTT). They proposed three models for the VRPSSTT: chance constrained model, 3-index recourse model, and 2-index recourse model. They presented a general branch-and-cut algorithm for all 3 models. The VRPSSTT model was applied to a banking context and an adaptation of the savings algorithm was used in [33]. Jula et al. [30] developed a procedure to estimate the arrival time to the nodes in the presence of hard time windows. In addition, they used these estimates embedded in a dynamic programming algorithm to determine the optimal routes. Hadjiconstantinou and Roberts [26] formulated a VRP with stochastic service times to model a company who receives calls (failure reports) from customers and dispatches technicians to customer sites. They used a two stage recourse model and a paired tree search algorithm to solve it.

3 Minimum Unmet Demand Routes

We now formulate our problem into a mixed integer programming (MIP) model. We first introduce the notation used and formulate the deterministic version of the problem. We then compare different uncertainty models for this problem.

3.1 Notation

We consider a set K of vehicles and a set D of demand nodes. We identify an additional node, node 0, as the supply node (depot) and let $C = D \cup \{\text{node } 0\}$ represent the set of all nodes. Indexed on sets K and C, we define the following *deterministic parameters*:

n: initial number of vehicles at the supply node (depot)
s : amount of supplies at the supply node (depot)
c_k: load capacity of vehicle k
dl_i: service deadline at demand node i.

We use M as a large constant used to express nonlinear relationships through linear constraints. We also consider the following two parameters to represent the uncertain travel time and demand, respectively

$\tau_{i,j,k}$: time required to traverse arc(i,j) for vehicle k
ζ_i : amount of demand needed at node i.

Finally, we define the binary and non-negative decision variables as follows, indexed on sets K, C:
Binary:

$X_{i,j,k}$: flow variables, equal to 1 if (i,j) is traversed by vehicle k and 0 otherwise
$S_{i,k}$: service variables, equals to 1 if node i can be serviced by vehicle k

Non-negative:

$Y_{i,j,k}$: amount of commodity traversing arc(i,j) using vehicle k
U_i: amount of unsatisfied demand of commodity at node i
$T_{i,k}$: visit time at node i of vehicle k
$\delta_{i,k}$: delay incurred by vehicle k in servicing i.

3.2 Deterministic Model

The deterministic, minimize unmet demand problem can be expressed as follows

$$\textbf{DP}: \quad \begin{array}{c} \text{minimize} \sum_{i \in D} U_i + \kappa \sum_{i \in D, k \in K} T_{i,k} \\ \text{subject to constraints } (1) - (17) \,, \end{array}$$

where the constraints are explained in detail below.

The objective of model DP is to minimize the weighted sum of the total unmet demands over all demand nodes and the total visit time at demand nodes of all vehicles. The κ value usually is set to be very small to make the total travel time a secondary objective compared with the unmet demand quantity. However, the travel time is a necessary term in the objective function to guide the route generation after the deadline. Since we model the routing problem in response to a large-scale emergency, the service start times (arrival times) directly associate with when the supply will be shipped and used at the dispensing sites. We would like to serve the dispensing sites as early as possible for life-saving purposes, so the arrival time is a much more important indicator of the service quality than the conventional objectives such as travel times or operational time.

We group the constraints into four parts: route feasibility constraints, time constraints, demand flow constraints and node service constraints. The following constraints (1)-(6) characterize the vehicle flows on the path and enforce the route feasibility.

$$\sum_{i \in D} \sum_{k \in K} X_{0,i,k} \leq n \tag{1}$$

$$\sum_{i \in D} \sum_{k \in K} X_{i,0,k} \leq n \tag{2}$$

$$\sum_{j \in D} X_{0,j,k} = \sum_{j \in D} X_{j,0,k} = 1 \qquad (\forall k \in K) \tag{3}$$

$$\sum_{j \in C} \sum_{k \in K} X_{i,j,k} = 1 \qquad (\forall i \in D) \tag{4}$$

$$\sum_{j \in C} \sum_{k \in K} X_{j,i,k} = 1 \qquad (\forall i \in D) \tag{5}$$

$$\sum_{j \in C} X_{i,j,k} = \sum_{j \in C} X_{j,i,k} \qquad (\forall i \in D \quad k \in K) \tag{6}$$

Constraints (1) and (2) specify that the number of vehicles to service must not exceed the available quantity ready at the supply node at the beginning of the planning horizon. The number of vehicles to service is stated by the total number of vehicles flowing from and back to the depot. Constraint (3) represents each vehicle flow from and back to the depot only once. Constraints (4) and (5) state that each demand node must be visited only once. Constraint (6) requires that all vehicles who flow into a demand point must flow out of it.

