

A Heuristic-Based Procedure for the Weighted Production Cell Formation Problem

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Abstract : A key issue in the design of a cellular manufacturing system is the formation of the machines and parts into groups or production cells. The production cells are designed to minimize the costs of inter-cell part movements and intra-cell processing, while balancing the workload within each cell. Most of the prior research represents the cell formation problem as a binary machine-part incidence matrix. The workload balance within each production cell may be precisely calculated only if the processing times and demand rates are included in the analysis. For this reason, a heuristic-based procedure that uses processing times and demand rates to form the production cells is proposed. The procedure considers the cell imbalance costs as well as the costs associated with the inter-cell part movements and intra-cell processing. The efficiency and effectiveness of the heuristic is compared to other methods, and an industrial application of the proposed heuristic is presented.

1.0 Introduction

Successful implementation of a cellular manufacturing system (CMS) can significantly improve factory performance in terms of setup cost, lead time, work-in-process (WIP), throughput time, material handling, tooling, labor cost, and job satisfaction (Wemmerlöv and Hyer [1989]). The three major steps in implementing a cellular manufacturing system (CMS), as outlined by Askin and Vakharia [1990], are developing a parts coding scheme, forming the production cells, and laying out the machines. This paper focuses on the second step, production cell formation. The motivation of this research arose from a study of a cellular manufacturing project at Champion Irrigation Products (CIP). CIP manufactures irrigation products. The company's market includes professional organizations such as golf-courses and retail outlets such as HomeDepot. Demand is highly

seasonal reaching its peak during the winter (often exceeding the plant's capacity) and reaching its minimum during the summer. The proportion of demand for each part type is the same in each season. Because of the significant costs associated with material handling, set-ups, and WIP with the old functional layout of the plant, CIP is incrementally transforming its functional layout to a CMS. A pilot CMS implementation demonstrated a significant reduction in inventory and number of set-ups. It was relatively straightforward to form the production cells for the pilot CMS due to the natural grouping of the candidate parts and machines. However, CIP is hesitant to expand the CMS because the remaining parts have a more variable processing and demand requirements. This variability in processing and demand requirements may lead to unbalanced cells if they are not taken into consideration when designing the cells.

Most of the prior research represents the cell formation problem as a binary machine-part incidence matrix. A disadvantage of this representation is that it ignores the workload balance within each production cell which can only be precisely calculated if the processing times and demand rates are included in the analysis (Kusiak *et al.* [1985], Vakharia and Wemmerlöv [1987], and Vakharia and Kaku [1993]). For this reason, a heuristic-based procedure that uses processing times and demand rates to form the production cells is proposed. This procedure considers the cell imbalance costs as well as the costs associated with the inter-cell part movements and intra-cell processing. The efficiency and effectiveness of the heuristic is compared to other methods. Finally, the proposed heuristic is applied to the expansion of the CMS for CIP.

2.0 Background

Each production cell consists of several functionally dissimilar machines that are grouped into machine cells and parts with similar processing requirements that are grouped into part families. The grouping of similar parts into part families reduces lead times by decreasing the number of setups. If there is little inter-cell part movement, the grouping of machines into machine cells simplifies the material flow and fewer material handling movements are required. A simplified material flow makes the scheduling task easier. A cell with a balanced workload realizes a reduction

in machine idle time and WIP. For these reasons, the objective in production cell design is to form the cells such that the costs of inter-cell part movements and intra-cell processing are minimized, while balancing the workload within each cell.

Three approaches used to formulate production cells (sometimes referred to as the group technology cell formation problem) are *matrix analysis*, *optimization models*, and *graphical analysis*. The matrix formulation procedures include *similarity coefficient* (Seiffodini and Wolfe [1986]), *direct clustering* (Chan and Milner [1982]), *sorting-base* (Chandrasekharan and Rajagopalan [1986b]), *bond energy* (McCormick *et al.* [1972]), *production flow* (Burbidge [1989]), *cost-base* (Askin and Subramanian [1987]), and *cluster identification* (Kusiak and Chow [1987]). In these procedures, a binary machine-part incidence matrix is created. Row i refers to machine i while column j refers to part j . An entry of 1 (0) in the matrix indicates that machine i is used (not used) to process part j . These procedures transform the initial incidence matrix into a more structured form where block diagonals can be identified. The block diagonals identify the production cells.

