

Mechanisms for Cooperative Freight Routing: Incentivizing Individual Participation

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Abstract—The efficient use of the road network for freight transport has a big impact on travel times, pollution and fuel consumption, as well as on the mobility of passenger vehicles. In today’s road network truck drivers make uncoordinated selfish routing decisions which may easily congest an initially uncongested route as many truck drivers make the same selfish decision by choosing the same route in an effort to minimize their travel time without accounting for the fact that others do the same given the same available traffic information. In this paper we propose a coordinated system for truck drivers, using monetary incentives and fees, to balance the traffic load and improve the overall traffic conditions and time delays experienced by both truck and passenger vehicle drivers. The basic characteristics of the mechanisms presented are that they are budget balanced, do not penalize the truck drivers compared to the user equilibrium, and they assume voluntary participation. Two models of voluntary participation are considered: weak and strong voluntary participation. In the first, each one of the drivers prefers all the drivers (including self) to participate in the mechanism than not. In the second model, each one of the truck drivers prefers to participate in the system, provided that all the others do. For each model of voluntary participation, an incentive mechanism is designed. A special emphasis is given to the fairness of the proposed mechanisms. Numerical examples are used to demonstrate the results and the efficiency of the solution techniques.

Index Terms— Freight, Load Balancing, Mechanism Design, User Equilibrium, Routing

I. INTRODUCTION

Trucks represent an important portion of the road network loads. For example, in the United States there is an estimated number of 15.5 million trucks and 3.5 million truck drivers, while the fuel consumption of trucks accounts for 12.8% of all the fuel consumed [1]. The average share of utility vehicles in all types of vehicles within the European Union was 14.5% in 2013 [2]. Furthermore, each truck has a much larger influence on the surrounding traffic than a passenger vehicle and this impact becomes even larger as congestion increases [3], [4].

It is well known that in the absence of coordination, i.e. in the case where the drivers make selfish decisions, these decisions lead to inefficient use of the road network. A typical example is when each driver from the same origin to

destination chooses the route of minimum travel time without being aware what others are choosing. If many of them choose the same minimum travel time route that route may no longer be the minimum travel time route, contributing to an unbalanced network. In the literature, usually the situation where the drivers are making selfish uncoordinated decisions is analyzed, under the additional assumption that they anticipate correctly the behaviors of the others. This situation is modeled as a User Equilibrium (UE).

This paper provides coordination-fee mechanisms for the truck drivers which aim to balance the traffic loads and improve the overall traffic conditions and time delays experienced by all the network users. The coordination-fee mechanisms will induce cooperative routing behaviors closer to the System Optimal (SO) solution. Trucks as compared to passenger vehicles are an ideal candidate subclass of vehicles for coordinated routing since truck drivers routinely use varying routes for the same journey, depending on the traffic conditions. Furthermore, since their profits depend directly on travel time and fuel consumption, their value of time can usually be quantified. Trucks are also technologically more advanced, and localization and navigation equipment is more common in them. We thus expect that with enough economic incentives provided, truck drivers would be both capable and willing to participate in a coordinated routing scheme.

Equilibrium modeling of traffic assignment dates back to 1920 [5]. The same equilibrium principles were introduced by Wardrop in [6] and systematically studied in [7]. These models consider a continuum of drivers and essentially correspond to non-atomic games. Numerical solutions to equilibrium traffic assignment problems were proposed in [8] (see also [9]). More general game theoretic models were considered in [10] and [11]. Upper bounds for the price of anarchy, i.e. the ratio of the efficiency of the UE outcome over the efficiency of the SO outcome are presented in [12]. In the literature, there are several efforts involving models with uncertain demand. For example, in [13] models with a single OD pair are considered. In [14] there are several OD pairs, but the proportion of demand among the OD pairs is fixed. In [15] the demands to the different OD pairs could have varying ratios. The effects of the provision of additional information to the drivers are studied in [16]–[20].

The inefficiency of the UE is usually addressed in the literature using some fee (toll)-incentive schemes. The first idea proposed was to make each driver pay (internalize) for the additional cost its presence causes to the others, an idea called marginal cost pricing, or more generally congestion pricing [7], [21]. Other works on toll design that take into

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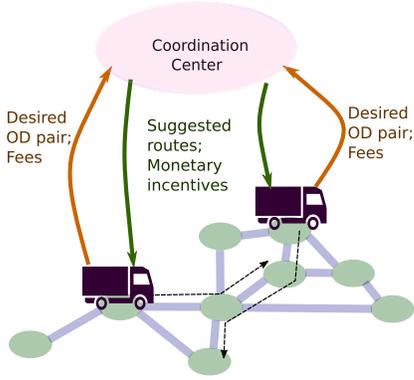


Fig. 1: The structure of the proposed mechanisms. There is a movement of material, while in parallel there is a bi-directional flow of information and money.

account the heterogeneity of the sensitivity of several users to taxes are [22]–[26]. Some robust toll designs are proposed in [27], [28] and some negative results are presented in [29]. Toll design for stochastic demand models were proposed in [30], under statistical independence conditions. A very related stream of research is the study of models with multiple classes of vehicles ([31], [32], [33]). The basic model is described in [31]. [32] proposes a toll design problem with equity constraints and [33] suggests schemes for which all the drivers are better off, in the most fair possible way.

The basic contribution of this work is the design of coordination incentive mechanisms, for the truck drivers, in which both information and money flow in both directions. A schematic of the proposed mechanism is given in Figure 1. A distinctive feature, compared to the most of the literature is that we consider models with stochastic demands that exploit the drivers’ uncertainty. First, as a benchmark for the mechanism design, we present a game theoretic traffic model with stochastic demands, describing the behavior of the truck drivers in the absence of coordination. It turns out that the equilibrium is not necessarily unique and different equilibria may induce different expected costs, for the same routes. Some uniqueness results are then proved, for special conditions of the problem. We use an equilibrium solution as a benchmark for the mechanism design. That is, the participating drivers will have a cost smaller or equal to their cost in the UE. We then investigate the possibility of using monetary incentives, as well as informed suggestions for a portion of the road network users, namely the truck drivers in order to improve the overall traffic conditions and costs experienced by all the road users. We choose to study solutions that do not harm the ones who contribute to the improvement of the traffic conditions of the others. Thus, we consider models where the truck drivers would voluntarily participate in the mechanism. There are two levels of voluntarily participation: collective and individual. For each of these levels of voluntary participation an incentive scheme is proposed. Furthermore, we focus on budget balanced schemes, where no money is put into or taken from the system.

The mechanism operates as follows. At first the drivers who

want to participate, send their OD pairs to the coordinator of the mechanism. Then, the mechanism computes the routes that minimize a “social cost function” and sends to each one of the participating players his/her proposed route and the incentives or fees. For all the participating drivers the fees are obligatory, while the proposed routes are not. However, given the incentives/fees each driver has an incentive to follow the suggested route. The players not participating in the mechanism pay no fees and receive no incentives or advice. Inspired from [32] and [33] we allocate the fees and incentives in such a way that the cooperation benefits are distributed in the participants in the most fair possible way.

In the first model of voluntary participation, the mechanism optimizes for the social cost under the constraint that the proposed solution collectively benefits the truck drivers in terms of their operation costs. Then, an amount of money is redistributed among the truck drivers such that the total cost of each one of them (operation cost and fees) is better when compared with that in the uncoordinated case (UE). Under this mechanism, the truck drivers have a collective motivation to participate. That is, each one of them prefers the outcome proposed by the mechanism than the UE. The second mechanism, which ensures individual voluntary participation, minimizes the social cost under the constraint that the proposed solution collectively benefits the truck drivers and that each one of them is better off by participating in the mechanism assuming that all the others do. This scheme exploits the traffic demand uncertainties.