Constraints (7)-(10) guarantee schedule feasibility with respect to time considerations.

$$T_{0,k} = 0 \qquad (\forall k \in K) \tag{7}$$

$$(T_{i,k} + \tau_{i,j,k} - T_{j,k}) \leq (1 - X_{i,j,k})M \qquad (\forall i, j \in C \quad k \in K) \tag{8}$$

$$0 \leq T_{i,k} \leq \sum_{j \in C} X_{i,j,k} M \qquad (\forall i \in D \quad k \in K) \tag{9}$$

$$0 \leq T_{i,k} - \delta_{i,k} \leq dl_i \sum_{j \in C} X_{i,j,k} \qquad (\forall i \in D \quad k \in K) \tag{10}$$

The fact that all vehicles leave the depot at time 0 is specified by constraint (7). Constraint (8) enforces the time continuity based on the node visiting sequence of a route. Constraint (9) sets the visit time to be zero if the vehicle does not pass a node. The variable $\delta_{i,k}$ represents the delay of the visit time if a vehicle reaches the node after the deadline and is set to zero if it arrives before the deadline in constraint (10).

This model primarily accommodates the emergency situation where late deliveries could lead to fatalities. To maximize the likelihood of saving lives, medication should be received by the affected population within the specified hours of the onset of symptoms to impact the patient survival. This is the rationale behind the preference of using a hard deadline constraint instead of the soft deadline. However, for problems where late deliveries are possible we can translate the proposed model to soft deadlines, having the penalty on the violation represent the worsening in patient condition due to late arrival.

Constraints (11)-(13) state node service constraints.

$$\delta_{i,k} \leq (1 - S_{i,k})M \qquad (\forall i \in D \quad k \in K) \tag{11}$$

$$S_{i,k} \leq \sum_{j \in C} X_{i,j,k} \qquad (\forall i \in D,\ k \in K) \tag{12}$$

$$S_{i,k}M \geq \left(\sum_{j \in C} Y_{j,i,k} - \sum_{j \in C} Y_{i,j,k} \right) \qquad (\forall i \in D,\, k \in K) \tag{13}$$

Binary decision variables $S_{i,k}$ are used to indicate whether a node i can be serviced by vehicle k (when it equals to 1). That is, if the vehicle k visits node i before the deadline, then the vehicle can drop off some commodities at this node. However, the vehicle does not necessarily do it when $S_{i,k}$ equals to 1 since there might not be enough supply at the depot so the vehicle may not carry any commodities when it visits a later node in the route. We use these binary variables to keep the feasible region of this problem non-empty all the time. Constraints (4) and (5) will still enforce each node to be visited once and only once no matter before or after the deadline; however, those visits after the deadline cannot service the node any more. Constraint (11) states the deadline constraint and it can only be violated when $S_{i,k}$ equals to zero. Constraint (12) illustrates the relationship between the binary flow variables and the binary service variables. It implies the service variable can only be true when a vehicle physically passes a node. Constraint (13) requires that no commodity flows in a node after the deadline. On the other hand, there is no compulsory dropping-off commodities at nodes visited before the deadline since there may not be enough supplies to meet the demand. It establishes the connection between the commodity flow and the vehicle flow.

Constraints (14)-(16) state the construction on the demand flows.

$$s - \sum_{k \in K} \left[\sum_{j \in C} Y_{0,j,k} - \sum_{j \in C} Y_{j,0,k} \right] \geq 0 \tag{14}$$

$$\sum_{k \in K} \left[\sum_{j \in C} Y_{j,i,k} - \sum_{j \in C} Y_{i,j,k} \right] + U_i - \zeta_i \geq 0 \qquad (\forall i \in D) \tag{15}$$

$$X_{i,j,k} c_k \geq Y_{i,j,k} \qquad (\forall i, j \in C,\, k \in K) \tag{16}$$

Constraint (14) requires the total shipment of commodity from the depot not exceeding its current supply inventory level. Constraint (15) enforces the balanced material flow requirement for the demand nodes. Constraint (16) allows the flow of commodities as long as there is sufficient vehicle capacity. It also connects the commodity flow and the vehicle flow.

$$X_{i,j,k}, S_{i,j} \text{ binary}; \qquad Y_{i,j,k} \geq 0; \qquad U_i \geq 0; \qquad T_{i,k} \geq 0; \qquad \delta_{i,k} \geq 0. \tag{17}$$

Constraint (17) states the binary and non-negativity properties of the decision variables.

3.3 Stochastic Model

The parameters $\tau_{i,j,k}$ in constraint (8) and ζ_i in constraint (15) represent the uncertain travel time and demand parameters of our problem, respectively. If we ignore the uncertainty and replace these random quantities by representative values, such as their mean $\mu^{\tau}_{i,j,k}$ and μ^{ζ}_i or mode values, we can solve a deterministic problem DP to obtain a simple solution for this problem. This deterministic solution will be helpful as a benchmark to compare the quality of routes and demonstrate the merits of other more sophisticated methods we discuss next. There are two other ways to handle uncertainty that for this problem lead to the solution of a single deterministic problem DP: chance constrained programming and robust optimization. The solution of this routing problem through other methods of representing uncertainty, such as stochastic programming and markov-decision processes require more involved solution procedures and will not be explored in this paper.