A disadvantage of the matrix formulation procedure is that the incidence matrix only considers information on the relationship between the parts and the machines. It does not consider the unique processing time and demand requirement of each part on the different machines, but instead implicitly assumes a uniform workload (demand rate \times standard processing time) for each part on each machine. As a result the production cells generated by these methods may not be balanced. The advantage of matrix formulation is that the solution procedures are computationally efficient (Ng and Lin [1993]).

The optimization models include *linear programs* (Purcheck [1975]), *integer programs* (Kusiak [1987]), and *quadratic programs* (Kumar *et al.* [1986]). The optimization models are capable of considering non-equal processing times, but they consider only the grouping of parts into part families. These models do not consider the grouping of machines into machine cells and can only be applied to small and simple problems due to their complexity (Kusiak [1990]).

The last approach for production cell formation is graphical analysis. In this approach the machine-part incidence matrix is represented by a graph. The nodes typically represent the

machines and the weights on the edges represent the similarity between the machines. The techniques used in graphical analysis are *boundary graph* (Vannelli and Kumar [1986]), *shortest spanning path* (Slagle *et al.* [1975]), *Hamiltonian path* (Askin *et al.* [1991]), and *minimum spanning tree* (Ng [1989]). Each technique differs in the calculation of the edge weights. For example, the minimum spanning tree technique defines the edge weights as the degree of dissimilarity between any two rows in the incidence matrix. The production cells are then determined by deleting edges in the graph until no further improvements in the design can be obtained.

3.0 Model Formulation

An appropriate formulation for the production cell formation problem needs to consider all the costs in the design of production cells such as material handling, machine processing, set-up and changeover, WIP inventory, and machine cell balance delay (Oliva-Lopez and Purcheck [1979], Wemmerlöv and Hyer [1986], and Vakharia [1986]). To explicitly account for machine cell balance delay, processing times and demand must be included in the formulation.

The production cell formation problem that considers processing times and demand requirements can be represented by a nonnegative $m \times n$ machine-part matrix $A = (w_{ij})$, where $i=1, \dots, m$ represent the machines, $j=1, \dots, n$ represent the parts, and w_{ij} is the workload (demand rate \times standard processing time) of part j on machine i . If machine i does not process part j , w_{ij} is equal to zero. The development of production cells is equivalent to decomposing matrix A into diagonal blocks of submatrices. Each diagonal block identifies a production cell. Some elements in matrix A may not be in diagonal blocks and represent inter-cell part movements.

Let K be the number of production cells. Associated with each production cell h ($h=1, \dots, K$) is a machine cell and a part family, which are uniquely assigned to it, and each given its index h . In production cell h , let R_h be the set of machines assigned to machine cell h , and let C_h be the set of parts assigned to part family h . Clearly, $K \leq m \leq n$. In most practical cases, it is not possible to design a CMS that is completely decoupled. That is, there may be an operation for a part that requires a machine that is outside the assigned production cell. Recognizing this situation, the

objective is to determine K , R_h , and C_h that minimize the costs associated with inter-cell part movements and intra-cell processing while balancing the workload within each cell.

Let W be the total weights contained in matrix A . Then W_a represents the total weights in the diagonal blocks, W_e represents the total weights outside the diagonal blocks, and W_d represents the total line balance delay. The variables W_a , W_e , and W_d are decision variables that depend on the decomposition of matrix A into diagonal blocks. Given a decomposition of matrix A , it is relatively straight-forward to determine W_a and W_e . Since balancing the workload within each cell is similar to the mixed-part line balancing problem, the total line balance delay (W_d) calculations are derived from the mixed-part line balancing literature.

The objective of the mixed-part line balancing problem is to allocate the work equally across all machines on a daily or shift basis (Macaskill [1972]). Smoother machine balance is desirable because individual operators are able to work at a steadier pace, and part sequencing inefficiencies (i.e., congestion, idle time, and work deficiency) are not as severe (Thomopoulos [1967]). Thomopoulos [1970] derived a performance measure for the mixed-part line balancing problem based on the consistent loading of machines on a part-by-part basis as well as even loads on each machine. Thomopoulos defines the performance measure as the total line balance delay (W_d), or the total difference between the amount of time that machine i is processing part j and the average intra-cell processing time of part j . The performance measure W_d is incorporated in the proposed weighted production cell formation model as the total imbalance in the production cells.

