Collaborative optimization of last-train timetable with passenger accessibility: a space-time network design based approach

Lixing Yang[a]*, Zhen Di[a,b], Maged M. Dessouky[c], Ziyou Gao[a]*, Jungang Shi[b]

[a] *State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing, 100044, China*

[b] *College of Transportation and Logistics, East China Jiaotong University, Nanchang 330013, China*

[c] *The Daniel J. Epstein Department of Industrial and Systems Engineering, University of Southern California, Los Angeles, CA 90089-0193, USA*

Abstract

To improve the accessibility of the metro network during night operations, this study aims to investigate a collaborative optimization for the last train timetable in an urban rail transit network. By using a space-time network framework, all the involved transportation activities are well characterized in an extended space-time network, in which the train space-time travel arcs, passenger travel arcs, transfer arcs, etc., are all taken into account. Two performance measures are proposed to evaluate the network-based timetable of the last trains. Through considering the route choice behaviors, the problem of interest is formulated as 0-1 linear programming models from the perspective of a space-time network design. To effectively solve the proposed models, we dualize the hard constraints into the objective function to produce the relaxed models by introducing a set of Lagrangian multipliers. Then, the sub-gradient algorithm is proposed to iteratively minimize the gap of the lower and upper bounds of the primal models. Finally, two sets of numerical experiments are implemented in an illustrative network and the Beijing metro network, respectively, and experimental results demonstrate the efficiency and performance of the proposed methods.

Keywords: Last train timetable; Space-time network design; Accessibility; Lagrangian relaxation

1 Introduction

Nowadays, more and more urban rail transit systems have been constructed in large cities. Due to their wide accessibility, large capacity, low carbon emissions, high security and reliable services (Krasemann, 2012; Yang et al., 2016; D'Acierno et al., 2017), the urban rail transit is widely regarded as an indispensable transportation mode for satisfying passenger travel demands and relieving serious traffic congestion. As the backbone of urban traffic, it also plays an irreplaceable role in improving the accessibility of long distance commuting and reducing the reliance on private cars. For instance, the total milage of urban rail transit system has increased up to 5000 km in China, and more than 30 cities own their urban rail transit systems. In Beijing, the urban rail transit system consists of 18 operational lines with a total length of 608 km, and it usually carries more than 10 million passengers to their destinations each day (Yin et al., 2017).

To maintain high-quality services for demand, the passenger-oriented train timetabling problem has attracted tremendous attention from a variety of researchers in the literature. In general, three types of

*Corresponding authors. E-mail addresses: lxyang@bjtu.edu.cn (L. Yang), zygao@bjtu.edu.cn (Z. Gao)

schedules are usually considered in practical operations to meet dynamic passenger demands: peak-hour train timetables, off-peak-hour train timetables, and last-train timetables. For these schedules, different operation modes are considered in accordance to the dynamic arrival rates of passengers. For instance, more train services should be executed in peak-hours to transport a large amount of passenger flow, while less train services should be taken into operations in off-peak hours so as to save operations costs (Canca et al., 2014; Yin et al., 2017; Guo et al., 2017). In addition, another significant concern for the metro system is the last-train timetabling process since it determines the service quality in night operations, which is also an important evaluation criterion from the perspective of passengers since it specifies whether they can go to their destinations successfully at night (e.g., going home). In recent years, the last-train timetabling problem has been investigated by a significant amount of researchers (Xu et al., 2008; Kang et al., 2015a,b; Kang and Zhu, 2017; Kang and Meng, 2017; Yang et al., 2017; Chen et al., 2019). The key concern of this problem is to determine the last-train timetable on each line of the urban rail transit network, in which the quality of the connection of the different trains should be taken into account as a major criterion to evaluate the performance of the schedules.

In the literature, the majority of existing studies mainly focus on the optimization for either the number of successful connections of the involved trains (e.g., Kang et al. (2015a)) or the volume of successful transfer passengers at the designated transfer stations (e.g., Yang et al. (2017)), in which no passenger route choice behaviors are taken into consideration. That is, if the connection of a pair of trains fails, then it indicates that all the transfer passengers fail to arrive at their destinations, leading to inaccessibility of some demands. However, we note that this is not the case in reality, since passengers might be able to choose alternative routes to reach their destinations if there exist other accessible transfer strategies in the urban rail transit network. In this sense, the passenger-oriented micro route choice should be considered to measure the accessibility of the network-based last-train timetable. Two advantages exist with this method. That is, (1) the transportation accessibility can be evaluated over the entire urban rail transit network instead of just focusing on individual transfer stations; (2) an accessible transfer plan can be generated for each passenger if there exist feasible routes, which corresponds to the best passenger-oriented transfer plan with the least travel time. The following discussion will address this issue in more detail.

1.1 Literature review

As we aim to investigate the accessibility based timetabling problem of the last trains from the perspective of network design, we organize this part from three aspects: last train timetabling optimization, public transit accessibility, and discrete network design.

(1) Last-train timetabling optimization. The last-train timetable is the significant guidance for night operations of urban rail transit, and it has received much attention from a variety of researchers in recent years. Up to now, several studies have provided excellent surveys associated with the collaborative optimization of the last-train timetables. In the earlier research, Xu et al. (2008) proposed and studied this problem to determine the departure time domains of the last trains. According to the locations of the lines or stations and the amount of demand, they first determined different importance levels for all metro lines and transfer stations, and then designed an optimization policy and algorithm for multi-layer coordination. For maximizing passenger transfer connection headway that reflects last-train connections and transfer waiting time, Kang et al. (2015a) formulated a last-train network transfer model and developed a genetic algorithm to solve their proposed model. Subsequently, when considering train delays caused by train operations, Kang et al. (2015b) proposed a rescheduling model for last trains in order to minimize the running and dwelling time, and meanwhile maximizing the average transfer redundant time and network transfer accessibility. Through minimizing the standard deviation of transfer redundant times and balancing the last train transfers, Kang and Zhu (2017) dealt with the last train operating problem under practical significance. To optimize the transfer connection and times, Kang and Meng (2017) formulated a linear mixed-integer programming model and developed a two-phase decomposition method to solve large-scale instances in practice. Under the transfer demand uncertainty, Yang et al. (2017) reformulated a mean-variance model for the last-train timetabling problem, which explicitly considers the amount of successful transfer passengers and running time of the last trains. Zhou et al. (2019) investigated the destination-reachability based last train timetabling problem in the urban rail transit network, and CPLEX solver is used to solve the proposed model. In addition, with the given last train schedules, Dou et al. (2015) studied a feeder bus schedule coordination problem to reduce transfer failures in an intermodal bus-and-train transport network.

(2) Public transit with accessibility. The accessibility of public transit modes is becoming increasingly important due to high levels of car travel and equity issues of society (Mavoa et al., 2012). The measurement of public transit accessibility (PTA) has been receiving a lot of attention in the management and planning process of public transport agencies. Gleason (1975) used the location model to explore the layout of public transport stations. Due to the diversity of understanding to PTA and the different purpose of use, subsequent researchers mainly focused on the measure of PTA. For instance, according to the definition of accessibility given by Handy and Niemeier (1997), PTA can be roughly described as the ease with which a given destination can be reached from an origin location using public transit systems. Murray et al. (1998) defined PTA as the suitability of the public transport network to get individuals from an entry station to an exit station in a reasonable amount of time. The literature distinguishes between the accessibility to transit and the accessibility by transit (Murray et al., 1998; Moniruzzaman and Paez, 2012; Li et al., 2017). Note that the PTA in what follows mainly refers to the accessibility by transit.