Let us then comment on the differences between the current work and [31], [32], [33]. First, we analyze stochastic models and thus the system coordinator has different information from the truck drivers. This creates additional opportunities for coordination, which we exploit in the second mechanism. Furthermore, while on average our mechanisms do not collect or distribute money, they are allowed (for some realizations of the random variables) to allocate an amount of money in a way such that the overall traffic conditions are improved, while the operation cost of the trucks is worse off compared to the UE. In these realizations the truck drivers are compensated monetarily, and this amount of money is collected in other realizations of the traffic conditions. Another important difference is that the formulation of our first Mechanism is convex and allows for a very efficient solution (see the run times in a medium scale example in the Numerical Results section). Finally, the proposed mechanisms are in some sense proactive. They ask a part of the road users to provide them with demand information, before this demand is realized. This may lead to improved suggested routes.

II. MOTIVATING EXAMPLE

In this section, we describe a very simple example (Example 1) which illustrates the non-efficiency of the User Equilibrium, the mechanism design objectives (particularly: efficiency, budget balance, fairness, and voluntary participation) and motivates the problem formulation of the following sections.

Example 1: Consider a simple road network describing the transportation of freight from a port to the center of a nearby

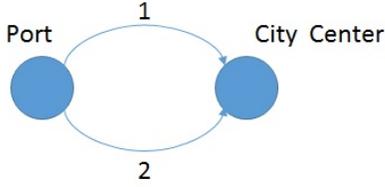


Fig. 2: The road network of Example 1.

city and assume that the port is connected to the city center by two alternative routes, described by links 1 and 2 of the graph in Figure 2.

Assume that links 1 and 2 are already used by a number of passenger vehicles denoted by X_{1p} and X_{2p} respectively. The total number of trucks is denoted by d . To simplify the calculations, we assume that the variables X_{1p} , X_{2p} and d can take continuous values. In this example the truck drivers aim to minimize the expected total time spent on the road (there are various other possibilities, such as fuel consumption, safety etc.). The time to traverse paths 1 and 2 of the graph depend on the total number of passenger vehicles and trucks using each path. In the current example, the time needed to traverse each path is the same for both the freight transport and the passenger vehicles and for paths 1 and 2 is given by:

$$\bar{J}_1 = a_1 + b_1(X_{1p} + X_{1T})^2, \text{ and} \quad (1)$$

$$\bar{J}_2 = a_2 + b_2(X_{2p} + X_{2T})^2, \quad (2)$$

respectively, where X_{1T} and X_{2T} are the total number of trucks in routes 1 and 2 respectively.

In the absence of coordination, the truck drivers are not aware of the total number of trucks d , and choose the route which minimizes the expected time. A situation, where all the truck drivers manage to minimize their expected time spent, anticipating correctly the decisions of the others is called a User Equilibrium (UE). If a percentage α chooses route 1, then $X_{1T} = \alpha d$ and $X_{2T} = (1 - \alpha)d$. To compute the UE, observe that in the UE we will have one of the following three cases: a) all the truck drivers will choose route 1 and route 2 will be more time consuming, b) all the truck drivers will choose route 2 and route 1 will be more time consuming and c) both routes have the same expected cost. Cases (a) and (b) are possible, due to the fact that the road network is used by the passenger vehicles, as well.

For example if $a_1 = b_2 = 1$, $a_2 = 2$ $b_1 = 1/2$, the (scaled) number of passenger vehicles is $X_{1p} = 1$, $X_{2p} = 0$ and the total demand is deterministic with value $d = 1$, then the equilibrium value of α will be $\alpha^{UE} = 3 - \sqrt{6} \simeq 0.55$ (to simplify the computations, we assume that all the variables are scaled with an order of magnitude of 1). In the UE the cost of both routes is equal, having a value of 2.202. The total time spent by all the trucks and all the passenger vehicles is given by:

$$T_S = (X_{1T} + X_{1p})\bar{J}_1 + (X_{2T} + X_{2p})\bar{J}_2 \quad (3)$$

and has a value of 4.4041. We may observe that in this case the UE is not efficient. Particularly, it is easy to find the value of α which minimizes the total time cost (i.e. T_S) which is called

the System Optimal (SO) solution. In the current example the system optimal solution α^{SO} has a value of $3 - \sqrt{\frac{22}{3}} \simeq 0.292$. In this case the total cost T_S has a value of 4.1412.

In order to improve the overall efficiency, we consider a coordinator who collects and distributes payments among the truck drivers. We will design coordination schemes in which the coordinator is *budget balanced* i.e., he/she imposes zero net payments. We assume that the objective of each truck driver is to minimize a cost function given by the sum of his/her expected time and the monetary cost imposed by the coordinator:

$$J_1 = \bar{J}_1 + \tau_1, \quad J_2 = \bar{J}_2 + \tau_2, \quad (4)$$

where τ_1 and τ_2 are the payments imposed to the truck drivers following paths 1 and 2 respectively.

We are interested to design mechanisms that the truck drivers would voluntarily accept. In this particular example, the total truck cost for the SO solution given by:

$$T_{tr} = X_{1T}\bar{J}_1 + X_{2T}\bar{J}_2 \quad (5)$$

has a value of $T_{tr}^{SO} = 2.3066$, whereas in the equilibrium the total truck cost would be $T_{tr}^{UE} = 2.202$. Thus, in this case there is no budget balanced scheme imposing the SO solution which the truck drivers would voluntarily accept, and the SO solution would favor the passenger vehicles in expense of the truck drivers. In order to design budget balanced schemes inducing voluntary participation of the truck drivers, we should minimize the total cost T_S under the constraint that $T_{tr} \leq T_{tr}^{UE}$. Solving this problem we get $\alpha^M = 0.412$ and the total cost is reduced to $T_S^M = 4.1989$, while the total truck cost remains unchanged compared to the UE. The payments can be easily calculated by the following system:

$$\tau_1 - \tau_2 = \bar{J}_2 - \bar{J}_1, \quad \alpha\tau_1 + (1 - \alpha)\tau_2 = 0,$$

where the first equation ensures that, in equilibrium, the truck drivers will be split according to the desired α and the second represents the budget balance.

The values of the payments are $\tau_1 = 0.2051$ and $\tau_2 = -0.1437$, meaning that the truck drivers following road 2 are getting paid whereas the truck drivers following road 1 are paying to the coordinator.

In this example, the payments are uniquely determined and serve a dual role. First, they make the desirable value of α an equilibrium i.e., the truck drivers have a motivation to follow the proposed roads and second, the drivers in the road with the higher travel time (road 2) are compensated for their extra cost. Thus, the solution is fair among the truck drivers. For a network with a single origin destination (OD) pair, these two goals coincide. In more general networks, having multiple OD pairs, a set of budget balanced payments making the desirable value of α an equilibrium, is not unique. Therefore, there is extra room to optimize for the *fairness* of the proposed solution. Such a fairness measure will be presented in Section IV-B.

Remark 1: (i) As we see in Example 2, when the demand is not deterministic, the UE is not necessarily unique. (ii) In Example 1, the trucks and the passenger vehicles are

Variable	Meaning
$G = (V, E)$	The graph representing the transportation network
N	The number of OD pairs
m	The number of roads
d_j	The total demand for OD pair j
\vec{d}	The vector of demands (d_1, \dots, d_N)
R_j	The set of possible routes for a player with OD pair j
X_{lp}	Flow of passenger vehicles traversing road l
X_{lT}	Flow of trucks traversing road l
α_r^j	The fraction of vehicles with OD pair j following route r
\bar{J}_i	The operation cost of player i
J_i	Total cost of player i , i.e. operation cost plus the fees
T_S	Total social operation cost
T_{tr}	Total operation cost for the truck drivers
\bar{A}_j	Average operation cost for the truck drivers with OD pair j
A_j	Average total cost for the truck drivers with OD pair j
τ_i	Total fees paid by player i
p_j	Total fees charged to players with OD pair j

TABLE I. Notation: The variables used.