Let a be the operation cost of machine i processing part j when machine i is in the assigned machine cell, or the unit time intra-cell processing cost. Let e be the operation cost of machine i processing part j when machine i is not in the assigned machine cell, or the unit time inter-cell processing cost. e includes the machine processing cost and the additional material handling, set-up changeover, and WIP costs due to processing the part outside of its assigned production cell. Let d be the costs associated with idle time and WIP inventory, or the unit time cell balance delay cost.

The objective of the decomposition (weighted production cell formation problem) is to minimize the overall cost (Z), or the sum of the total intra-cell processing costs ($a W_a$), the total

inter-cell processing costs ($e W_e$), and the total cell balance delay costs ($d W_d$). The complete nonlinear integer programming model formulation of the problem (D) is:

$$(D) \text{ Min } Z = a W_a + e W_e + d W_d$$

s.t.

$$\sum_{h=1}^K \sum_{j=1}^n x_{jh} = \sum_{h=1}^K \sum_{i=1}^n y_{ih} \quad h = 1, \dots, K \quad (9)$$

$$\sum_{h=1}^K y_{jh} = 1 \quad j = 1, \dots, n \quad (6)$$

$$y_{jh} = 0 \text{ or } 1 \quad j = 1, \dots, n, \quad h = 1, \dots, K \quad (10)$$

$$W_a \geq 0, W_e \geq 0, W_d \geq 0, K \geq 0 \quad (11)$$

where

a = unit time intra-cell processing cost

e = unit time inter-cell processing cost

d = unit time balance delay cost

$$W = \text{total weights in matrix } A, W = \sum_{i=1}^m \sum_{j=1}^n w_{ij}$$

K^f = user-specified maximum number of production cells ($K^f \leq m$)

w_{ij} = workload (demand rate \times std. processing time) of part j on machine i

K = number of production cells

W_a = total intra-cell weights

W_e = total inter-cell weights

W_d = total balance delay

\bar{w}_j = average intra-cell processing time for part j

$$x_{ih} = \begin{cases} 1 & \text{if machine } i \text{ is assigned to machine cell } h \\ 0 & \text{otherwise} \end{cases}$$

$$y_{jh} = \begin{cases} 1 & \text{if part } j \text{ is assigned to part family } h \\ 0 & \text{otherwise} \end{cases}$$

The decision variables x_{ih} and y_{jh} define the sets R_h and C_h . Note that the product $x_{ih}y_{jh}$ is a binary variable that designates the membership of machine i and part j in the same cell if it is unity. Constraints (1), (2), and (3) define W_a , W_e , and W_d , respectively. Constraint (4) defines the average intra-cell processing time for the j th part. Constraint (5) restricts each machine to exactly one machine cell, while Constraint (6) restricts each part to exactly one part family. Constraint (7) specifies the number of formed production cells K . The maximum number of production cells, K^f , may be specified (Constraint 8) and K^f must be less than or equal to m . Constraints (9) and (10) restrict the decision variables x_{ih} and y_{jh} to be binary.

As Kusiak [1990] points out, the production cell formation problem is an NP-hard problem (i.e., exponential time complexity). The complexity of the direct enumeration solution procedure for the production cell formation problem is $O(m!n!)$ (McCormick *et al.* [1972]). The model formulation is a nonlinear integer program which is generally difficult to solve. For this reason, a heuristic solution procedure is proposed to solve the model.

4.0 Heuristic Solution Procedure

The heuristic solution procedure is based on constructing a minimum spanning tree (MST). Ng [1989] proposes the MST method for the binary cell formation problem and refers to it as BMST. The MST method is used for the weighted cell formation problem because the solution procedure explicitly considers imbalance delay costs as well as inter-cell and intra-cell costs in forming the production cells and places the greatest emphasis in minimizing the highest cost elements. In this manner, the procedure is flexible and the user can place the highest cost to the factors that are most important in their particular application. In addition, the solution procedure

automatically identifies both the machine cells and part families. Finally, the solution procedure is computationally efficient.