The measure of PTA can be classified into two levels: the point level and the route/network level. The former mainly refers to how many public transit services can go across a zone or station, while the latter concerns how easy it is to get between any two zones or stations using transit. At the point level, Polzin et al. (2002) proposed a measure of transit accessibility across a zone as the daily trips per capita in this zone exposed to transit service. The index of PTA level developed by Kerrigan and Bull (1992) and Hillman and Pool (1997) essentially indicated a density measure of the transit service via a station. On the route or network level, Schoon et al. (1999) provided an accessibility index between an origin-destination (OD) pair through calculating the proportion of travel time by transit and average travel time across all modes. For evaluating transit quality of service, Fu and Xin (2007) proposed to use the proportion of the door-to-door travel time by auto and by transit as a performance indicator. Mavoa et al. (2012) presented an accessibility index to measure potential access to destinations via public transit and walking modes. Based on full travel chains provided by Internet mapping open platform service, Chen et al. (2017) developed a multimodal door-to-door approach with temporally sensitive accessibility measures. Considering accessibility as outputs of the multi-modal transport model, Geurs et al. (2016) examined the impacts of bicycle-train integration policies on train ridership and job accessibility for public transport users. Lee and Miller (2018) used a high-resolution space-time accessibility measure to assess the influence of new public transit services on the number of jobs and healthcare facilities that residents in an underserved neighborhood can reach within given travel time budgets. Bhat et al. (2006) presented an utility-based accessibility measure in their research report and developed an application software, where its computation needs multidimensional data, involving user-specified sets of origins and destinations, trip purpose and demographic groups.

(3) Discrete network design. The discrete network design problem (DNDP) is a long-term and widely concerned issue in the field of transportation planning and management. DNDPs can be classified into the road network design problem (Gao et al., 2005; Wang et al., 2013; Tong et al., 2015; Di et al., 2018) and transit network design problem (Cancela et al., 2015; An and Lo, 2016; Liu and Zhou, 2016; Fan et al., 2018). The former intends to find the optimal combination scheme from a candidate set of roads so that a certain index of the entire system performs well, while the later aims to determine the station locations, route alignments, frequencies and ticket prices of public transit lines to serve the passengers. Interested readers can refer to the comprehensive surveys on the modeling and algorithmic development of network design problems, for instance, Friesz (1985), Yang and Bell (1998), Farahani et al. (2013), etc. Here, we aim to introduce some closely related literature with respect to network accessibility. As one of the first papers that embedded the accessibility index into the network design processes, Antunes et al. (2002) formulated a network design model to maximize the accessibility of the inter-urban road network planning. Subsequently, Santos et al. (2008) and Murawski and Church (2009) also studied road network design problems associated with accessibility. To maximize the space-time accessibility within travel time budgets, Tong et al. (2015) formulated the transportation network design problem as a linear 0-1 programming model through introducing the space-time network. Di et al. (2018) focused on maximizing the flow-based accessibility through considering the trade-off between individual travel time budgets and actual congestion levels.

We can deduce the following characteristics from the literature review mentioned above. (1) For the last-train timetabling problem, most studies focus on the optimization of the number of successful connections of trains at different transfer stations. From the perspective of the entire network and passenger route choice behaviors, these evaluations are typically insufficient to assess the accessibility of the metro system. The main difficulty of characterizing the route choice behaviors is how to transform the physical network and travel activities together into a well-structured space-time network, which can reasonably capture the route choice behaviors in a metro network once the last-train timetable is determined. With this concern, this paper investigates this problem by proposing an integrated space-time network representation for the train and passenger activities. (2) The accessibility measure is a significant index in the transportation field. This study not only considers the space-time accessibility, but also further considers the flow-based accessibility. (3) For the network design problem, the majority of existing studies focus on the network design in the sense of traditional geographic dimensionality. This study aims to solve the timetable collaboration problem of the last trains in a metro network. Since the physical network is predetermined, the planners need to focus on the trajectory design for all the last trains in a space-time network. Accordingly, we call this process as the network design in the sense of space-time dimensionality, i.e. space-time DNDP.

1.2 Focus of this study

Note that, since the current studies cannot effectively characterize the accessibility of the entire network from the perspective of time-dependent and OD-based passenger demands, this paper will consider a last-train timetabling problem from the perspective of network design in a space-time network (i.e., space-time DNDP), in which the route choice behavior of passengers will be considered to measure the accessibility of the operation strategies. In detail, the specific contributions of this study can be summarized as follows.

(1) A space-time network framework is proposed to characterize the movements of trains and passengers, which consists of space-time running arcs, space-time dwelling arcs, space-time transfer arcs and space-time waiting arcs with respect to trains and passengers. With this representation, the problem of interest can be easily transformed into a network design problem, where the space-time trajectories of non-last trains constitute the existing space-time network and the potential space-time trajectories of last trains

constitute the candidate set of the network design problem.

(2) With the network design methods, two 0-1 linear integer programming models are proposed to measure the accessibility of the overall network, where the route choice behavior for demands is considered. In the proposed models, the decision variables are associated with the space-time network design variables and passenger route choice variables. Also, two types of constraints are formulated, including train traversing trajectory constraints and passenger route choice constraints. In addition, the trajectory overlapping constraints are also developed to characterize the trip relationship between trains and passengers.

(3) To solve the proposed models, a dual decomposition approach is proposed based on a Lagrangain relaxation framework. The hard constraint is first dualized through introducing a set of Lagrangian multipliers. Then, the relaxed models are decomposed into a set of shortest path problems with multiplier-related weights. A sub-gradient algorithm is designed to update the lower and upper bound iteratively. Finally, two sets of numerical examples in a small network and a large-scale Beijing metro network, respectively, are performed to test the effectiveness of the proposed approaches.

To better highlight the characteristics of our proposed model and algorithm, a systematic comparison among some closely related studies on this topic is provided in Table 1.

Table 1: Comparison between this paper and the existing literature

Publication	Objective function	Model	Algorithm
Xu et al. (2008)	Waiting time of all passengers	Linear	Heuristic algorithm
Kang et al. (2015a)	Transfer connection headways	Nonlinear	Genetic algorithm
Kang et al. (2015b)	Running and dwelling time	Nonlinear mixed-integer	Genetic algorithm
Kang and Zhu (2017)	Standard deviation of transfer redundant times	Nonlinear mixed-integer	Heuristic algorithm
Kang and Meng (2017)	Transfer connections Transfer connection times	Linear mixed-integer	Two-phase decomposition
Yang et al. (2017)	Transfer passengers and running time	Nonlinear mixed-integer	Tabu search CPLEX
This paper	Space-time accessibility Flow-based accessibility	Linear 0-1	Lagrangian relaxion

From the literature review and Table 1, our proposed method is essentially different from the current existing approaches from several aspects. (1) Optimization Goal. The aim of our models is to improve the accessibility of the metro system in the night operations, in which a time-dependent OD pair is accessible if its origin and destination are connected in the generated space-time network of last trains. Thus, for each time-dependent OD pair, it is needed to find a connected path in the network so as to identify its connectivity. In comparison, the other existing works (e.g., Kang and Meng (2017)) mainly focus the connectivity between trains in the transfer stations (i.e., the focus is mainly on the local optimization in comparison to our approach). For instance, at a transfer station connecting two metro lines, only one direction of two last trains can be transferred successfully, and the other direction must be failed, where the purpose is either to improve the number of transfer passengers or minimize the transfer waiting time. However, note that optimizing the transfer activity at the transfer station is not equivalent to the optimization of the accessibility over the network, since even if the transfer activity is guaranteed at some local stations, the involved passengers might still have no paths to reach their destinations. In this sense, our work has its superiority to measure the accessibility characteristics. (2) Demand Data Structure. In our models, the input data are the time-dependent or dynamic passenger demands at each origin station, where the demand OD information and the favorite departure time are pre-known. However, in the literature (e.g., Kang and Meng (2017)), the demands are assumed to be static and pre-given at each transfer station. That is, the demand data are the numbers of passengers who want to transfer to another metro line at each transfer station. In essence, the different demand data lead to the significant difference of our formulated model and other models. (3) Model Formulation. In our study, we use a space-time network to capture the dynamics of passengers and trains. Then, the problem of interest can be simplified into 0-1 linear integer programming models, which can handle the case with dynamic passenger demands. In comparison, the existing works usually handle the problem with static demands. This is another key difference between our models and other works.

The remainder of this paper is organized as follows. Section 2 provides a detailed statement of the problem. In Section 3, two 0-1 linear integer programming models are first proposed. Then, in Section 4, a model decomposition scheme based on an efficient Lagrangian relaxation algorithm is discussed. Section 5 conducts two sets of numerical examples, and the last section concludes this study and provides further research directions.