Function	Meaning
C_{lp}	operation cost of a passenger vehicle traversing road l
C_{lT}	The operation cost of a truck traversing road l
F_r^j	The expected cost of route r for OD pair j , as a function of the fractions α

TABLE II. Notation: The functions used.

equally weighted. In the following sections, we will consider a weighted form of the total costs:

$$T_S = (1 - w)T_{tr} + wT_p, \quad (6)$$

where $w \in [0, 1]$ and T_p is the total cost for the passenger vehicles.

In the following sections we generalize the concepts and approach described in the above simple example to arbitrary road networks and number of OD pairs by developing the appropriate mathematical models.

III. MATHEMATICAL MODEL

In this section, we review and extend a static non-atomic game theoretic model (i.e. a model with a continuum of users) for the case of stochastic demands first presented in [15]. For easy reference, the notation used is summarized in Tables I, II. The players of the game represent the truck drivers.

We consider a road network described by a graph $G = (V, E)$, where V is the set of nodes and $E = (1, \dots, m)$ is the set of links. Each link corresponds to a road segment. This network serves both passenger and freight transport vehicles. The players of the game correspond to the truck drivers. Each one of the players has an origin destination (OD) pair and the set of OD pairs is $\{1, 2, \dots, N\}$. For each OD pair j there is a total demand d_j describing the total number of trucks. We assume that $[d_1 \dots d_N]^T$ is a random vector with bounded support and a commonly known distribution (that is, we have a symmetric information model). A player i having an OD pair j can choose among several different routes $r_i \in R_j$ connecting the origin and destination nodes.

Let us denote by X_{lp} the number of passenger vehicles and by X_{lT} the number of trucks traversing road segment l , in a given time interval. The cost of a player i may be separated in an operation cost \bar{J}_i and a positive or negative fee cost τ_i . The operation cost may depend on travel time, fuel consumption,

etc. We assume that the operation cost associated with the traversing of road segment l is a known nonlinear function of X_{lp} and X_{lT} denoted by $C_{lT}(X_{lp}, X_{lT})$. We assume that C_{lT} is strictly increasing in both its arguments, has continuous first partial derivatives and $\partial C_{lT}/\partial X_{lT}$ is bounded away from zero. For a passenger vehicle the corresponding function is denoted by C_{lp} . We also assume that X_{lp} is known to the truck drivers and it is independent of their choices.

The operation cost of a route r_i for player i has the form:

$$\bar{J}_i = \sum_{l \in r_i} C_{lT}(X_{lp}, X_{lT}), \quad (7)$$

and the total cost of player i is given by:

$$J_i = \bar{J}_i + \tau_i, \quad (8)$$

where τ_i is the payment that he/she may owe or collect (i.e. can be either positive or negative).

For an OD pair j and a route $r \in R_j$ let us denote by α_r^j the fraction of the trucks having OD pair j , following route r . The variable X_{lT} may be expressed as:

$$X_{lT} = \sum_{j=1}^N \sum_{r \in R_j: l \in r} d_j \alpha_r^j. \quad (9)$$

Using this formula the operation costs can be expressed as a function of the vector $\alpha = (\alpha_r^j)_{j=1, \dots, N, r \in R_j}$.

Assume that in the absence of a coordination mechanism the players will follow a User Equilibrium (UE). That is, the players within each one of the OD pairs are split among the routes in such a way that no one has the motivation to unilaterally change his/her route, in order to reduce his/her expected operation cost. Equivalently, a vector α is a UE if for every used route connecting the origin and destination of j , there is no other route with smaller expected cost. Denoting by $F_r^j(\alpha)$ the expected cost for a driver with route r connecting OD pair j , the condition for a UE is expressed mathematically by:

$$F_r^j(\alpha) \leq F_{r'}^j(\alpha), \quad (10)$$

for every $r \in R_j$ such that $\alpha_r^j > 0$, every $r' \in R_j$ and every OD pair j . In game theoretic language each one of the drivers is modelled as a player in a game, the possible actions of each player is the choice of a route, the cost function is the expected operating cost and the UE corresponds to the Nash equilibrium of the game.

In order to state the main result of this section we first recall a notion of monotonicity of vector functions (see [34]).

Definition 1: A vector function F is pseudo-monotone plus in a convex set K , if for any $\alpha_1, \alpha_2 \in K$, the conditions $(\alpha_1 - \alpha_2)^T F(\alpha_2) \geq 0$ and $(\alpha_1 - \alpha_2)^T F(\alpha_1) = 0$, imply $F(\alpha_1) = F(\alpha_2)$.

Theorem 1: Assume that there are no fees and the players care only for their expected operation cost. Then:

- (i) There exists a UE.
- (ii) Consider the vector function $F(\alpha) = (F_r^j)_{r \in R_j}^{j=1, \dots, N}$ and assume that there exist positive constants ζ_1, \dots, ζ_N such that the vector function $\tilde{F}(\alpha) = ZF(\alpha)$ is pseudo-monotone plus, where Z is the diagonal matrix

$\text{diag}(\zeta_1 I_{|R_1|}, \dots, \zeta_N I_{|R_N|})$ and I_n is the $n \times n$ identity matrix. We will call this condition, scaled pseudo-monotonicity plus. Then, the cost of each route is the same for all the equilibrium solutions.

Proof: See Appendix A. \square

Part (i) of the Theorem was already proved in [15]. Appendix A gives a simpler proof.

Let us give a simple example where the equilibrium is unique and an example with multiple equilibria.

Example 2: Consider the road network of Figure 3 which has only 4 nodes and 5 links. In the beginning consider only one OD pair from node n_1 to node n_4 . The routes of this OD pair are $r_1^1 = (1, 4)$, $r_2^1 = (1, 3, 5)$ and $r_3^1 = (2, 5)$. Assume that the flow of passenger vehicles is given by: $X_{1p} = X_{2p} = 1$, $X_{3p} = 0.1$, $X_{4p} = 0.3$ and $X_{5p} = 0.5$. The demand in the single OD pair is 0.3 or 0.5 with equal probabilities. Assume further that the cost for the use of each road segment for a passenger vehicle or a truck is given by:

$$\begin{aligned} C_{1p} &= C_{1T} = 1 + (X_{1p} + X_{1T}) + (X_{1p} + X_{1T})^2 \\ C_{2p} &= C_{2T} = 1 + (X_{2p} + X_{2T}) + (X_{2p} + X_{2T})^2 \\ C_{3p} &= C_{3T} = 0.5(X_{3p} + X_{3T})^2 \\ C_{4p} &= C_{4T} = 2 \\ C_{5p} &= C_{5T} = 0.5(X_{5p} + X_{5T})^2 \end{aligned}$$

Straightforward computations show that condition (ii) of Theorem 1 is satisfied using $Z = I_3$. Hence, in equilibrium the cost for each route is unique. This equilibrium may be computed using the complementarity formulation (for details see Section VII) and it is given by: $\alpha_1^1 = 0$, $\alpha_2^1 = 0.484$ and $\alpha_3^1 = 0.516$.