The rationale of the proposed weighted cell formation method is based on the principle that similar machines and similar parts are organized in the same production cell. *Similar machines* are defined to be the set of machines that process a group of similar parts with nearly the same amount of processing times. *Similar parts* are defined to be the set of parts that have similar machine routings. A quantitative measure of the machine difference is defined as the dissimilarity coefficient, $D(i1, i2)$, for any two rows $i1$ and $i2$ in the machine-part matrix.

$$D(i1, i2) = \frac{\sum_{j=1}^n |w_{i1,j} - w_{i2,j}|}{\sum_{j=1}^n (w_{i1,j} + w_{i2,j})}$$

The dissimilarity coefficient measures the absolute difference in weights between any two rows, and is bounded below and above by 0 and 1. The smaller the dissimilarity coefficient between any two rows, the more likely the machines will belong in the same production cell.

The proposed heuristic follows two broad stages. The first stage determines an initial decomposition of the machine-part incidence matrix into production cells. The second stage iteratively searches for improvements in the design of production cells. The first stage involves constructing a network where the nodes of the network represent the machines. Each machine has a direct adjacency relationship with another machine and each node is connected by $m-1$ edges. The edge weight between any pair of nodes is the dissimilarity coefficient. The next step reduces the network into a spanning tree.

A *tree* is a connected graph without circuits. A *spanning tree* is a connected graph that contains all the nodes. A *minimum spanning tree* is a spanning tree that has minimum sum of edge weights. A spanning tree represents the adjacency relationships between all nodes. The summation of the dissimilarity coefficients is minimized by determining the minimum spanning tree.

It is desirable to have a tree structure because a deletion of any edge creates two disjoint

subgraphs (each subgraph is also a tree). Each subgraph h in the network then identifies a machine cell h . The machines grouped in machine cell h are the nodes in subgraph h . The initial tree is the minimum spanning tree (refer to Kruskal [1956] for a discussion on how to construct a minimum spanning tree). Since the edge weights are the dissimilarity coefficients, the remaining edges in the network identify the adjacency relationships that minimize the sum of the dissimilarity coefficients in the tree.

The initial feasible solution is identified as the single production cell design. In this solution $K=1$, the set R_I contains all the machines, and the set C_I contains all the parts. Improvements to this design are found by deleting an edge from the minimum spanning tree. The deletion of an edge generates two subgraphs (machine cells). The nodes in each subgraph identify the machines in each production cell. Part j is assigned to the part family h that has the maximum sum of the weights. This assignment ensures that part j is assigned to the part family h belonging to the production cell that processes the part the longest.

The edge that is deleted from the minimum spanning tree is the edge that, if removed, creates the best production cell design in terms of the objective Z . If this solution is not better than the single production cell design then the heuristic procedure stops. Otherwise the current solution is updated. Let the current solution be identified by K_h^t , R_h^t , and C_h^t . The total cost of the current solution is Z^t .

The next stage iteratively decomposes each subgraph h to search for improvements in Z^t . The solution procedure stops if there are no further improvements in Z^t or if $K^t = K^f$. The deletion of any edge in a subgraph generates two subgraphs, and the deleted edge is the one that generates the largest improvement in Z^t . If the decomposition of subgraph h into two subgraphs $h1$ and $h2$ generates an improvement in Z^t , the number of production cells (K^t) is incremented by one, the sets R_{h1}^t and R_{h2}^t are created, and the sets C_h for $h=1, \dots, K^t$ are updated. Subgraph $h1$ or $h2$ is further analyzed for possible decomposition and the procedure stops when all subgraphs have been analyzed for decomposition. Finally, the outcomes for K^* , R_h^* , and C_h^* are set to K^t , R_h^t , and C_h^t , respectively. A summary of the proposed heuristic and complexity analysis of the solution

procedure is contained in the Appendix. The overall complexity of the proposed heuristic is $O(m^2n\log m)$.

Note that the single production cell design is the initial feasible solution. It might be more advantageous to start with a different feasible solution. One such solution can be any solution found by a binary production cell formation method.

5.0 Comparison with Other Procedures

To analyze the effectiveness of using processing times and demand requirements in the production cell formation problem, the proposed heuristic solution procedure (WMST) is compared with three binary procedures that perform reasonably well and are simple to implement. These procedures are Bond Energy Algorithm (BEA), Shortest Spanning Path (SSP), and Binary Minimum Spanning Tree (BMST).