2 Problem statement

Consider a directed metro network, which consists of a set of independent metro lines, denoted by $L := \{l|l = 1, 2, \cdots, l_{max}\}$. The set of stations along the lth line is represented as N_l, and the set of all involved stations in the entire network is denoted by N_L. These stations are successively indexed by a sequence of natural numbers, i.e., $N_L := \bigcup_{l\in L} N_l = \{i|i = 1, 2, \cdots, i_{max}\}$. In general, there are many transfer stations in the network, where passengers can transfer in between different lines through the connected transfer channels. In this case, there possibly exists a nonempty intersection between two station sets N_l and $N_{l'}$ $(l, l' \in L, l \neq l')$ in the network. To avoid confusion in the modelling process, we set $N_l \bigcap N_{l'} = \emptyset$ for all $l, l' \in L$ and $l \neq l'$ in the input data preparation stage, through giving different indexes to a transfer station crossing both metro lines l and l'. For instance, Figure 1(a) illustrates a simple metro network consisting of two lines and six stations. Stations 2 and 5 are the same station physically (i.e., transfer station for these two lines). We use two indexes to represent this transfer station in the network. With this treatment, passengers are allowed to transfer from line 1 to line 2 through transfer channel (2, 5), and from line 2 to line 1 through transfer channel (5, 2). In addition, if station $j \in N_l$ is the next station of station $i \in N_l$ according to the running direction, then directed link (i, j) is an operational link over the lth line.

This study aims to investigate a timetabling problem of the last trains with the consideration of the OD-based accessibility or flow-based accessibility. In practice, once a network-correlated last train timetable is generated, we need to identify the accessibility performance of the demands through producing the least time paths. For formulating convenience, we regenerate the generalized physical network to clearly demonstrate the passenger transfer process. For this purpose, at each transfer station, we use different indexes to differentiate the transfer arc from the physical link inside the transfer station. For instance, suppose that metro line l crosses with metro line l' at transfer station i. Then, we use two distinctive nodes to represent different functions. Here, without the loss of generality, we use indexes i and i' to denote the same platform, in which node i represents the transfer origin node on line l, and node i'

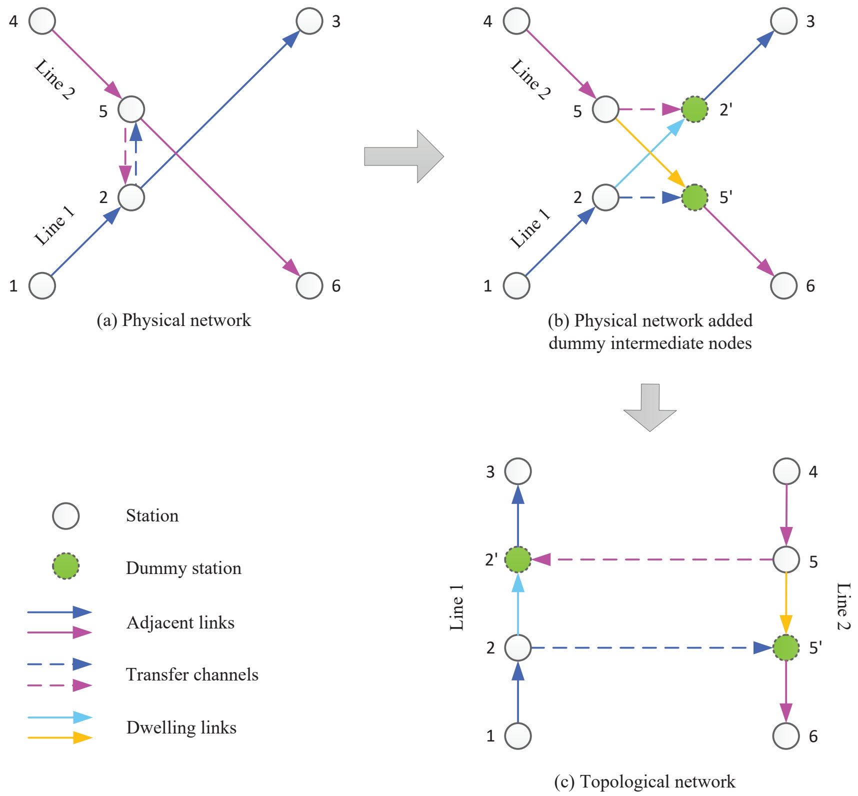

Figure 1: Illustration of a simple network

represents the transfer destination node from another line. As shown in Figure 1(b), although node 2 and node $2'$ represent the same station over line 1, the former represents the transfer departure node from line 1, while the latter is the transfer arrival node from line 2. We denote the set grouping all the transfer destination nodes on line l by N_l', and let $N_L' = \bigcup_{l \in L} N_l'$. In addition, the transfer activity between two metro lines corresponds to some transfer time since two lines might have individual platforms. Thus, if there exists a potential transfer activity at transfer stations from line l to line l', a successful transfer refers to that passengers can reach transfer arrival node of line l' from departure node of line l. By this representation, there are two potential transfer links in Figure 1(b), i.e., (2, $5'$) and (5, $2'$). For the convenience of description, we reconstruct Figure 1(b) into the form in Figure 1(c), which does not change the topological structure of a metro network. Taking Figure 1(c) as an example, in a physical metro network all operational links along the lth line are grouped in set A_l, all transfer links are grouped in set A_{tran}, and we let $A = \bigcup_{l \in L} A_l \bigcup A_{tran}, N = N_L \bigcup N_L'$. Then, a physical metro network can be represented by (N, A).

In this problem, we need to specify the timetable of each last train and passenger service choices in the metro network. Since both of these two processes need to take the time dimension into account, the following discussion intends to embed operations of trains and passengers into a space-time network, which can be used to characterize the travel of trains and passengers.

2.1 Construction of space-time network

The space-time network is the generalization of physical network when we embed the time dimension, which is used to characterize train and passenger time-dependent movements or trajectories in this study. In the literature, the concept of a space-time network is widely used in both traffic geography and traffic network modeling (Yang and Zhou, 2014; Tong et al., 2015; Yang et al., 2016; Tong et al., 2019). Theoretically, a physical network on a plane can be transformed into a space-time network represented by a three-dimensional coordinate system, where one coordinate denotes the time horizon and the other two coordinates denote the space position. Thus, each point in this space-time coordinate system can be used to represent the specific position of a train or traveler at the corresponding time. Generally, in the process of constructing a space-time network, we need to discretize the time horizon into a number of small time slots, during which no perceptible change of network status is assumed to occur. Without a loss of generality, we use a time interval with length δ to discretize the time axis into different timestamps. Then, the time horizon can be discretized into a set of timestamps, denoted by $T = \{u|u = t_0, t_0 + \delta, t_0 + 2\delta, \cdots, t_0 + M\delta\}$, in which t_0 and $t_0 + M\delta$ are the left and right end points of the considered time horizon.

By this method, a space-time vertex is defined as the status of a physical node at a specific timestamp, denoted by a two-element array (i, u). Here, we introduce and interpret space-time arcs, which need two criteria. That is, (1) there exists a link (i, j) in the physical network (N, A); (2) if a passenger or train enters link (i, j) at timestamp u, then the departure time from this link should be just over a timestamp v. In this case, these two space-time vertices (i, u), (j, v) can only be connected by a space-time arc, denoted by a four-element array (i, j, u, v), whose cost c_{ijuv} is equal to the travel time $v - u$. Typically, space-time arc (i, j, u, v) aggregates two features, i.e., connection of physical nodes, and entering and departure times over this link.

The space-time network has become a powerful tool for characterizing the operation diagram of metro and rail networks (Yang et al., 2014; Yin et al., 2017; Shang et al., 2018; Zhou et al., 2017). To demonstrate how to use a space-time network to describe travel characteristics, Figure 2 is given to show the transformation of a physical network to a space-time network. In this figure, we still use the topological network in Figure 1(c) as an illustration (i.e., left two-dimension space). To generate a space-time network, we introduce four types of arcs for the involved trains and passengers, including the space-time running arc, space-time dwelling arc, space-time transfer arc and space-time waiting arc, and they are uniformly expressed as four-element arrays. As shown in the right of Figure 2, the black and red solid lines are space-time running arcs of trains, e.g., arc $(1, 2, 1, 7)$. The blue and orange solid lines are space-time dwelling arcs of trains, e.g., arc $(2, 2', 7, 10)$. The trajectory of a train is made up of a sequence of space-time running arcs and space-time dwelling arcs. For the non-last trains, their space-time trajectories constitute the existing space-time network, i.e., their space-time running arcs and space-time dwelling arcs are deterministic and pre-given. The last trains are required to choose their individual space-time running trajectories from all feasible arcs on their metro lines.