In *chance constrained programming* (**CCP**) we assume that the parameters $\tau_{i,j,k}$ and ζ_i are unknown at the time of planning but follow some known probability distributions. We assume they are uniformly and independently distributed. We let α_D and α_T represent the confidence level of the chance constraints defining the unmet demand at each node and the arrival time of each vehicle at each node respectively. Thus, the constraints with stochastic parameters must hold with these given probabilities. For a given distribution on $\tau_{i,j,k}$ and ζ_i, we can rewrite constraint (8) and constraint (15) in the chance constrained fashion with levels α_T and α_D as follows:

$$P\left\{\tau|(T_{i,k}+\tau_{i,j,k}-T_{j,k}) \leq (1-X_{i,j,k})M\right\} \geq 1-\alpha_T \quad (\forall i,j \in C\ k \in K) \tag{18}$$

$$P\left\{\zeta|\sum_{k\in K}\left[\sum_{j\in C}Y_{j,i,k}-\sum_{j\in C}Y_{i,j,k}\right]+U_i-\zeta_i \geq 0\right\} \geq 1-\alpha_D \quad (\forall i \in D) \tag{19}$$

We call this *chance constrained model* (**CCP** model), which is modified based on the DP model in section 3.2, by replacing constraints (8) and (15) with constraints (18) and (19). Under some assumption of their distribution, constraint (18) and constraint (19) can be transformed to their deterministic counterpart. From this point onward in this paragraph, we use short notation τ and ζ to substitute $\tau_{i,j,k}$ and ζ_i for simplicity. For example, we assume τ and ζ follow a lognormal distribution with mean μ_τ and standard deviation σ_τ and mean μ_ζ and standard deviation σ_ζ respectively. The logarithm $\log(\tau)$, $\log(\zeta)$ are normally distributed as normal$(\mu'_\tau, \sigma'_\tau)$ and normal$(\mu'_\zeta, \sigma'_\zeta)$. The relationship between the parameters of lognormal distribution and normal distribution is stated as: $\mu' = \log\mu - \frac{1}{2}\sigma'^2$, $\sigma'^2 = \log(\frac{\mu^2+\sigma^2}{\mu^2})$. We let κ_T and κ_D represent the Z value for the normal distribution corresponding to the confidence level α_T and α_D and we call them "safety factors" in the later experimental results section. Therefore, the deterministic counterpart of constraint (18) and constraint (19) can be expressed as:

$$(T_{i,k} + e^{\mu'_{\tau_{i,j,k}} + \kappa_T \sigma'_{\tau_{i,j,k}}} - T_{j,k}) \leq (1 - X_{i,j,k})M \qquad (\forall i, j \in C \quad k \in K) \tag{20}$$

$$\sum_{k \in K} \left[\sum_{j \in C} Y_{j,i,k} - \sum_{j \in C} Y_{i,j,k} \right] + U_i \geq e^{\mu'_{\zeta_i} + \kappa_D \sigma'_{\zeta_i}} \qquad (\forall i \in D) \tag{21}$$

The *robust optimization model* assumes that the uncertain parameters $\tau_{i,j,k}$ and ζ_i are only known to belong to a given uncertainty set $\mathcal{U}$. The robust optimization approach, introduced by Ben-Tal and Nemirovski [2] and more recently extended to integer programming [8] and VRP [43], requires that the solution satisfy the constraints with uncertain parameters for all possible values in the uncertainty set $\mathcal{U}$. That is, we rewrite constraint (8) and constraint (15) as follows:

$$T_{i,k} + \tau_{i,j,k} - T_{j,k} \leq (1 - X_{i,j,k})M \quad \forall \tau_{i,j,k} \in \mathcal{U}, \qquad (\forall i, j \in C \;\; k \in K) \tag{22}$$

$$\sum_{k \in K} \left[\sum_{j \in C} Y_{j,i,k} - \sum_{j \in C} Y_{i,j,k} \right] + U_i - \zeta_i \geq 0 \qquad \forall \zeta_i \in \mathcal{U}, \qquad (\forall i \in D) \tag{23}$$

These infinitely many constraints, indexed over every $\tau_{i,j,k} \in \mathcal{U}$ and $\zeta_i \in \mathcal{U}$ can be expressed by single constraints by substituting the maximum possible uncertainty parameters $\tau^*_{i,j,k} = \max_{\tau_{i,j,k} \in \mathcal{U}} \tau_{i,j,k}$ and $\zeta^*_i = \max_{\zeta_i \in \mathcal{U}} \zeta_i$. With these maximum uncertainty parameters, these robust constraints can be written as:

$$T_{i,k} + \tau^*_{i,j,k} - T_{j,k} \leq (1 - X_{i,j,k})M \qquad (\forall i, j \in C \quad k \in K) \tag{24}$$

$$\sum_{k \in K} \left[\sum_{j \in C} Y_{j,i,k} - \sum_{j \in C} Y_{i,j,k} \right] + U_i - \zeta^*_i \geq 0 \qquad (\forall i \in D) \tag{25}$$