The WMST method explicitly represents the cell balance delay. The binary methods account for cell balance by assessing a cost for the number of voids in a cell design. If all parts have the same processing time, then the number of voids is an accurate representation of cell balance delay. The BMST and WMST methods automatically generate the production cells whereas the BEA and SSP methods require manual intervention to form the production cells.

A comparison of the 4 procedures in terms of the weights (W_a , W_e , and W_d) and the total cost (Z) for 11 sample problems found in the literature is performed. Table 1 lists the source of the 11 sample problems. Data sets 1, 8 and 10 are well-structured problems meaning that most of the elements in the machine-part incidence matrices belong to diagonal blocks. Data sets 5, 6, and 9 are ill-structured problems. To test a large data set, a data set with 76 machines and 468 parts is randomly generated (data set 11). The weights (workloads) are randomly sampled from a uniform random number between 1 and 9 since all the data sets reported in the literature are binary. Ten replications are made for each combination of solution procedure and data set. The total number of experimental runs performed is $4 \times 11 \times 10$.

The BEA and SSP procedures do not require input cost data to perform the analysis whereas

the BMST and WMST procedures require the user to input the cost data r and q , where r is the ratio of unit inter-cell cost to unit intra-cell cost (e/a), and q is the ratio of unit balance delay cost to unit intra-cell cost (d/a). The values r and q depend on the particular application. The procedures are compared using a low r/q ratio of 3 (cell imbalance costs dominate) and a high r/q ratio of 6 (inter-cell costs dominate).

Table 2 shows the average of the 10 replications of W_a , W_e , and W_d for the four different procedures on the 11 different data sets using an r/q ratio of 3. When cell imbalance costs are high, WMST gives the desired result of a low total cell balance delay. In all the data sets, the solution using the WMST method had the lowest W_d . Furthermore, the WMST solution procedure outperforms all the other methods in terms of the total cost (Z) especially for the ill-structured data sets (data sets 5, 6, and 9). The Newman-Keuls (Anderson and McLean [1974]) pairwise statistical test shows that Z for the WMST is statistically significant at a 95% confidence level over the other procedures for Data Sets 1-6, 8 and 9. An underline in the Newman-Keuls test means that the procedures yield a Z that is statistically insignificant. Note that BEA and SSP solution procedures are not performed for data sets 10 and 11 because for large size problems it is difficult to identify the production cells from the BEA and SSP output.

Table 3 shows the average of the 10 replications of W_a , W_e , and W_d for the four different procedures on the 11 different data sets using an r/q ratio of 6. In this case when the inter-cell costs dominate, WMST gives the lowest W_e in all data sets except data set 2 and outperforms all the other methods in terms of the total cost (Z).

The experimental runs were performed on a 386 IBM personal computer (33 MHz CPU speed without math-coprocessor) using Turbo C++. The BMST procedure found the solution using the least amount of CPU time. However, for practical purposes there is little difference between the computational requirements of BMST and WMST. For large problem sizes, WMST found a solution in 50 CPU *seconds* on an old-technology personal computer.

To demonstrate the sensitivity of W_d and W_e with the ratio r/q , Figures 1 and 2 plot W_e and W_d as a function of r/q for data set 4, respectively. The BEA and SSP solution procedures are not

influenced by the ratio r/q . In general, as the ratio r/q increases, W_d increases and W_e decreases in both BMST and WMST. However, WMST gives a lower W_e when inter-cell costs dominate and a lower W_d when cell imbalance costs dominate. For this data set, the values of W_d and W_e are sensitive to the ratio r/q for values less than 4.50 for WMST and 8.00 for BMST.

It is not surprising that the WMST outperforms BEA and SSP since they do not consider r and q . WMST outperforms BMST which suggests that an improved production cell design results when weights are considered. The disadvantage of using the weights is that the standard processing times and demand requirements may not be known at the cell design stage. To test the sensitivity of the results on the distribution of weights, the same experiments with a weight distribution from 1-5 were performed and the same results were found.