Now, assume that there is a second OD pair with origin n_2 and destination n_4 . For the demand, assume that there are two possible realizations with equal probabilities. In the first realization the demand for the first OD pair is 0.5 and 2 for the second. For the second realization the demand for the first OD pair is 0.3 and for the second 2.5. In this case there are several possible user equilibria. To obtain an equilibrium we may follow a simple procedure. At first fix a value for α_1^2 , i.e. fix a splitting of the players within the second OD pair. Then, consider the players of the first OD pair. It is not difficult to see that given α_1^2 (and thus also α_2^2), there is a unique set of fractions $(\bar{\alpha}_1^1, \bar{\alpha}_2^1, \bar{\alpha}_3^1)$ such that the players within OD pair 1 are in equilibrium, namely all the used routes joining the end points of OD pair 1 have equal expected costs, lower than the unused routes. Now if it turns out that both $\bar{\alpha}_1^1$ and $\bar{\alpha}_2^1$ are different than zero, then the routes 1 and 2 of the first OD pair have the same cost. Observing that routes 1 and 2 of the first OD pair have road 1 joining n_1 and n_2 in common, we conclude that the routes 1 and 2 of the second OD pair have the same expected costs. Thus, the tuple $(\bar{\alpha}_1^1, \bar{\alpha}_2^1, \bar{\alpha}_3^1, \alpha_1^2, 1 - \alpha_1^2)$ constitutes a UE. For different choices of α_1^2 , we end up with different equilibria. In fact the set of user equilibria is a line segment in the 5-dimensional space of $(\alpha_1^1, \alpha_2^1, \alpha_3^1, \alpha_1^2, \alpha_2^2)$. For example if we start with $(\alpha_1^2, \alpha_2^2) = (0.6, 0.4)$ we may compute $(\bar{\alpha}_1^1, \bar{\alpha}_2^1, \bar{\alpha}_3^1) = (0.224, 0.080, 0.696)$. In this equilibrium the total truck cost is 6.682.

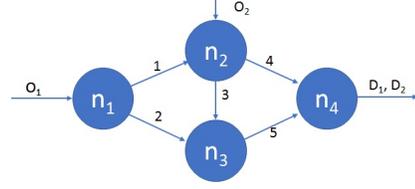


Fig. 3: The road network of the Example.

The solution that minimizes the total truck cost is given by $(0, 0.305, 0.695, 0.639, 0.361)$. In this case the total truck cost is 6.677. We may observe that different equilibria of the game with uncertain demand may lead to different expected costs for the players.

Let us then study the efficiency of the UE considering the efficiency metric, for the example of Figure 3:

$$T_S = \sum_{l=1}^m (0.5X_{lT}C_{lT}(X_{lp}, X_{lT}) + 0.5X_{lp}C_{lp}(X_{lp}, X_{lT})),$$

which stands for the total cost of the trucks plus the total cost of the rest of the vehicles averaged with equal weights. We will call this quantity the “social cost”. The social cost in the second equilibrium described is 7.68. If the fractions were chosen centrally in order to minimize T_S , then the social cost would be 7.091. Furthermore, the total truck cost would be 6.003. This value is less than 6.677 which is the corresponding value in UE. \square

The example illustrates that the uncoordinated selfish decisions leading to the UE do not induce an efficient use of the road network. Truck drivers have a **collective interest** to coordinate their actions. Furthermore, this coordination will benefit also the other road users. The problem considered in this work is how to get the truck drivers to participate in the coordination mechanism or in other words how to turn this collective interest into an **individual interest**. In the following sections, we come up with budget balanced incentive schemes that induce a voluntary participation of the players. That is, the truck drivers will exchange money among themselves and among different instants of time through a coordination mechanism, in such a way that each one of them has a motivation to participate and follow the mechanism suggestions.

Remark 2: As Example 1 suggests the conditions of Theorem 1 are not always satisfied, and we may not expect a unique solution in general. Thus, the system performance, in the absence of a coordination mechanism, is not predictable from the game description, and we should resort to our experience or collected data to describe the outcome. We do not consider this to be a serious drawback of the model, since even in simpler models a long observation of the system is needed in order to measure the costs of the players, as well as the demand distributions.

In the current work, we use the equilibrium solutions as a benchmark in order to design mechanisms for coordination. Particularly, the participating drivers should be better off compared to their equilibrium cost. For the numerical examples, we choose the user equilibrium with the least total truck cost.

IV. MECHANISM DESIGN OBJECTIVES

We first describe the basic characteristics and design objectives for the coordination mechanisms:

- The participation to the mechanism is optional.
- The participating truck drivers send their OD pairs to the coordinator of the mechanism.
- The mechanism computes the routes that minimize a “social cost function”.
- The mechanism sends to each one of the participating players his/her proposed route and the incentives or fees.
- For all the participating drivers the fees are obligatory, while the proposed routes are not.
- The mechanism is budget balanced. That is it redistributes the money among the participating truck drivers.
- The players not participating in the mechanism pay no fees and receive no incentives or advice.

The basic objectives for the mechanisms designed are: efficiency, voluntary participation, budget balance, fairness and non-exploitability. In the following subsections we introduce some efficiency and fairness measures.

A. Social Cost and Efficiency

Let us first introduce the “social cost” (or total cost) function T_S as a weighted sum of the costs of the truck and passenger vehicles:

$$T_S = \sum_{l=1}^m ((1-w)X_{lT}C_{lT}(X_{lp}, X_{lT}) + wX_{lp}C_{lp}(X_{lp}, X_{lT})), \quad (11)$$

where w is a weighting factor for passenger vehicles. For $w = 0$ the social cost depends only on the average cost of the trucks, while for $w = 1$ the social cost depends only on the cost of the passenger vehicles.

Assume that the outcome in the absence of a mechanism is described by a particular UE. The social cost of the UE is denoted by T_S^{UE} . That is, T_S^{UE} is computed using (11) with X_{lT} given by (9) and α by the UE. Define T_{tr} as:

$$T_{tr} = \sum_{l=1}^m X_{lT}C_{lT}(X_{lp}, X_{lT}). \quad (12)$$

This quantity corresponds to the total operation cost of the truck drivers. In the UE, this quantity is denoted by T_{tr}^{UE} . Furthermore, let us denote by \bar{A}_j the average operation cost of all the players with OD pair j and by \bar{A}_j^{UE} the corresponding quantity in the UE.

B. Fairness

An important characteristic of any social outcome is fairness. Several fairness measures could be defined. In the current work, the measure of fairness of a coordination mechanism depends on the distribution of the benefits compared to the UE. It is not always considered as fair for the players to have equal total costs or equal shares to the coordination benefits. As an example, consider the case of two players for which the origin/destination distance is very different; for the first one it is very long while for the second is very short. Assume also that the absolute amount of reduction of the operation

cost for the first player is much larger than for the second. It is considered as unfair to transfer money from the first player to the second one to equalize the coordination benefits. Therefore, the distribution of the total costs of the players is considered as fair if it is approximately proportional with a reference operation cost distribution. The reference cost for a player with an OD pair j should quantify the amount of resources of the network that are on average needed in order for that player to travel from the origin to the destination. In the current work we quantify the amount of resources as the average operation cost that the players with OD pair j would have if they complied with the mechanism’s proposed routes.