For clarity, we illustrate three space-time running trajectories on the space-time plane 1 to represent the potential options of the last train on metro line 1, while the three space-time trajectories on space-time plane 2 are potential choices for the last train on metro line 2. Practically, some passengers usually need to transfer from one line to another during their trips. Those black and red dashed lines in between the two space-time planes indicate space-time transfer arcs with the corresponding transfer time (here, we set $\delta = 1$ minute in this illustration), e.g., arc $(2, 5', 7, 8)$. Additionally, some passengers who arrive at departure platform should wait for the approaching train, and the green solid lines represent space-time waiting arcs for passengers, e.g., arc $(5', 5', 9, 10)$. In the problem of interest, once the last-train trajectories are determined (i.e., the space-time corridors are given in the space-time network), the OD-based passenger transfer strategies are taken into consideration for evaluating the accessibility of the last-train operation plans.

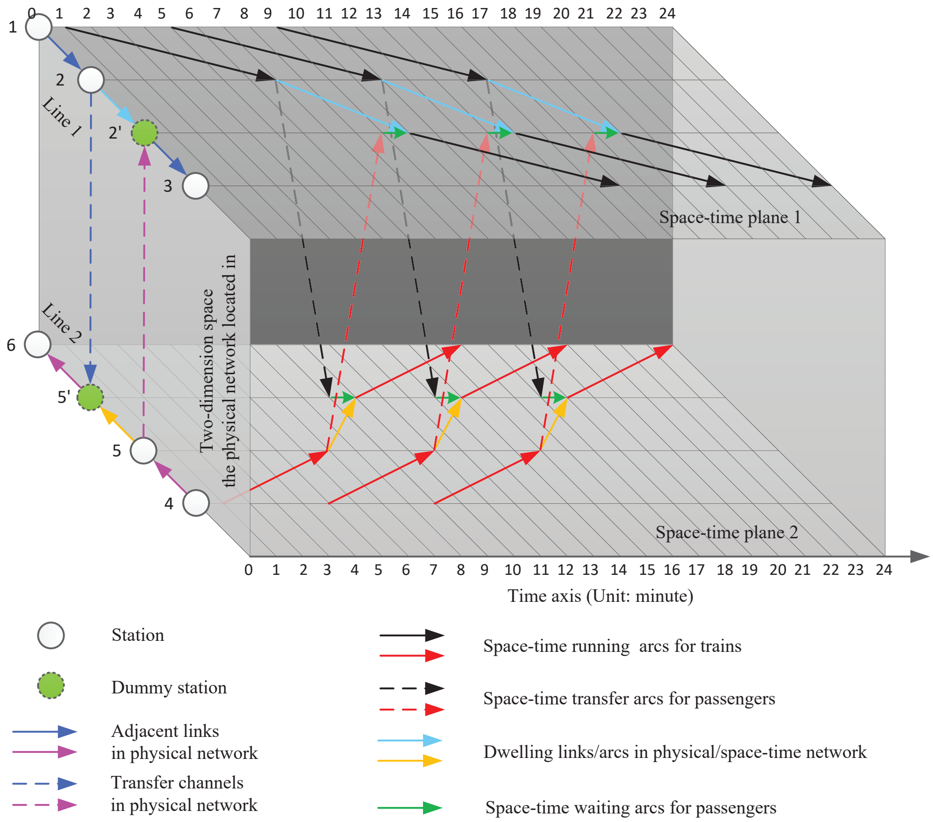

Figure 2: Illustration of a space-time network

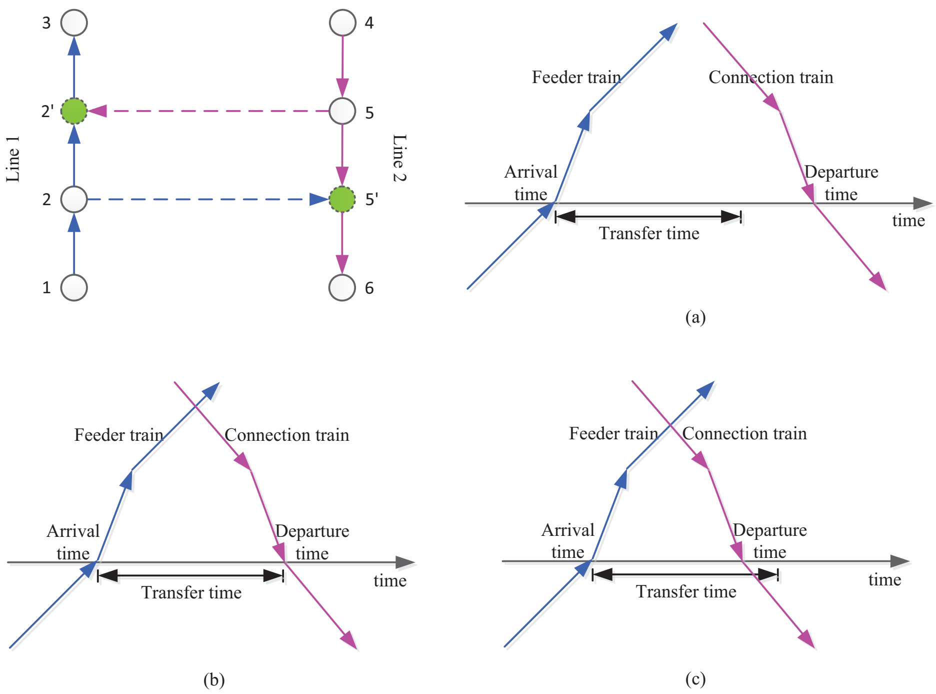

Figure 3: Graphical description of transfer success and transfer failure

2.2 Timetable collaboration of last trains

The collaboratively optimized timetable of the last-trains in the metro network aims to either make more passengers transfer successfully, or make more trains connect successfully (i.e., producing more accessible space-time corridors in the metro network). Although these two objectives are slightly different, a successful transfer or connection at a transfer station always requires that the time difference between the departure of the connection train and arrival of the feeder train is not less than the passenger transfer time, as shown in Figure 3(a) and (b). Otherwise, passengers will miss the connection train due to insufficient transfer time window, leading to the failure of the transfer activity, as shown in Figure 3(c).

In fact, the timetable collaboration of the last trains involves two aspects. On one hand, it might refer to the connection of the last trains over all the metro lines. That is, the involved trains taken by passengers in their entire journeys are all last trains over their corresponding lines. For example, Figure 4 shows timetables of the last trains over Line 1 and Line 2, in which the black and red space-time trajectories correspond to the last trains on metro Line 1 and Line 2, respectively. If some passengers from station 1 to station 6 reach station 1 at timestamp 7 and intend to transfer to Line 2 at transfer station 2, then they should take the last train of Line 1 first, and then transfer to the last train of Line 2. In this process, their space-time trajectories can be characterized as follows: waiting arc $(1, 1, 7, 9)$→ running arc $(1, 2, 9, 15)$→ transferring arc $(2, 5', 15, 17)$→ waiting arc $(5', 5', 17, 18)$→ running arc $(5', 6, 18, 24)$, shown as bold lines in Figure 4. On the other hand, the timetable collaboration of the last trains implies that only part of trip involves last trains of some metro lines, while the other parts will be on non-last trains. For example, Figure 5 shows the running trajectories of the last two trains in accordance to the different timetables of Line 1 and Line 2. If some passengers from station 1 to station 6 reach station 1 at timestamp 4, then they can take the penultimate train of Line 1, and transfer to the last train of Line 2. Their space-time trajectories can be characterized as: waiting arc $(1, 1, 4, 5)$→ running arc $(1, 2, 5, 11)$→ transfer arc $(2, 5', 11, 13)$→ waiting arc $(5', 5', 13, 15)$→ running arc $(5', 6, 15, 21)$, shown as bold lines in Figure 5.