6.0 Industrial Application

CIP is concerned with simplifying and improving the production process of its high volume production line. Figure 3 contains the blueprint of 70% of the company's products. The boxes with numbers represent the individual machines. The initial layout of the machine shop followed a functional machine layout approach where the machines are arranged on the factory floor according to their type (e.g. lathes, drills, etc.).

With the functional layout, the company had significant costs associated with material handling, set-ups, and WIP. For these reasons, the company wanted to change the layout to a CMS, but did not want to change the entire layout of the factory without developing a test system first. A pilot CMS with 7 machines manufacturing 9 products in 3 production cells is easily created due to its natural grouping. A cost/benefit analysis of the pilot CMS demonstrated a significant reduction in inventory and number of set-ups.

Based on the results of the CMS pilot, the company opted to arrange 22 of the remaining 35 machines not used in the pilot CMS into a separate CMS. The machine-part incidence matrix for the proposed CMS is shown in Figure 4. The rows of the matrix represent machines while the columns represent products. The proposed CMS is to manufacture 62 part types. The entries of the matrix (

w_{ij}) represent the workload (demand rate times the standard hour of processing 1,000 workpieces) of product j on machine i .

Figure 5 shows the grouping of parts and machines using the proposed heuristic procedure as outlined in Section 5.0. A value of $r = 3$ and $q = .5$ is used to form the production cells and are based on observing the pilot CMS. The expanded CMS has 7 production cells. The number of inter-cell part movements is 9. All of the inter-cell part movement is to the production cell containing machine 403 (cell_27). Therefore it is proposed to locate production cell_27 in a central location. Figure 6 shows the layout of the proposed CMS. Cells numbered by 1 are for the Pilot CMS while the cells numbered by 2 are for the expanded CMS.

To compare the weighted method with the binary method for CIP, Figure 7 shows the grouping using the BMST method. The grouping based on the BMST method has a longer total idle time by 54 hours but a shorter inter-cell processing time by only 4 hours than the grouping based on the WMST method. Therefore, the production cells formed by the WMST method yields a lower total cost (Z).

The savings in labor can be estimated by the reduction in total set-up hours and material handling hours. The capital savings in labor equals the total savings in labor times the hourly rate. The savings in WIP inventory can be estimated by the total number of containers removed after the new cell design is implemented. The capital savings in WIP inventory equals the total number of removed containers times the inventory rate of Return-On-Investment (18% ROI). The expenses of implementing the system (one time investment) include electrical and pneumatic layouts, conveyors to tie up machines, and labor costs. The first year's net capital savings (\$52,000) equals the sum of the savings in labor (\$44,000) and WIP inventory (\$27,000), minus the expenses of implementing the new cell layout (\$19,000). In summary, the estimated benefits of implementing the expanded CMS is 70% savings in material handling time, 50% reduction in inventory, and 40% reduction in set-up time. The cost savings do not include the savings in quality improvement and decreased part tracking that ensued in the pilot CMS.

7.0 Conclusion and Future Direction of Research

To represent machine imbalances in the machine cells due to different processing times and demand requirements, it is necessary to model the design of a CMS as a weighted production cell formation problem. This paper presents a model and heuristic solution procedure for the weighted production cell formation problem. The proposed solution procedure outperformed other methods intended for the binary cell design problem in terms of the total cost (Z). An automatic grouping of machines into machine cells and parts into part families is provided. Finally, the procedure is computationally efficient for large-size problems.

A disadvantage of the proposed procedure is that it develops the production cells based on a static representation of the factory. It may be necessary to consider the dynamic nature of the factory (i.e., operation sequencing) when developing the production cells. Future research can focus on the development of an integrated framework that performs both a static and dynamic analysis. Dynamic analysis tools include simulation and queuing modeling.

Moily *et al.* [1992] note that frequent product mix changes can require frequent changes in the production cell design. It may be too costly to reconfigure the factory with each change in product mix. For this reason they propose the development of virtual cells. Future research can consider the application of the proposed solution procedure using virtual cells.

This research considered only the identification of part families and machine cells in the production cells. However, part families must also be matched with tools. A tool clustering strategy is needed to reduce the number of tool exchanges and eliminate tool duplication across a part family. Future research can consider the third dimension, tools, in the proposed weighted production cell formation procedure.

Appendix

The following steps summarize the heuristic solution procedure.