In order to introduce a fairness measure, denote by J_j^{UE} the expected cost of a player with OD pair j . For a mechanism M which suggests fractions $\alpha_r^j(d)$ for the routes and payments depending on j, r and d , let us denote by J_{jr}^M the total cost of a player with OD pair j , assuming that he or she complies with the mechanism suggestions. Furthermore, denote by \bar{A}_j^M the average cost that players with OD pair j will have if they comply with the mechanism suggestions and T_{tr}^M the corresponding total cost. The total expected benefit for the players is given by:

$$\Delta = E \left[\sum_{j=1}^N \sum_{r \in R_j} d_j \alpha_r^j (J_j^{\text{UE}} - J_{jr}^M) \right]. \quad (13)$$

A mechanism is considered as fair, if the total benefits are distributed among the players proportionally to the operation cost that they would have if they follow the mechanism suggestions. Then a fairness measure, quantifying the amount of unfairness, is described by:

$$\Phi = E \left[\sum_{j=1}^N \sum_{r \in R_j} d_j \alpha_r^j \left((J_j^{\text{UE}} - J_{jr}^M) - \frac{E[d_j \bar{A}_j^M]}{E[d_j]E[T_{tr}^M]} \Delta \right)^2 \right]. \quad (14)$$

Remark 3: Note that:

$$E \left[\sum_{j=1}^N \sum_{r \in R_j} d_j \alpha_r^j \left((J_j^{\text{UE}} - J_{jr}^M) - \frac{E[d_j \bar{A}_j^M]}{E[d_j]E[T_{tr}^M]} \Delta \right) \right] = 0.$$

Thus, the fairness measure is defined as the variance of the distribution of benefits of cooperation among the several players. The quantity $\frac{E[d_j \bar{A}_j^M]}{E[d_j]}$ in fact represents the operation cost of an average trucker in OD pair j over the several routes and the several realizations of the random demands. That is, the operation cost is averaged over the several possible realizations, weighted by the number of trucks in each realization. We would like the players to benefit from cooperation in proportion to their operation cost. The proportion of the operation cost of an average player in OD pair j is $\pi_j := \frac{E[d_j \bar{A}_j^M]}{E[d_j]E[T_{tr}^M]}$. Thus, the mechanism is completely fair if the benefits of each player are equal to a fair share, which for a truck with OD pair j are $\pi_j \Delta$.

There are several alternative definitions. For example, the expected quadratic deviation may be substituted by an expected absolute value or the maximum value. Another alternative is to consider as fair a distribution of benefits according to the UE costs.

Proposition 1: Consider a mechanism with payment rule $\tau_i = g(r_i, d)$ where r_i is the route of player i and d is the demand vector. Then, there is another mechanism with payment rule $\tau_i = \tilde{g}(r_i, d)$ such that the following conditions are satisfied:

- (i) For any i_1, i_2 with OD pair j :

$$\bar{J}_{i_1}(r_1, \alpha) + \tilde{g}(r_{i_1}, d) = \bar{J}_{i_2}(r_2, \alpha) + \tilde{g}(r_{i_2}, d), \quad (15)$$

where the operation costs are computed assuming that the drivers comply with the mechanism directions.

- (ii) They allocate the same amount of money to the drivers of each OD pair:

$$\sum_{r \in R_j} \alpha_r^j(d) \tilde{g}(r, d) = \sum_{r \in R_j} \alpha_r^j(d) g(r, d). \quad (16)$$

The payment scheme described in (i) has the properties:

- (iii) Compliance to any proposed routes becomes a Nash equilibrium.
 (iv) Of all the payment rules satisfying (16), the payment rule \tilde{g} which satisfies also (15), minimizes the fairness criterion Φ .
 (v) For the payment scheme \tilde{g} , the fairness measure Φ is written as:

$$\Phi = E \left[\sum_{j=1}^N d_j \left((A_j^{\text{UE}} - \bar{A}_j^M - p_j/d_j) - \frac{E[d_j \bar{A}_j^M]}{E[d_j] E[T_{tr}^M]} \Delta \right)^2 \right], \quad (17)$$

where $p_j = d_j \sum_{r \in R_j} \alpha_r^j \tilde{g}(r, d)$ is the total amount of money charged to the drivers of OD pair j .

Proof: See Appendix B \square

V. MECHANISM FOR WEAK VOLUNTARY PARTICIPATION

This section describes an incentive scheme for the truck drivers to achieve weak voluntary participation. That is, each one of the players prefers all the players (including himself/herself) to participate. We first assume that everyone participates and then derive conditions ensuring that this happens.

Let us first consider the following optimization problem:

$$\begin{aligned} & \underset{\alpha(\cdot)}{\text{minimize}} && E[T_S(\alpha)] \\ & \text{subject to} && E[T_{tr}(\alpha(d))] \leq E[T_{tr}^{\text{UE}}] \\ & && \sum_{r \in R_j} \alpha_r^j(d) = 1, \quad j = 1, \dots, N \\ & && \alpha_r^j \geq 0 \end{aligned} \quad (18)$$

where $\alpha(\cdot)$ is a function of the actual demand d . Problem (18) corresponds to the minimization of the social cost subject to the constraint that on average the total operation cost of the truck drivers is not increased. Note that in Problem (18) the coordinator knows the demands given by all the participants and computes the optimal value of α depending on the demand realization d . However, the different realizations of d are coupled through the first constraint of (18). The problem is

clearly feasible due to the fact that the UE belongs to the constraint set.

Let us now describe the mechanism. At first, the OD pairs of the players are used to find the demand vector d . Then, the solution to the optimization problem (18) is used to compute $\alpha(d)$. The players having an OD pair j are split among the possible routes according to $(\alpha_r^j)_{r \in R_j}$.

In order to describe a payment scheme we first determine the total amount of money charged to the drivers of each OD pair p_j , $j = 1, \dots, N$. In this scheme the payments are:

$$p_j = d_j \left[\bar{A}_j^{\text{UE}} - \bar{A}_j^M - \frac{E[d_j \bar{A}_j^M]}{E[d_j] E[T_{tr}^M]} \Delta \right]. \quad (19)$$

It is not difficult to show that $E[\sum p_j] = 0$. To do so observe that the expected total payment is:

$$E \left[\sum_j p_j \right] = E \left\{ -\Delta + \sum_j d_j (\bar{A}_j^{\text{UE}} - \bar{A}_j^M) \right\} = 0. \quad (20)$$

That is, the coordinator of the mechanism does not make money on average, but redistributes the participants own money among them and over time instants. However the coordinator has absorbed the risks of the players. In order to implement such a mechanism the coordinating organization should have a reserve of money.

Proposition 1 is then used to derive the individual payments τ_i , given the total amount of money charged to the drivers of each OD pair p_j . The results are summarized by the following Theorem:

Theorem 2: Consider the payment scheme obtained by substituting (19) to (33). Under this scheme:

- (i) Compliance becomes a Nash equilibrium.
 (ii) Each player prefers the application of the mechanism ex-post, compared to the UE.
 (iii) The mechanism is budget balanced on average.
 (iv) The mechanism is fair, i.e. the fairness measure Φ vanishes.
 (v) Assume that the following holds:

$$E[\bar{A}_j^{\text{UE}}] \geq \frac{E[d_j \bar{A}_j^M]}{E[d_j] E[T_{tr}^M]} \Delta. \quad (21)$$

Then, the mechanism is non-exploitable in the sense that no player has an incentive to declare an OD pair if he/she would not be willing to travel with zero delay.

- (vi) Assume that the functions $C_{IT}(X_{lp}, \cdot)$ are convex. Then, (18) is a convex optimization problem.

Proof: See Appendix C \square

Remark 4: Modifying (19) as follows:

$$p_j = d_j (\bar{A}_j^{\text{UE}} - \bar{A}_j^M) - \frac{\bar{A}_j^M}{T_{tr}^M} \Delta \quad (22)$$

we obtain a scheme, which at every time step is budget balanced (ex-post budget balance). However, (iv) - (vi) of Theorem 2 do not necessarily hold. Particularly, (iv) of Theorem 2 is weakened as: (iv)' each player prefers the mechanism before he/she learns d (ex-interim) [35].