To describe train operations, we need to consider two sets of arcs in the space-time space. One set corresponds to space-time arcs for non-last trains over the time horizon. Additionally, in the timetable of each line, trajectories of these non-last trains have been predetermined (i.e., the space-time corridor already exists in the space-time network). For convenience, we use set N_l^+ to collect all space-time vertexes past by trains on line l, use A_l^+ to collect all space-time running arcs of non-last trains on line l, and let $N_L^+ = \bigcup_{l \in L} N_l^+$, $A_L^+ = \bigcup_{l \in L} A_l^+$, while all space-time transfer arcs and waiting arcs are collected by sets A_{tran}^+ and A_{wait}^+, respectively.

This study is devoted to collaboratively designing timetables for all the last trains. In the framework of a space-time network, we might consider all the potential trajectories for these last trains, as shown in Figure 6. Typically, this approach can simultaneously optimize the speeds and dwelling time of last trains. In this figure, the black dashed lines are the potential running arcs corresponding to different speeds of the train, and the blue dashed lines are the potential dwelling arcs associated with different dwelling times. When the trajectory of the last train is determined, we can easily deduce its detailed speed on each segment and dwelling time at each station. In addition, for modeling convenience, we introduce a pair of dummy space-time origin (O_l', t_l^O) and dummy space-time destination (D_l', t_l^D) (e.g., corresponding to nodes $1'$ and $3'$ in Figure 6) for the last train on line l to connect all potential space-time trajectories of this last train, shown as orange dashed lines in Figure 6. The corresponding arcs are respectively called space-time origin arcs and space-time destination arcs, which only indicate the adjacent relationship between the real stations to dummy nodes, and no costs are associated with these arcs. We use set B_l^+ to collect all potential space-time arcs (involving space-time origin arcs and space-time destination arcs)

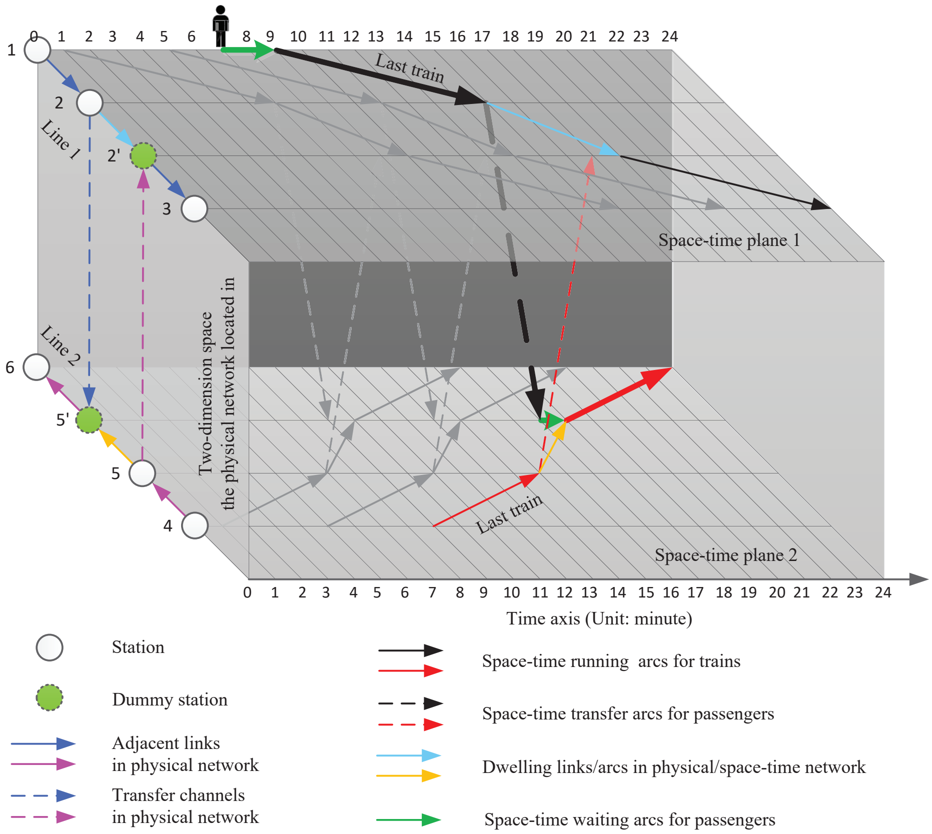

Figure 4: The collaboration between two last trains

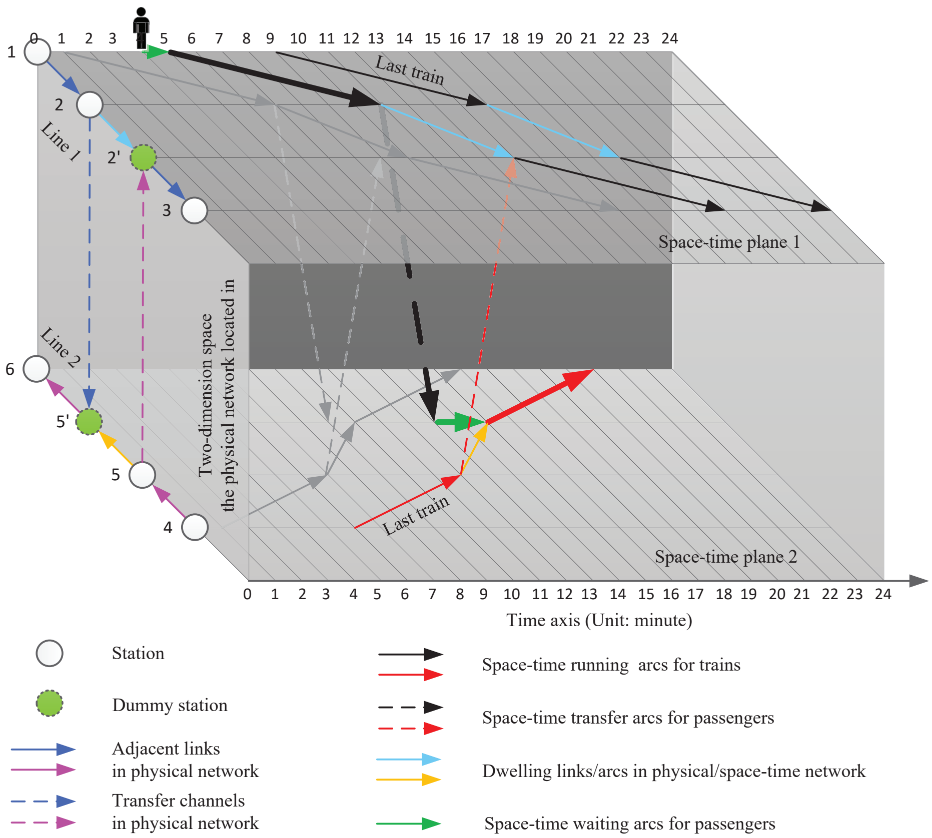

Figure 5: The collaboration between a non-last train and a last train

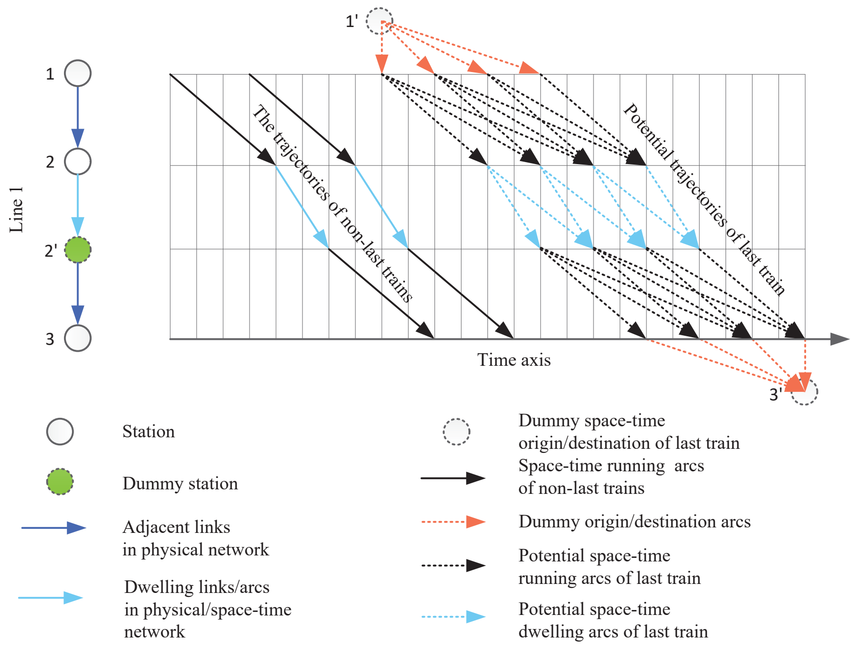

Figure 6: All potential trajectories of the last train

of the last train on line l, and let $B_L^+ = \bigcup_{l \in L} B_l^+$.