As we mentioned earlier, both the chance constrained model and the robust optimization model can be expressed as a deterministic equivalent problem, with changing demand and travel time values. Therefore these problems are just as difficult to solve as a single deterministic routing problem. The only difference between these two models of addressing uncertainty is the value of the uncertainty used, for instance $e^{\mu'_{\tau_{i,j,k}} + \kappa_T \sigma'_{\tau_{i,j,k}}}$ versus $\tau^*_{i,j,k}$ for the time continuity constraints. Note that varying κ_T and κ_D we can represent $\tau^*_{i,j,k}$ and ζ^*_i values as well as the mean values $\mu^{\tau}_{i,j,k}$ and μ^{ζ}_i, thus in the remainder of the paper we only consider the chance constrained model for different confidence levels.

4 Tabu Heuristic Solution Approach

Tabu search is a local search procedure which iteratively moves from a solution to its best neighbor until some stopping criteria have been satisfied. The search keeps a tabu list which prohibits revisiting a recently explored node unless some aspiration criteria have been met to avoid cyclic movement. It allows a solution to temporarily move to a worse position to escape a local optimum. Tabu search has been successfully used in solving hard optimization problems in many fields. A more comprehensive review on this technique and its applications can be found in [25]. Tabu search was first introduced for solving VRP by Willard [49]. Later on, different groups of researchers designed a variety of neighborhood/moves and adopted some problem-specific mechanism to significantly improve the performance, see for instance [39, 20, 50, 47]. Tabu search has also been applied to major variants of VRP, e.g. VRP with time windows [45], VRP with split delivery [1], the pick up and delivery problem [10] as well as the stochastic VRP [24]. It has been shown that tabu search generally yields very good results on a set of benchmark problems and some larger instances [21].

Applying tabu search to a particular problem requires a fair amount of problem-specific knowledge. Given the success of tabu heuristic in the classical VRP and its variants, and the similar structure of our model, we believe this approach holds much promise in solving our problem. The algorithm we propose here uses some ideas from the standard VRP, and incorporates new features taking into account the unmet demand objective of our problem.

Because the problem has insufficient supplies at the depot in most scenarios, we use an *UnassignNodeManager* list of nodes to keep track of all the nodes with unmet demand. We also use a *RouteManager* list of solutions to keep track of all the incomplete (may not include all the nodes) but feasible (meet both capacity and time constraints) routes. We initiate a solution with a visit-nearest-node heuristic and put all the unvisited nodes into the *UnassignNodeManager* list.

A key element in designing a tabu heuristic is the definition of the neighborhood of a solution or equivalently the possible moves from a given solution. Beside adopting the standard 2-opt exchange move and the λ-interchange move, we implement a new DEM-move to accommodate the special needs of the potentially unmet demands. The DEM-move will insert an unassigned demand node into a current route or exchange an unassigned node with some (one to three) consecutive node(s) in a route by abiding both deadline and capacity constraints. In a λ-interchange move, with $\lambda = 3$, it includes a combination of vertex reassignments to different routes and vertex interchanges between two routes; the reassigning/interchanging segment is up to three consecutive vertices in a route.

We also maintain a *MoveManager* list to record all possible moves from the current solution. After a solution moves to its neighbor, instead of reconstructing the whole neighborhood of the new solution, we only update the non-overlapping neighbors. That is, we eliminate those moves who are relevant (sharing the same route or sharing the same unassigned node, etc.) to the move that just has been executed, and generate new feasible moves that are relevant to the updated route(s) only. This will significantly reduce the computational effort of exploring the neighborhood. In the search process, we apply a random tabu duration (uniformly random generated between 5 to 10 steps) and a "best-ever" aspiration scheme.

We stop the heuristic after a given number of non-improving steps. After we obtain the tabu search solution, we complete the route by adopting a variation of a "next-earliest-node" heuristic to insert all unassigned nodes after the deadline of each route (keeping the before-deadline part intact). In this post-processing heuristic, each unassigned node picks a route to locate itself after the deadline, where the summation of its arrival time and the arrival time to the next node is minimized at the time of assignment. The quality of this tabu heuristic is evaluated in the next section.

5 Experimental Results

In this section, we demonstrate how the proposed MIP model in section 3 and the tabu heuristic solution approach in section 4 can be used to solve our problem at hand.

The purpose of the experiments is to compare the performance of the DP and CCP routes in terms of the unmet demand through simulations. We define a *problem scenario* for every randomly generated network consisting of a depot and 50 demand nodes, with a randomly generated mean demand quantity for each demand node. We perform simulations on 10 problem scenarios and average the results. Our interests are primarily focused on comparing the alternatives between short-and-risky routes and long-and-safe routes in the first set of experiments. Another experiment investigates how the value of the safety factors influences the quality of the routes.