WMST Heuristic Algorithm:

Step 1 Construct a network with the nodes representing machines. The edge weight between nodes $i1$ and $i2$ is the dissimilarity coefficient $D(i1, i2)$:

$$D(i1, i2) = \frac{\sum_{j=1}^n |w_{i1,j} - w_{i2,j}|}{\sum_{j=1}^n (w_{i1,j} + w_{i2,j})}$$

Step 2 Determine the minimum spanning tree.

Step 3 Set $K^t \leftarrow 1$, assign all parts j to C_l , and all machines i to R_l . Set

$$Z^t = a \sum_{i=1}^m \sum_{j=1}^n w_{ij} + d \sum_{i=1}^m \sum_{j=1}^n \left| \frac{\sum_{l=1}^m w_{ij}}{m} - w_{ij} \right|$$

Step 4 Let Z^u be the total cost of the design if edge u is deleted from the minimum spanning tree, for $u=1, \dots, m-1$.

$$\text{Set } Z^t \leftarrow \min_{u=1, \dots, m-1} (Z^u)$$

Step 5 If $Z^t > Z^u \Rightarrow$ Set $Z^t \leftarrow Z^u$ and $K^t \leftarrow 2$. Let subgraphs 1 and 2 be the subgraphs created by deleting edge u , where $Z^u = Z^u$. Define set G as the set of subgraphs.

Let the nodes in subgraph 1 be contained in set R_1 and the nodes in subgraph 2 be contained in set R_2 . Go to Step 6.

If $Z^t \leq Z^u \Rightarrow$ Go to Step 10.

Step 6 Determine the sets C_h^t for $h=1, \dots, K^t$. Assign part j to part family l , where part family l belongs to the production cell that yields the maximum total weight:

$$d(j, R_l) \leftarrow \text{Max}_{h=1, \dots, K^t} \left(\sum_{i \in R_h} w_{ij} \right)$$

Arbitrarily break ties.

Step 7 If G is an empty set or $K^t = K^f \Rightarrow$ Go to Step 10.

If G is not an empty set \Rightarrow Go to Step 8.

Step 8 Arbitrarily select subgraph h from set G . Let $|R_h|$ be the number of nodes in subgraph h and $|R_h|-1$ be the number of edges in subgraph h . Let Z^u be the total cost of the design if edge u is deleted from subgraph h for $u=1,\dots,|R_h|-1$.

$$\text{Set } Z'' \leftarrow \min_{u=1,\dots,|R_h|-1} (Z^u)$$

Step 9 If $Z' > Z'' \Rightarrow$ Create two subgraphs $h1$ and $h2$ from subgraph h by deleting edge u , where $Z'' = Z^u$. Set $Z' \leftarrow Z''$, $K' \leftarrow K'+1$, and $R_{h1} \cup R_{h2} \leftarrow R_h$. Determine the set C_h^t for $h=1,\dots,K'$ by the same calculation as in Step 6. Remove subgraph h from set G and add subgraph $h1$ to G if $|R_{h1}| > 1$. Do the same for subgraph $h2$. Go to Step 7.

If $Z' \leq Z'' \Rightarrow$ Remove subgraph h from set G . Go to Step 7.

Step 10 Set $Z^* \leftarrow Z'$, $K^* \leftarrow K'$, $R_h^* \leftarrow R_h^t$, $C_h^* \leftarrow C_h^t$ for $h=1,\dots,K^*$, and stop.

The actual implementation of the proposed heuristic is based on Pointer data type on the Tree structure. The access scheme uses the First-Child-Forward-Twin List. The sorting technique uses Heap Sort.

To construct the network and determine the dissimilarity coefficient matrix, the time required is $O(m^2)$. To find the minimum spanning tree, the total time required is $O(m \log m)$. To calculate the objective Z for the initial feasible solution requires $O(mn)$ time. There are at most $m-1$ edges that can be deleted at each iteration, and there can be at most m iterations (m machine cells formed) during the decomposition process. Each edge deletion requires only updating W_d and W_e . The time for each edge deletion test is $O(n \log m)$ (this includes assigning all machines to machine cells and all parts to part families). Hence, the worst case overall complexity of the proposed heuristic is $O(m^2 + m \log m + mn + m^2 n \log m)$ which is formally $O(m^2 n \log m)$.

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