2.3 Metro network accessibility

Next, we aim to analyze the route choice behavior of the passengers in the metro network once the last-train timetable for all the lines is determined, and accordingly measure the accessibility of the metro network from two aspects. Here, we consider a metro network consisting of three lines, i.e.,

Line 1: $1 \to 2 \to 2' \to 3 \to 3' \to 4$;

Line 2: $5 \to 6 \to 6' \to 7 \to 7' \to 8$;

Line 3: $9 \to 10 \to 10' \to 11 \to 11' \to 12$.

The physical transfer arcs between different lines are shown as follows.

Line 1 $\to$ Line 2: $2 \to 6'$;

Line 1 $\to$ Line 3: $3 \to 10'$;

Line 2 $\to$ Line 3: $7 \to 11'$.

In this simple network, we first give space-time trajectories of the last trains on three space-time planes, as shown in Figure 7. Let us consider a demand pair between stations 1 and 12. Note that these two stations are located on lines 1 and 3, respectively. According to the transfer strategy with the least number of transfer activities (see Huang et al. (2017)), the preferred routing plan turns out to be "Line 1 $\to$ Line 3" since only one transfer activity is needed. In this strategy, passengers should first take the last train on Line 1 from station 1 to station 3, and then transfer to Line 3 at station $10'$ to arrive at station 12. However, with the current timetable of the last trains, they cannot transfer to Line 3 at station $10'$ since the last train on this line leaves station $10'$ before their arrival. In this sense, this transfer strategy is infeasible.

Nevertheless, we note that there exits a feasible transfer strategy in this space-time network, namely "Line 1 $\to$ Line 2 $\to$ Line 3", which needs two transfer activities from the origin to destination. As shown by the thick lines in Figure 7, passengers can first take Line 1 from station 1 to station 2, and transfer to Line 2 at station $6'$; at station 7, they can transfer to station $11'$ of Line 3, and then arrive at their destination station. In this strategy, the physical path turns out to be $1 \to 2 \to 6' \to 7 \to 11' \to 12$. Although this transfer strategy needs two transfer activities, it provides a feasible transfer plan in the network for this demand.

In a space-time network, we now define a time-dependent demand pair (TDDP), which refers to an origin-destination (OD) pair with a departure time. For example, OD pair $rs (\in W)$ with departure time $t (\in T')$ is a TDDP, and it is denoted by $(rs, t) (\in W \times T')$, where W groups all OD pairs and T' groups all departure times. If there is at least one feasible route between OD pair rs with departure time t, then this TDDP is called accessible; otherwise, it is called inaccessible. The number of all accessible TDDPs over this network is called OD-based accessibility or space-time accessibility. The number of passengers of all accessible TDDPs is called flow-based accessibility. Here, we note that these two evaluation indices of accessibility may conflict with each other because of the difference of passenger distributions over different TDDPs. This method corresponds to two advantages. That is, (1) we can use a path generation method to identify the accessibility of each given TDDP; (2) it can provide a feasible transfer strategy for passengers with the given timetable, which is possibly different from the preferred strategy with the least number of transfers.

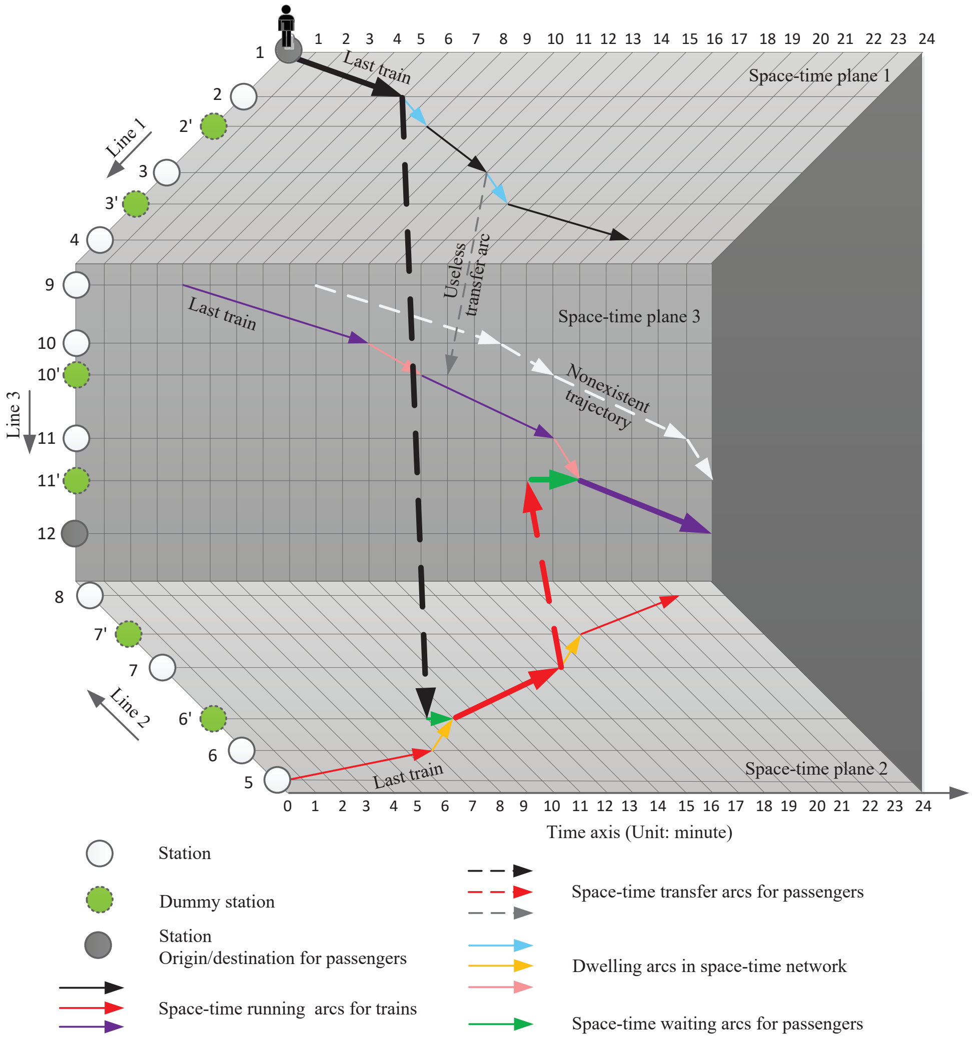

Figure 7: Accessibility in a space-time network of the last trains

In addition, we mention that some TDDPs might be inaccessible in the space-time network. For instance, in Figure 7, if the trajectory of Line 2 moves three minutes backwards along the time axis, then no feasible route exists from station 1 to station 12 in this space-time network. In this case, such TDDPs and the corresponding passengers will become inaccessible. To capture such TDDPs and passengers who cannot reach their destinations by metro transfers, we need to add a super space-time arc (as shown in Figure 8) for passengers of TDDP (rs, t). The form of super space-time arcs is also a four-element array (r, s, t, v'), where (s, v') is the dummy space-time destination for passengers whose destination is at station s. If passengers cannot take trains to reach their destinations in the network, then they will go through super space-time arcs, by which we can formulate the accessibility evaluation as a path finding process. For modeling convenience, the space-time vertices that trains can possibly stop at station s (denoted by (s, v_l^s)) should be connected to space-time vertex (s, v'), and the generated arcs are called dummy arrival arcs (as shown in Figure 8).