Remark 5: Up to now we assume that all the players participate in the mechanism. Furthermore, each one of them prefers all of the players (including himself/herself) to participate in the mechanism than not. We can restate the mechanism such that each one of the players has a motivation to participate. In order to do so, we introduce an intermediate step. The mechanism first receives the OD pairs. Then, it computes the suggested routes and the fees and sends them to the drivers. Then, in the intermediate step, the players are asked if they approve the mechanism. Each player has two choices {‘Approve’, ‘Veto’}. If every player approves the mechanism, then the payments from and to the mechanism become obligatory. If at least one player vetoes, then the mechanism is not used. Theorem 2 part (ii) implies that it is a dominant strategy for each player to approve the mechanism (i.e. ‘Approve’ is his/her optimal action regardless of the actions of the others). Thus, in the two stage mechanism we have also **voluntary participation** (individual rationality).

One drawback of the introduction of the intermediate step is that the outcome is sensitive to the choices of each single player. A practical alternative is to have a status quo situation, where the mechanism is applied. Then, ask the participants if they would like to suspend it. If a player wants to suspend the mechanism she can file an application, paying a small amount of money (e.g. \$1). If a small portion of the players (e.g. 2%) files applications to stop the mechanism, then the mechanism is suspended, and they receive their money back.

Remark 6: The collective motivation of the truck drivers to have the mechanism in force is $E[T_{tr}^{UE} - J_{tr}^M]$. Thus, we can probably restate the first constraint in (18) as $E[T_{tr}(\alpha)] + \delta \leq E[T_{tr}^{UE}]$ for a properly chosen δ .

A complete description of this mechanism is summarized in Algorithm 1.

Algorithm 1

Input: The graph G , the stochastic description of the demand, the functions C_{lp} and C_{lT} and a reference UE α^{UE} .

- 1: Receive OD pairs of the players and use them to compute d .
 - 2: Solve (18) to obtain $\alpha^M(d)$.
 - 3: Use (19) and (33) to compute the suggested payments.
 - 4: Send the suggested routes and payments to the participants.
 - 5: Make the payments according to the realized routes.
-

VI. MECHANISM FOR STRONG VOLUNTARY PARTICIPATION

In this section we describe a mechanism in which there is voluntary participation and the mechanism is implemented even if some players do not participate. In the spirit of Proposition 1, consider the following optimization problem:

$$\begin{aligned}
& \underset{\alpha(\cdot), p_j(\cdot)}{\text{minimize}} && \lambda E[T_S(\alpha)] + (1 - \lambda)\Phi(\alpha, p) \\
& \text{subject to} && E \left[\sum_{r \in R_j} \alpha_r^j \bar{J}(r, \alpha) + p_j(d)/d_j \right] \leq \min_{r \in R_j} E[\bar{J}(r, \alpha)], \\
& && E[T_{tr}(\alpha(d))] \leq E[T_{tr}^{UE}] \\
& && \sum_{r \in R_j} \alpha_r^j = 1, \alpha_r^j \geq 0, E \left[\sum_j p_j \right] = 0,
\end{aligned} \tag{23}$$

where $\lambda \in [0, 1]$. In this problem both efficiency and fairness are part of the objective function with a weighting factor λ and voluntary participation is a constraint. The problem is feasible because the UE with $p_j = 0$, for all j is a feasible solution.

Theorem 3:

- (i) Denote by $\alpha(\cdot), p(\cdot)$ any (sub-)optimal solution of (23) and denote by $\tau_i(r, j)$ the payments obtained using p_j in Proposition 1. Then, compliance becomes a Nash equilibrium, the mechanism is budget balanced on average and there is strong voluntary participation ex-ante (before truck drivers learn the actual demand vector).
- (ii) Assume that $\lambda = 1$ and that there is an OD pair j_0 such that d_{j_0} takes at least two different positive values $d_{j_0}^1$ and $d_{j_0}^2$ with positive probabilities. Then, there is a solution to (23) with value coinciding with the value of (18) which in addition satisfies:

$$E \left[\sum_{r \in R_j} \alpha_r^j \bar{J}(r, \alpha) + p_j(d)/d_j \right] \geq 0 \tag{24}$$

- (iii) Any solution of (23) satisfying the conclusions of (ii), is non-exploitable in the sense that no player has an incentive to declare an OD pair if he/she would not be willing to travel with zero delay.

Proof: See Appendix D. \square

Remark 7: In the case of full information, i.e. when d is deterministic the only feasible solution of (23) is the UE. Thus, this scheme depends on the ‘‘collective value of incomplete information’’.

Assuming that $\lambda = 1$ and that the conditions of part (ii) of Theorem 3 are satisfied then the routing decisions can be obtained from the optimization problem (18) and the payments according to the solution of the problem:

$$\begin{aligned}
& \underset{p_j}{\text{minimize}} && E \left[\sum_{j=1}^N d_j \left((A_j^{UE} - \bar{A}_j^M - p_j/d_j) - \frac{E[d_j \bar{A}_j^M]}{E[d_j]E[T_{tr}^M]} \Delta \right)^2 \right] \\
& \text{subject to} && E[p_j(d)/d_j] \leq \min_{r \in R_j} E[\bar{J}(r, \alpha)] - E[\bar{A}_j^M], \\
& && E[p_j(d)/d_j] \geq -E[\bar{A}_j^M], \\
& && E \left[\sum_j p_j \right] = 0,
\end{aligned} \tag{25}$$

Observe that problem (25) is a quadratic program.

Remark 8: Let us then move to the other extreme where $\lambda = 0$. In that case we may choose to charge no fees, i.e. $p_j = 0$ and suggest the UE routes. With this scheme the total cost in (23) which depends only on Φ will be zero. Indeed, $A_j^{\text{UE}} = \bar{A}_j^M$ and $\Delta = 0$. This observation along with part (iii) of Theorem 3 show the trade-off between efficiency and fairness.

A complete description of this mechanism is summarized in Algorithm 2.

Algorithm 2

Input: The graph G , the stochastic description of the demand, the functions C_{lp} and C_{lT} and a reference Nash equilibrium α^{UE} .

- 1: Receive the OD pairs of the players and use them to compute d .
 - 2: Solve (23) to obtain $\alpha^M(d)$ and p_j .
 - 3: Use (33) to compute the incentives.
 - 4: Send to the players the proposed routes and the fees assigned to each route.
 - 5: Make the payments according to the players actions.
-

VII. COMPARISON OF THE MECHANISMS

In this section, we compare the proposed mechanisms, the UE solution and the System Optimal (SO) solution in terms of their efficiency, freedom of choice and fairness.

Denote by T_S^{SO} the minimum value of the problem:

$$\begin{aligned} & \underset{\alpha(\cdot)}{\text{minimize}} && E [T_S(\alpha)] \\ & \text{subject to} && \sum_{r \in R_j} \alpha_r^j = 1, \quad \alpha_r^j \geq 0, \end{aligned} \quad (26)$$

i.e. the social cost if routing were centrally planned.

The following proposition compares the two mechanisms described with the Nash outcome and the centrally planned solution.

Proposition 2: The following inequalities hold:

$$T_S^{\text{SO}} \leq T_S^{M_1} \leq T_S^{M_2} \leq T_S^{\text{UE}}. \quad (27)$$

Furthermore, if $w = 0$ then $T_S^{\text{SO}} = T_S^{M_1}$ and for $\lambda = 1$, under the assumptions of Theorem 3 part (ii), $T_S^{M_2} = T_S^{M_1}$.

Proof: The proof follows immediately from the analysis above. \square

The proposition states that the first mechanism is more efficient than the second and they both improve the social cost compared to the UE. However, it is easier to implement Mechanism 2 than Mechanism 1 and Mechanism 2 is implemented even if some of the truck drivers are not participating. Furthermore, in Mechanism 2 each player is more free to chose his/her actions and these actions do not have an immediate consequence on the behavior of the other drivers. Of course the choice of players is less free in both mechanisms when compared with the UE but much more free when compared with a SO solution controlled by a central planner. Finally, if the SO solution is more efficient than Mechanism 1, then the SO solution would collectively harm the truck drivers compared to the UE.