We first describe the input parameters for our example and then we show the quality of the tabu heuristic. Finally, we demonstrate and discuss the results from the simulation experiments.

5.1 Data Generation of Input Parameters

We test our model and heuristic on 10 different problem scenarios. For each scenario we uniformly generate: 50 demand nodes and 1 depot node in a 200 by 200 square domain; and a mean demand quantity for each demand node ranging from 5 to 15. We service this demand with a fleet of 10 uncapacitated vehicles. We use the Euclidean distances between any two of the nodes and assume a symmetric complete graph topology. The mean travel time between any two nodes is proportional to the distance.

In the DP model, we use the mean value of the demand quantities and travel times as its parameters. In the CCP model, we use a lognormal distribution with the same mean value as was used in the DP model; the standard deviation is set to be proportional to the mean value for demand (20% of the mean value) and inversely proportional to the mean value of the travel time ($\sigma = \frac{UB-\mu}{100}\mu$, UB is the upper bound for the inversely proportional transformation, whose value is dependent on the graph topology, which must be greater than the longest arc in the graph). We restrict our analysis to this type of travel time distribution because we aim to compare the tradeoffs between short-and-risky routes and long-and-safe routes. Note that the DP routes in our experiments, tend to be short-and-risky since they do not take into account the variability, while the CCP routes tend to favor longer routes that have smaller variance. For the chance constrained model, we set the confidence level as 95%, thus setting the values κ_D and κ_T in constraints (20)-(21) to 1.65, which we refer to as the "safety factor". Since the routes generated from the model are sensitive to both the deadline and the total supply at the depot, we vary these two parameters and observe the results. We use 70%, 80%, 90%, 100% and 120% of the *base quantity* as the available supply quantity. The *base quantity* of the total supply at the depot is defined to be the summation of the demand quantity at all the demand nodes, which is 500 on average. The deadline is set to 40%, 50%, 60%, 80%, 100% and 120% of the *base route length.* The *base route length* is defined as the average length of all the edges in the graph times 5 (50 demand nodes are served by 10 vehicles; so on average, each vehicle serves 5 demand nodes). We call one combination of the deadline and the total supply parameters a *test case.* So for each problem scenario, we have 30 *test cases*, which are identified by the different combinations of the deadline on the demand nodes (6 types: 40%, 50%, 60%, 80%, 100%, 120%) and the total supply at the depot (5 types: 70%, 80%, 90%, 100%, 120%). For each *test case*, both the deterministic and the CCP version of the problem are solved by the tabu heuristic.

5.2 Quality of Tabu Heuristic

To evaluate the quality of our tabu heuristic, we run this search process over 10 random problem scenarios as described above. The average of the percentage of unmet demand as well as the remaining supply quantity level at the depot for the deterministic formulation is recorded for each *test case.* Note that each entry in the table is an average of 10 routes. The results are presented in Table 1 and Table 2.

These results show that, in terms of minimizing the unmet demand, the 6 cases which are indicated by a "*", reach the minimum. They either meet all the demands (0% unmet demand) or deliver all the supplies (zero remaining supply at the depot) over the 10 problem scenarios we tested. From the first three rows of the above tables, the results are very close to the best we can do

Table 1. Percentage of unmet demand over total demand for the deterministic model.

	DL200 (40%)	**DL250 (50%)**	**DL300 (60%)**	**DL400 (80%)**	**DL500 (100%)**	**DL600 (120%)**
sup350(70%)	30.78%	30.63%	30.63%	30.61%	30.59%	30.59%
sup400(80%)	21.19%	20.75%	20.73%	20.71%	20.64%*	20.64%*
sup450(90%)	12.76%	11.12%	10.82%	10.82%	10.82%	10.78%
sup500(100%)	5.31%	2.20%	1.87%	1.77%	1.75%	1.75%
sup600(120%)	4.98%	0.20%	0.00%*	0.00%*	0.00%*	0.00%*

Table 2. Remaining supply at depot for the deterministic model.

	DL200 (40%)	**DL250 (50%)**	**DL300 (60%)**	**DL400 (80%)**	**DL500 (100%)**	**DL600 (120%)**
sup350(70%)	0.9	0.2	0.2	0.2	0.1	0.1
sup400(80%)	2.4	0.2	0.2	0.1	0*	0*
sup450(90%)	9.7	1.5	0.1	0.1	0.1	0.1
sup500(100%)	22.1	6.5	4.7	4.4	4.3	4.2
sup600(120%)	120.3	95.9	94.8*	94.8*	94.8*	94.8*

regarding to the unmet demand, since the remaining supplies are almost empty and the percentage of the unmet demand is near the simple lower bound given by the shortage of supplies to meet the total demand. Only the DL200-sup500 and DL200-sup600 grids give a higher percentage of unmet demand than what is due to the shortage in supplies. We suspect this is due to the tight deadline which might prevent the timely delivery even with enough supplies. A tighter lower bound with respect to a short deadline is a subject for future research. However, in general, we conclude that the tabu search method we proposed is very effective in minimizing the unmet demand, which is our primary concern in this model.