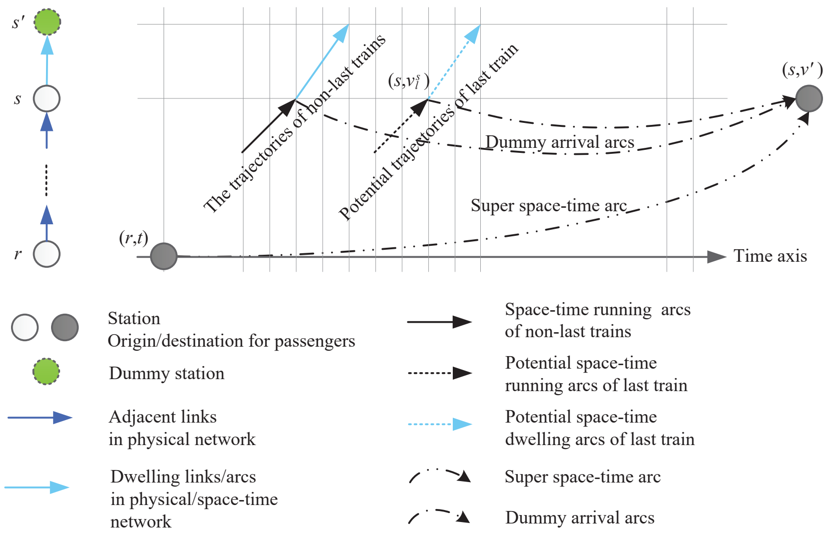

Figure 8: An illustration of super space-time arcs and dummy arrival arcs

By this representation method, each TDDP is connected in the space-time network. That is, if a feasible transfer strategy can be found by the route generation algorithm, it corresponds to a space-time route; if a TDDP is inaccessible, the corresponding super space-time arc will be selected. In the network design process, we need to find the best last-train timetables, in which the number of inaccessible TDDPs can be minimized. For clarity, we use sets A^+_{sup} and A^+_{dum} to collect all super space-time arcs and all dummy arrival arcs, respectively.

Now, we have constructed a space-time network $(\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the union set of all types of space-time vertex sets, i.e., $\mathcal{N} = N_L^+ \bigcup\{(O_l', t_l^O), (D_l', t_l^D)|l \in L\} \bigcup\{(s, v')|rs \in W\}$, $\mathcal{A}$ is the union set of all types of space-time arc sets, i.e., $\mathcal{A} = A_L^+ \bigcup B_L^+ \bigcup A^+_{tran} \bigcup A^+_{wait} \bigcup A^+_{sup} \bigcup A^+_{dum}$. Typically, space-time network $(\mathcal{N}, \mathcal{A})$ involves the information on timetables, last trains and passenger TDDPs.

3 Mathematical formulation

In the following discussion, we formulate the problem of interest as integer linear programming models. For the convenience of modelling, some necessary assumptions are first given as below.

Assumption 1: The urban rail transit network consists of a series of independent metro lines. That is, all the involved metro lines are separated from each other physically, and trains on one metro line are prohibited to travel on other lines (this is the case for urban rail transit systems in China).

Assumption 2: The loading capacity of trains will not be taken into consideration in this study since the night operations are during off-peak hours and the transportation capacity is sufficient to transport all passengers in the metro system.

Assumption 3: All the transfer passengers are required to walk from the feeder-line platform to the connection-line platform (i.e., transfer activities are conducted between different platforms). For simplicity, the transfer time on each transfer arc is assumed to be a constant since there is no congestion when passengers cross transfer channels during the night operation period.

Assumption 4: On each metro line, the train operations are usually bidirectional. If we have two last trains for different directions, the two directions can be regarded as independent lines, and thus the last trains are distinct in these two directions.

3.1 Notation and parameters

Next, for modelling convenience, we first give all the related subscripts and parameters used in the formulation of our problem, listed in Table 2. In addition, two types of decision variables are defined for this problem. One is associated with the space-time arc selection in the corresponding space-time network, which will constitute the timetables of all the last trains. The other corresponds to the route choice process in the space-time network for each considered TDDP. All the involved decision variables are listed in Table 3.

3.2 Optimization model

In the following, we give a formal formulation of the problem of interest in the aforementioned space-time network $(\mathcal{N}, \mathcal{A})$. The system constraints are first formulated according to the characteristics of the considered problem. In this study, the purpose is to choose the space-time trajectory of each last train on its metro line. In the space-time network, the trajectory of each train consists of a series of space-time arcs. Thus, for any metro line $l \in L$, the following flow balance constraint must hold for the last train on it.

$$\sum_{(i,j,u,v)\in B_l^+} y_{ijuv} - \sum_{(j,i,v,u)\in B_l^+} y_{jivu} = \begin{cases} 1, & \text{if } i = O_l', u = t_l^O \\ -1, & \text{if } i = D_l', u = t_l^D \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

For brevity, (O_l', t_l^O) and (D_l', t_l^D) in constraint (1) are dummy space-time origin and destination nodes respectively. Note that in the process of constructing the space-time network, we can only define the space-time arcs in one direction according to the traversing direction. Then, on each metro line, no cycle exists in the space-time network. Correspondingly, if a set of space-time arcs satisfying the aforementioned constraints are generated, they constitute the space-time trajectories for all the last trains.

As stated above, once the above-mentioned constraints generate a network-based timetable for all the last trains, its accessibility performance can be evaluated over the entire transportation network. To this end, the following methods are used to identify the accessibility of each involved TDDP. That is, we first find the least time path in the generated space-time network for each TDDP. If the generated path overlaps with the super link, it shows that the current TDDP is not accessible. Otherwise, it is an accessible TDDP (i.e., we can find a space-time transfer path in the constructed network). The route choice process of passengers of TDDP (rs, t) can be formulated as the following flow balance constraint.

$$\sum_{(i,j,u,v)\in \mathcal{A}} x_{ijuv}^{rs,t} - \sum_{(j,i,v,u)\in \mathcal{A}} x_{jivu}^{rs,t} = \begin{cases} 1, & \text{if } i = r, u = t \\ -1, & \text{if } i = s, u = v', \quad (rs,t) \in W \times T'. \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Table 2: Subscripts and parameters involved in the formulations

Notations	Definition
L	Set of lines of a metro system
l	Index of a line, $l \in L$
N_l	Set of stations on line l
N'_l	Set of transfer destination nodes on line l
N	Set of all stations in the metro system, $N = \bigcup_{l\in L} N_l \bigcup_{l\in L} N'_l$
A_l	Set of links on line l
A	Set of all arcs in the metro system, $A = \bigcup_{l\in L} A_l \bigcup A_{tran}$
(N, A)	Topological network of the metro system
N_l^+	Set of space-time vertexes past by trains on line l
N_L^+	Union set of all N_l^+
A_l^+	Set of running arcs of non-last trains on line l
B_l^+	Set of potential running arcs of the last train on line l
A_L^+	Union set of all A_l^+
B_L^+	Union set of all B_l^+
A_{tran}^+	Set of space-time transferring arcs
A_{wait}^+	Set of space-time waiting arcs
A_{sup}^+	Set of super space-time arcs
A_{dum}^+	Set of dummy arrival arcs
δ	Length of time interval in the process of discretizing time horizon
$[t_0, t_0 + M\delta]$	Considered time horizon in this problem
T	Set of discretized timestamps, i.e., $T = \{u \| u = t_0, t_0 + \delta, \cdots, t_0 + M \cdot \delta\}$
T'	Set of all the departure times, $T' \subset T$
i, j	Indices of nodes, $i, j \in N$
(i, j)	Index of physical link between stations i and j, $(i, j) \in A$
t, u, v	Indices of timestamps, $t, u, v \in T$
M	The number of discretized time interval
$(i, u), (j, v)$	Indices of space-time vertices, $(i, u), (j, v) \in \mathcal{N}$
(i, j, u, v)	Index of space-time arcs, $(i, j, u, v) \in \mathcal{A}$
(O'_l, t_l^O)	Dummy space-time origin for the last train on line l
(D'_l, t_l^D)	Dummy space-time destination for the last train on line l
(s, v')	Dummy space-time destination for passengers whose destination is at station s
$\mathcal{N}$	Set of vertices in the space-time network
$\mathcal{A}$	Set of arcs in the space-time network
$(\mathcal{N}, \mathcal{A})$	Space-time network
W	Set of OD pairs
rs	Index of OD pairs, $rs \in W$
$W \times T'$	Set of the involved TDDPs
(rs, t)	Index of a TDDP, $(rs, t) \in W \times T'$
q_{rs}^t	The amount of passengers of TDDP (rs, t)