Let us then compare the mechanisms in terms of fairness. From Theorem 2 and Remark 8 we conclude that:

Corollary 1: The outcomes of Mechanism 1 are more fair compared to the outcomes of Mechanism 2. When $\lambda = 0$, Mechanisms 1 and 2 are equally fair.

VIII. NUMERICAL EXAMPLES

In this section, the techniques presented are applied to the Sioux-Falls network. Before presenting the numerical results, let us describe the techniques used to solve numerically the problems involved and particularly the computation of the UE with the least total cost.

The equilibrium condition (10) can be written equivalently as a set of conditions:

$$0 \leq \alpha_r^j \perp F_r^j(\alpha) - v_j \geq 0, \quad \sum_{r \in R_j} \alpha_r^j = 1 \quad (28)$$

for $j = 1, \dots, N$, $r \in R_j$ where v_j is a set of free variables denoting the expected cost of any used route with OD pair j and the orthogonality notation \perp means that for each j and r , either $\alpha_r^j = 0$ or $F_r^j(\alpha) - v_j = 0$.

Then, the problem of finding the user equilibrium which induces the minimum total truck cost (as described in Remark 2) can be expressed as an optimization problem with complementarity constraints (e.g. [36]):

$$\begin{aligned} & \underset{\alpha(\cdot)}{\text{minimize}} && E \left[\sum_{l=1}^m X_{lT} C_{lT}(X_{lp}, X_{lT}) \right] \\ & \text{subject to} && 0 \leq \alpha_r^j \perp F_r^j(\alpha) - v_j \geq 0, \\ & && \sum_{r \in R_j} \alpha_r^j = 1. \end{aligned} \quad (29)$$

For the needs of this section, problem (29) is solved using the PATH solver on the NEOS server [37].

A. The Sioux Falls network

We apply the incentive schemes of the previous sections to a network with 24 nodes and 76 edges, called the Sioux Falls network, originally presented in [38]. The Sioux Falls network has been used extensively as a benchmark problem in transportation modeling.

For the needs of the current example, we assume that the cost of each road segment corresponds to travel time and it is given by a Bureau of Public Roads (BPR) function:

$$C_{lp}(X_{lp}, X_{lT}) = C_{lT}(X_{lp}, X_{lT}) = c_a + c_b \left(\frac{X_{lp} + 3X_{lT}}{c_k} \right)^4.$$

Furthermore, as passenger vehicles load, we use the solution presented in [39]. We then assume that there is a number of trucks having six possible OD pairs: $j_1 = (n_1, n_7)$, $j_2 = (n_1, n_{11})$, $j_3 = (n_{10}, n_{11})$, $j_4 = (n_{10}, n_{20})$, $j_5 = (n_{15}, n_5)$ and $j_6 = (n_{24}, n_{10})$. In order to have a computationally tractable model, for each OD pair we assume that there is a limited number of possible routes. Particularly, we assume that each one of the drivers have available only 10 possible routes which correspond to the 10 less congested routes if there were no trucks. For the demand we assume that it takes

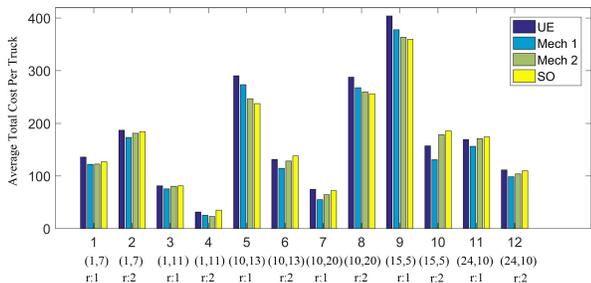


Fig. 4: Comparison of the costs for the UE and the Mechanisms 1 and 2. Each 4-tuple of bars represents the cost of a truck having a particular OD pair, in a realization of the demand for the UE and Mechanisms 1, 2 and SO respectively.

one of two equi-probable values (3, 4.5, 6, 2.4, 12, 3.6) and (3.6, 1.8, 3.9, 15, 5.4, 2.4).

Assume that $w = 0.5$. The social cost of the UE is 20407.33 and the total truck cost is 7412.24. Applying Mechanism 1, the social cost becomes 19679.49 and the total truck cost becomes 6827.4. Thus, compared with Mechanism 1 the UE has a total truck cost that is 8.57% higher and a social cost that is 3.7% higher. The SO social cost in this case coincides with the social cost under Mechanism 1. We then implement Mechanism 2 with $\lambda = 0.9995$. The total truck cost is 6822.43 and the social cost is 19682.56. In this solution, the Fairness measure has a value of 16433.58. We observe that the total cost is slightly higher, but the truck cost is slightly lower. A comparison of the total cost with Mechanism 1, with Mechanism 2 and the cost under the UE and SO solutions are illustrated in Figure 4. As expected, Mechanism 1 is always better than the UE, but Mechanism 2 is better for each OD pair only on average. For example, in realization 2 of the demand in the OD pair (15, 5), the cost under Mechanism 2 is larger than in the UE. Comparing the outcome of Mechanism 1 with the SO solution, we observe that they have the same social cost, but the SO solution may favor the drivers within some OD pairs over the others, while Mechanism 1 distributes the benefits of cooperation in a more fair way.

Regarding the run-time, the NEOS server reports an execution time of 0.002s for the equilibrium computation (solution of Problem (29)), 0.006s for the computation of traffic assignment under Mechanism 1 (solution of Problem (18)), and 0.046s for Mechanism 2 computations (solution of Problem (23)).

IX. CONCLUSION

The use of two coordination/fee mechanisms for truck routing in a road network is proposed. Both mechanisms are budget balanced. The mechanisms are designed such that they induce voluntary participation and they distribute the benefits of the cooperation as fairly as possible. To do so a fairness measure was first introduced. The first mechanism is always fair and efficient, but it allows only for either all of the players to participate or none. The second mechanism is designed by minimizing a weighted sum of the efficiency and fairness under stronger voluntary participation constraints, but it allows for some players to participate and some not. The mechanisms

are applied to the Sioux-Falls network and an improvement in terms of efficiency is observed.

APPENDIX

A. Proof of Theorem 1

(i) The equilibrium condition (10) can be equivalently written using a variational inequality. Particularly, it is not difficult to see that α corresponds to an equilibrium if and only if:

$$(\alpha' - \alpha)^T F(\alpha) \geq 0, \quad \text{for all } \alpha' \in K, \quad (30)$$

where $K = \{\alpha \in \mathbb{R}^{|R_1| + \dots + |R_N|} : \alpha_r^j \geq 0 \text{ and } \sum_{r=1}^{|R_j|} \alpha_r^j = 1, \text{ for all } j\}$ and $F(\alpha)$ is the route cost vector having components given by:

$$F_r^j(\alpha) = E \left[\sum_{l \in r} C_{lT} \left(X_{lT}, \sum_{j'=1}^N \sum_{r' \in R_{j'}: l \in r'} d_{j'} \alpha_{r'}^{j'} \right) \right]. \quad (31)$$

The component $F_r^j(\alpha)$ represents the expected cost of route r . We will denote by $VI(F, K)$ the variational inequality (30).

This result has been already proved in [15], using Kakutani fixed point theorem. For a simpler proof, observe that the variational inequality (30) satisfies the conditions of Corollary 2.2.5 of [34] which implies the existence of a solution to (30) and thus the existence of an equilibrium.