5.3 Simulation and Analysis

To evaluate the quality of the routes generated by the DP and CCP models, we observe how each type of route performs under randomly generated demand

and travel time situations. We now describe this simulation process and then present two sets of results with analysis.

LP model for Simulation

To evaluate the effectiveness of DP and CCP routes, we assume that they are planned prior to the realization of the uncertainty and have to operate in an environment where demand and travel times that occur may deviate from the ones considered in the planning phase. Thus we consider the following sequential events: first, we use the MIP problem to obtain DP and CCP routes with estimates (mean and distributions) of the uncertain demand and travel times. Then the uncertainty is realized, and these fixed routes have to meet as much of the demand that appears. Here we assume that we are able to take a recourse action on how to distribute the supplies among the vehicles to meet the realized demand. We solve a linear programming problem to determine the quantity of commodity to be carried and delivered by each vehicle based on the fixed routes. Therefore, we generate pre-planned routes with the MIP model and then at the time of the emergency run a linear programming problem to determine the delivery quantity. There are several advantages to having well established pre-planned routes for training and preparation purposes.

Since most of the input parameters are known and only the commodity flows are decision variables in the simulation stage, this optimization model is a deterministic linear programming problem (DLP) with the same basic structure as the MIP problem presented in section 3. Now the vehicle flow $X_{i,j,k}$ is fixed to the result of the MIP problem. With these variables fixed, the values of $T_{i,k}$ and $\delta_{i,k}$ are also fixed and can be calculated from the fixed routes $X_{i,j,k}$. We eliminate those demand nodes whose $T_{i,k}$ already missed the deadline and minimize the unmet demand by adjusting the commodity flow $Y_{i,j,k}$. We formally show the model as follows:

Model **DLP**:

$$\text{minimize} \sum_{i \in D} U_i + \kappa \sum_{i \in D, k \in K} T_{i,k}$$

subject to: constraints (13)-(14), (16)

$$\sum_{k \in K} \left[\sum_{j \in C} Y_{j,i,k} - \sum_{j \in C} Y_{i,j,k} \right] + U_i - d_i \geq 0 \qquad (\forall i \in D) \tag{26}$$

$$Y_{i,j,k} \geq 0; \qquad U_i \geq 0. \tag{27}$$

The d_i in constraint (26) is the actual demand quantity at each node after the scenario has been realized. Constraint (27) shows the non-negativity properties of the decision variables and the DLP is a linear programming problem.

Solution and Analysis I – the Deterministic Routes vs the CCP Routes

Our first set of experiments test the efficiency and robustness of our chance constrained model in determining pre-planned routes. We compare solutions from the CCP model against routes generated by the DP model that uses the mean values of the demand quantity and travel times as problem parameters.

For every problem scenario and test case, for given deadline and total supply, we compute the DP and CCP routes with the tabu heuristic described in section 4. We also generate 100 *instances* of the realization of the random travel time and demand quantity parameters based on the same lognormal distribution assumed by the CCP model. We evaluate how well the DP and CCP routes can meet the demand in each instance by solving the DLP problem above. We solved the linear program DLP with CPLEX 9.0 with default settings on a 3.2 GHz CPU with 2GB RAM. To summarize, in the experiments below we consider 10 problem scenarios, and for each we perform 30 different test cases, varying the deadline and total supply. In each test case we evaluate the performance of the DP and CCP routes using 100 different random instances. Hence, for each *test case*, the simulated results of the unmet demand are obtained from the average over 1000 instances. The results are summarized in table 3 and figure 1.

Table 3. Average percentage of the unmet demand over the total demand for DP and CCP models (lognormal distribution)

	DL200 (40%) **DP**	DL200 (40%) **CCP**	DL250 (50%) **DP**	DL250 (50%) **CCP**	DL300 (60%) **DP**	DL300 (60%) **CCP**	DL400 (80%) **DP**	DL400 (80%) **CCP**	DL500 (100%) **DP**	DL500 (100%) **CCP**	DL600 (120%) **DP**	DL600 (120%) **CCP**
sup350 (70%)	31.79%	35.07%	31.32%	31.94%	31.03%	30.91%	30.57%	30.68%	30.78%	30.63%	31.04%	30.63%
sup400 (80%)	24.28%	30.53%	23.20%	24.85%	22.69%	22.09%	21.83%	21.01%	22.13%	20.73%	21.87%	20.70%
sup450 (90%)	19.67%	28.75%	17.48%	21.00%	16.35%	15.63%	14.59%	12.16%	13.98%	11.21%	14.45%	11.00%
sup500 (100%)	17.85%	28.07%	14.78%	19.70%	13.23%	13.28%	11.11%	6.51%	10.15%	4.17%	9.36%	3.34%
sup600 (120%)	17.22%	28.38%	14.37%	19.69%	11.57%	13.00%	10.54%	5.53%	8.55%	2.55%	7.78%	1.44%