(ii) At first let us show that the variational inequalities: $VI(F, K)$ and $VI(\tilde{F}, K)$ have identical solution sets. To do this recall that α satisfies $VI(F, K)$ if and only if $F_r^j(\alpha) \leq F_{r'}^j(\alpha)$ for all j, r and r' such that $\alpha_{r'}^j > 0$. The corresponding condition for $VI(\tilde{F}, K)$ is $\zeta_j F_r^j(\alpha) \leq \zeta_j F_{r'}^j(\alpha)$ for all j, r and r' such that $\alpha_{r'}^j > 0$. Due to the fact that $\zeta_j > 0$, the two problems are characterized by equivalent conditions and thus they have identical solution sets.

Thus, it is sufficient to show that $\tilde{F}(\alpha_1) = \tilde{F}(\alpha_2)$, for any α_1, α_2 satisfying $VI(\tilde{F}, K)$. Corollary 2.3.7 of [34] applies and proves the desired result. \square

B. Proof of Proposition 1

Let p_j be the total amount of money paid by the players with OD pair j , under the payment scheme g :

$$p_j = d_j \sum_{r \in R_j} \alpha_r^j g(r, d). \quad (32)$$

Then \tilde{g} with:

$$\tilde{g}(r, \alpha) = \bar{A}_j^M - \bar{J}(r, \alpha) + p_j/d_j \quad (33)$$

satisfies both (i) and (ii). Particularly:

$$\bar{J}_i(r_i, \alpha) + \tilde{g}(r_i, d) = \bar{A}_j^M + p_j/d_j, \quad (34)$$

for all players i with OD pair j , which proves (i). Substituting (33) into (16) results in both terms becoming p_j/d_j which proves (ii).

(iii) Due to (15), no player has an incentive to change his/her strategy. Thus, compliance becomes an equilibrium

(iv) Assume that the payments follow the rule $\tau_i = g(r_i, d)$. Consider the (random) quantities:

$$\begin{aligned} \phi_j &= \sum_{r \in R_j} \alpha_r^j \left((J_j^{\text{UE}} - J_{jr}^M) - \frac{E[d_j \bar{A}_j^M]}{E[d_j]E[T_{tr}^M]} \Delta \right)^2 \\ &= \sum_{r \in R_j} \alpha_r^j \left([\tilde{g}(r, d) - g(r, d)] - (\bar{J}_j(r) + \tilde{g}(r, d)) + \right. \\ &\quad \left. + J_j^{\text{UE}} - \frac{E[d_j \bar{A}_j^M]}{E[d_j]E[T_{tr}^M]} \Delta \right)^2. \end{aligned} \quad (35)$$

Due to (15), the term $\bar{J}_j(r) + \tilde{g}(r, d)$ does not depend on r . Thus, the only term depending on r is $[\tilde{g}(r, d) - g(r, d)]$. Therefore, ϕ_j can be written in the form:

$$\phi_j = \sum_{r \in R_j} \alpha_r^j (z_{j,r} - \mu_j)^2,$$

where $z_{j,r} = \tilde{g}(r, d) - g(r, d)$, μ_j does not depend on r and it holds $\sum_{r \in R_j} \alpha_r^j z_{j,r} = 0$. It is not difficult to see that $z_{j,r} = 0$, i.e. $\tilde{g}(r, d) = g(r, d)$ minimizes ϕ_j , under the constraint $\sum_{r \in R_j} \alpha_r^j z_{j,r} = 0$. This holds true independently of d . Furthermore:

$$\Phi = E \left[\sum_{j=1}^N \phi_j \right]. \quad (36)$$

Thus Φ is minimized for $\tilde{g}(r, d) = g(r, d)$.

(v) Substituting (35) into (36) and using that $\tilde{g}(r, d) = g(r, d)$ and that $J_j^{\text{UE}} = \bar{A}_j^{\text{UE}}$, we get (17). \square

C. Proof of Theorem 2

(i) The payment scheme described satisfies the conditions of Proposition 1. Thus, compliance is a Nash equilibrium.

(ii) To show that each player prefers the application of the mechanism with payment rule (19) ex-post, observe that Δ is non-negative. Thus:

$$\begin{aligned} J_i^M &= \bar{J}_i(r_i, \alpha) + \tilde{g}(r_i, d) = \bar{A}_i^M + p_j/d_j = \\ &= \bar{A}_i^{\text{UE}} - \frac{E[d_j \bar{A}_j^M]}{E[d_j]E[T_{tr}^M]} \Delta \leq \bar{A}_i^{\text{UE}} = J_i^{\text{UE}}. \end{aligned} \quad (37)$$

(iii) The average budget balance property is a consequence of (20) and part (ii) of Proposition 1.

(iv) Substituting (19) into (17) and straightforward computations give the desired result.

(v) Straightforward.

(vi) We first prove that for given d the functions $T_S(\alpha)$ and $T_{tr}(\alpha)$ are convex. The function $T_S(\alpha)$ can be written as a composition $F_2 \circ F_1(\alpha)$ where $F_1(\alpha) = (X_{lT})_{l=1}^m$ is a linear function. It is not difficult to see that F_2 is a convex function. First, observe that the second partial derivatives $\partial F_2 / \partial X_{lT}$ are positive for every l and the mixed partial derivatives are zero. Thus, $T_S(\alpha)$ is convex since it is a composition of a convex with a linear function. Similar arguments can prove the convexity of $T_{tr}(\alpha)$. Taking the expectation with respect to d the functions $E[T_S(\alpha)]$ and $E[T_{tr}(\alpha)]$ are convex, which completes the proof.

D. Proof of Theorem 3

(i) The facts that there exist a solution to (23) and that compliance is a Nash equilibrium are already proved in the main text. The fact that there is ex-ante voluntary participation is implied by the first constraint of (23) and Proposition 1. The budget balance on average is equivalent with the last constraint of (23).

(ii): Consider an $\alpha(\cdot)$ solving (18). Choose $p_j(d)$, $j \neq j_0$ such that:

$$E \left[\sum_{r \in R_j} \alpha_r^j \bar{J}(r, \alpha) + p_j(d)/d_j \right] = \min_{r \in R_j} E [\bar{J}(r, \alpha)]. \quad (38)$$

For $j = j_0$ choose p_{j_0} to depend only on d_{j_0} . Furthermore, choose $p_{j_0}(d_{j_0}) = 0$ if $d_j \neq d_{j_0}^1$ and $d_{j_0} \neq d_{j_0}^2$. Denoting by π_1 and π_2 the probabilities of $d_{j_0}^1$ and $d_{j_0}^2$ respectively, the values of $p_{j_0}(d_{j_0}^1)$ and $p_{j_0}(d_{j_0}^2)$ are obtained from the solution of the following linear system:

$$\begin{aligned} \frac{\pi_1}{d_{j_0}^1} p_{j_0}(d_{j_0}^1) + \frac{\pi_2}{d_{j_0}^2} p_{j_0}(d_{j_0}^2) &= \\ &= \min_{r \in R_j} E [\bar{J}(r, \alpha)] - E \left[\sum_{r \in R_j} \alpha_r^j \bar{J}(r, \alpha) \right] \end{aligned} \quad (39)$$

$$\pi_1 p_{j_0}(d_{j_0}^1) + \pi_2 p_{j_0}(d_{j_0}^2) = -E \left[\sum_{j \neq j_0} p_j \right] \quad (40)$$

This linear system has a unique solution due to the fact that $d_{j_0}^1 \neq d_{j_0}^2$. Furthermore, (39) ensures that (38) holds for every j . Hence, both the first constraint of (23) and (24) are satisfied.

(iii) Immediate consequence of (24). \square

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