The results suggest that, when the deadline is very tight (between 40% to 60% of the *base route length*), the deterministic routes outperform the CCP routes. There is no benefit for utilizing a conservative routing strategy under tight deadlines since the longer less risky trips (arcs) in most generated in-

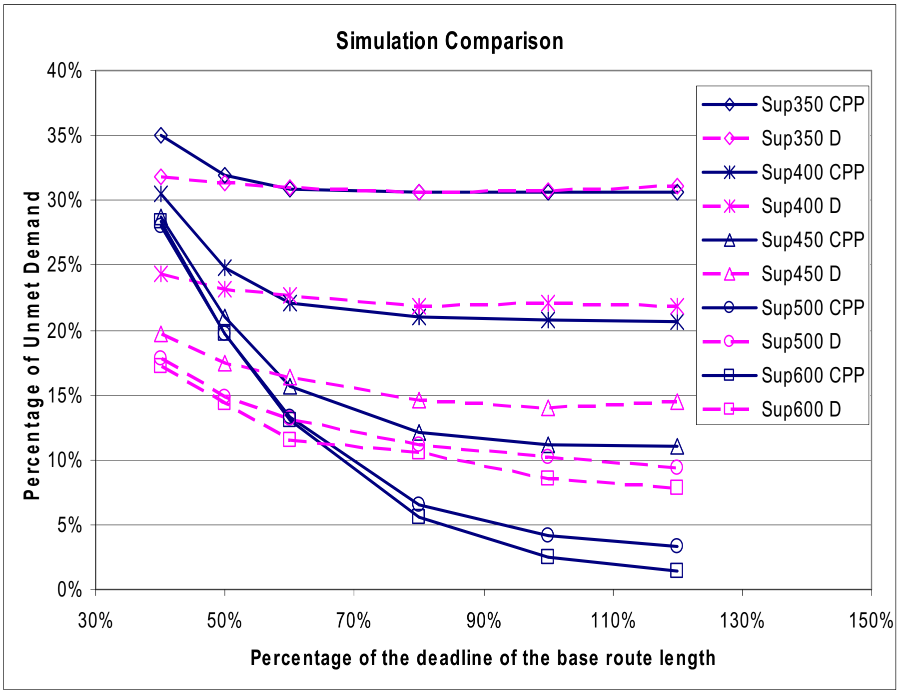

Fig. 1. Simulation comparison for the DP and CCP models

stances are longer than the deadlines. In this case, the short-and-risky routes generated by the DP model at least have a chance in some of the generated instances of arriving on-time especially for the demand points in the beginning of the route. In the situation where there is limited supply (e.g., when the total supply is only 70% of the total demand), the DP and CCP routes yield roughly the same amount of unmet demand even under a more relaxed deadline. Because of the insufficient supplies at the depot, the extended deadline cannot provide a better coverage.

Under moderate deadlines and supply quantities (when the percentage of the total supply over the total demand and the percentage of the deadline over the *base route length* both are between 80% and 120%), we can conclude that CCP routes outperform DP routes 2% to 6% in terms of the percentage of the unmet demand over the total demand, according to our simulation experiments. As the deadline and the total supply are relaxed, the percentage of the unmet demand is decreasing for both DP and CCP routes; at the same time, the advantage of CCP routes over DP routes gradually increases. The simulation results on the percentage of the unmet demand from the CCP routes are approaching to its lower bound due to the shortage of the supply, with an increasing deadline. We believe that it is because of, under the relaxed enough deadline and supply quantities, the conservative nature of the CCP model shows its merits. It leads to similar number of nodes in different routes in the CCP model. In contrast, the DP routes are more prone to have an uneven number of nodes between different paths leading to a higher chance of having unmet demand when they are tested over different realization *instances*.

Solution and Analysis II – the Role of the Safety Factor

Another experiment we conduct is to fix the deadline and the total supply at the depot and vary the confidence level, which has a one-to-one correspondence to the κ_T and κ_D values in constraints (20) and (21). We use the same value for both κ_T and κ_D. For one specific network topology and demand distribution (a *problem scenario*), we fix the deadline at 80% of the *base route length* and the total supply at 90% of the total demand. We generate different sets of CCP routes with different safety factors, and run the same simulation as described above. We plot the percentage of the unmet demand over the total demand in figure 2. Since we are using a lognormal distribution, the deterministic case (mean value) lies at $e^{\mu' + \frac{\sigma'^2}{2}}$. The corresponding safety factor is bigger than 0.5 as compared with exactly 0.5 for the normal distribution. The value of the safety factor for the deterministic case in the lognormal distribution is heavily dependent on both the mean and the variance of the distribution. For the specific problem scenario we are testing, the estimated safety factor is 0.75 by averaging the variance value of all the edges in the graph in a statistical sense. It is shown as a dot on the plot. The point pinned by a square gives the best average unmet demand ratio, which is also the value we used for the CCP model for the comparison purpose in section 5.3.2. It corresponds to the confidence level at 95% and the safety factor as 1.65. The plot in Figure 2 infers that with an increasing “contingency stuffing” (a bigger safety factor) starting from the deterministic point, it will provide better coverage. However, if we are too conservative as further increasing the safety factor after the “best CCP point”, the quality of the result deteriorates. Under a given deadline, if the safety factor is too big such that all the short and risky arcs have been “stretched” to overpass the deadline, then the route planning based on those “over-stretched” arcs is not very meaningful in providing any guidance since every node would miss its deadline. Hence it is crucial for planning with the “optimal” safety factor, which is neither too opportunistic nor too conservative.

As we discussed earlier, the robust optimization, which aims to optimize the worst-case scenario, also shares the same problem structure and computational complexity as the deterministic and the CCP formulations under a bounded uncertainty set or a given set of scenarios. Recently, different groups of researchers are interested in develop more elaborated uncertainty sets in order to address the issue of over-conservative in worst case models while maintaining the computational tractability. Also several works have established the link between the chance-constrained technique and robust optimization [32, 12, 19, 14]. In our work, the only difference between the deterministic counterpart of the robust model and the CCP model are the different safety factors used by each model. In chance-constrained optimization, the constraints are enforced up to a pre-specified level of probability. Robust optimization seeks a solution which satisfies all possible constraint instances. It achieves this by enforcing that the strictest version of every constraint is satisfied by substituting the largest uncertainty parameter value in each constraint. This maximal value for the uncertainty parameter has a one-to-one correspondence with a safety factor value in the chance-constrained formulation and lies somewhere on the plot in figure 2.

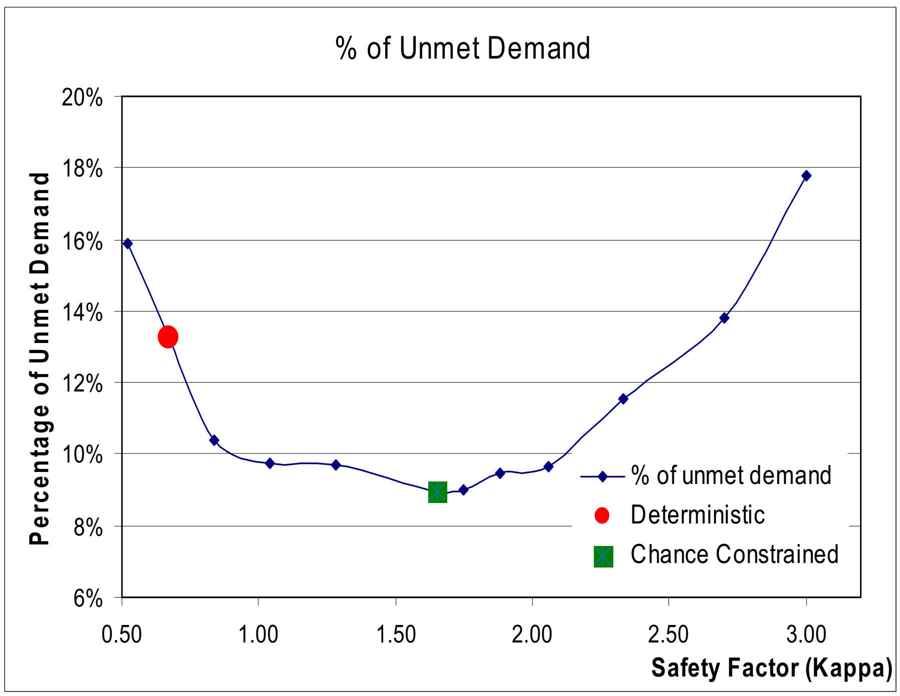

Fig. 2. Simulation comparison for different safety factors

6 Conclusions and Future Work

In this paper, we model a stochastic vehicle routing problem to minimize the unmet demand with a chance constrained programming technique, which is primarily motivated by routing medical supplies for large-scale emergency response. Unique features of this problem have been analyzed and embedded into our model. A tabu search heuristic is proposed to solve this model and its quality is evaluated by preliminary numerical experiments. Simulations are conducted to test the effectiveness of the CCP routes over the deterministic routes under different deadlines and supply quantities at the depot. The influence of the safety factor at different values is also demonstrated by the simulation results. We conclude that our model is effective in producing the pre-planned routes under moderate deadlines and supplies and can provide a better coverage of the overall demand with some uncertainty present.

We are interested in the following two ongoing research directions: first, due to the nature of the large-scale emergencies, we believe it is more pragmatic to incorporate a split delivery and multi-depot situation into our model to better deal with the huge amount of requests within a short time line. Second, we are interested in developing a tighter lower bound with short deadline, which does not only help to evaluate the effectiveness of our heuristic result, but could also facilitate to eliminate those infeasible routes in the earlier stage to improve the efficiency of the heuristic.

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