Table 3: Decision variables used in the formulations

Notation	Definition
$x_{ijuv}^{rs,t}$	=1, if space-time arc $(i,j,u,v) \in \mathcal{A}$ is used by passengers of TDDP (rs,t); =0, otherwise.
X	Vector grouped by all variables $x_{ijuv}^{rs,t}$.
y_{ijuv}	=1, if candidate space-time arc $(i,j,u,v) \in B_l^+$ is part of the space-time trajectory of the last train on line $l, l \in L$; =0, otherwise.
Y	Vector grouped by all variables y_{ijuv}.

In constraint (2), $\mathcal{A}$ groups the space-time arcs, $W \times T'$ groups all the involved TDDPs, and (r,t) and (s,v') are space-time origin and destination vertices on the corresponding TDDP (rs,t).

In addition, we need to ensure that, the path chosen by passengers in the space-time network should overlap with the existing space-time service arcs of some last trains if they ride these last trains. Therefore, we have the following passenger-train coupling constraints.

$$x_{ijuv}^{rs,t} \leq y_{ijuv}, \quad \forall (rs,t) \in W \times T', \quad (i,j,u,v) \in B_L^+ \tag{3}$$

In constraint (3), if a last train uses a space-time arc (i,j,u,v) on its trajectory (i.e., $y_{ijuv} = 1$), then the involved passengers are allowed to select this service arc on their routes (i.e., $x_{ijuv}^{rs,t}$ can take a value of either 1 or 0); otherwise, it is an infeasible service arc in the passenger route choice process (i.e., $x_{ijuv}^{rs,t}$ becomes 0). On the other hand, if there exist passengers using space-time arc (i,j,u,v) on their routes (i.e., $x_{ijuv}^{rs,t} = 1$), then constraint (3) guarantees that there must exist a train using the space-time arc (i,j,u,v) on its trajectory (i.e., $y_{ijuv} = 1$). Constraint (3) defines the matching relationship between the passenger demands and provided services.

Finally, the binary constraints for decision variables are imposed as below.

$$x_{ijuv}^{rs,t} \in \{0,1\}, \quad \forall (rs,t) \in W \times T', \quad (i,j,u,v) \in \mathcal{A}. \tag{4}$$

$$y_{ijuv} \in \{0,1\}, \quad \forall (i,j,u,v) \in B_L^+. \tag{5}$$

For a given network-based last train timetable, we measure it through considering the accessibility performance from the perspective of passenger demands. Here, we develop two definitions with respect to accessibility in the considered metro network, i.e., space-time accessibility and flow-based accessibility. The space-time accessibility performance focuses on maximizing the total number of accessible TDDPs or minimizing the number of inaccessible TDDPs during the operation period of the last trains. Since the later performance is easy to calculate by considering the selection of the super space-time arcs, the corresponding objective function can be written as follows.

$$\min: Z = \sum_{(i,j,u,v) \in A_{sup}^+} x_{ijuv}^{rs,t}. \tag{6}$$

In this objective function, $x_{ijuv}^{rs,t} = 1$ represents that super link over OD rs with favorite departure time t is selected, i.e., passengers of TDDP (rs,t) cannot reach their destination under the current network design scheme. Alternatively, Eq. (6) can also be written as follows.

$$\min: Z = \sum_{(rs,t) \in W \times T'} x_{rstv'}^{rs,t}. \tag{7}$$

On the other hand, the flow-based accessibility focuses on maximizing the amount of passengers with successful transfer activities, or minimizing the amount of transfer-failure passengers. Therefore, similar to Eq. (7), the objective function with flow-based accessibility can be formulated as follows.

$$\min : Z = \sum_{(rs,t)\in W\times T'} q^t_{rs} x^{rs,t}_{rstv'} \tag{8}$$

where q^t_{rs} is the amount of passengers of TDDP (rs, t).

These two evaluation measures can possibly lead to different characteristics of the optimal solutions if the passenger distribution is uneven. Space-time accessibility maximizes the number of TDDPs that can be connected by the last trains, while flow-based accessibility pays more attention to the volume of successful transfer passengers who need to ride the last trains. Thus, with these two different objective functions, we can formulate two 0-1 integer linear programming models for the problem of interest, listed below.

$$\text{Model P1:} \begin{cases} \text{Objective function (7)} \\ s.t. \ \ \text{Constraints (1)} - \text{(5)}. \end{cases} \tag{9}$$

$$\text{Model P2:} \begin{cases} \text{Objective function (8)} \\ s.t. \ \ \text{Constraints (1)} - \text{(5)}. \end{cases} \tag{10}$$

Essentially, the difference between these two models is associated with the weight of each TDDP. In Model P1, all the involved TDDPs have the same weights, while the TDDP with more passengers has larger weight in Model P2 since the corresponding weight equals to the total number of the involved passengers. In practice, if metro operators pay more attention to some TDDPs, much larger weights can be imposed in the objective function and the Model P2 can be employed.

4 Solution algorithm

With the representation of space-time network, we formulate the considered last train timetabling problem as two 0-1 integer linear programming models. With the linearity of the proposed models, a commercial optimization software (e.g., CPLEX or GRUBI) can be used to solve near-optimal solutions for small-scale problems. However, if the network scale is large and the demand structure is complex, the proposed models are actually a complex model since they have a large set of decision variables. Under this condition, a commercial optimization software is typically inefficient in handling this problem. Note that the hard constraint in the aforementioned models refers to as the coupling constraint between the train and passenger space-time path selection. Next, an effective heuristic algorithm based on Lagrangian relaxation is developed to find the approximate optimal solutions, which can reduce the computational intensity greatly.

4.1 Model decomposition

Note that, the high-dimension decision vector $X = [x^{rs,t}_{ijuv}]$ might be intractable for real-world applications in large-scale networks. With this concern, we next develop a Lagrangian relaxation approach to dualize the hard constraint, and then decompose the relaxed model into two relatively tractable and classical subproblems. Since the proposed Models P1 and P2 have the similar structure, we only discuss the decomposition of Model P1 through dualizing those coupling constraints generated by inequality (3). We introduce a series of nonnegative Lagrangian multipliers $\lambda^{rs,t}_{ijuv}(\forall (rs,t) \in W \times T', (i,j,u,v) \in B^+_L)$, which are grouped into a vector $\boldsymbol{\lambda}$ for notation convenience, and then dualize these constraints into the objective function, listed below.

$$\text{Model P1'}: \begin{cases} \min : L(X,Y,\boldsymbol{\lambda}) = \sum_{(rs,t)\in W\times T'} x_{rstv'}^{rs,t} + \sum_{(rs,t)\in W\times T'} \sum_{(i,j,u,v)\in B_L^+} [\lambda_{ijuv}^{rs,t}\cdot(x_{ijuv}^{rs,t} - y_{ijuv})] \\ s.t.\quad \text{Constraints (1), (2), (4) and (5).} \end{cases} \tag{11}$$

After regrouping all the variables in the relaxed model, we can further rewrite the objective function of Model P1' as follows.

$$L(X,Y,\boldsymbol{\lambda}) = \sum_{(rs,t)\in W\times T'} x_{rstv'}^{rs,t} + \sum_{(rs,t)\in W\times T'} \sum_{(i,j,u,v)\in B_L^+} \lambda_{ijuv}^{rs,t} x_{ijuv}^{rs,t} - \sum_{(rs,t)\in W\times T'} \sum_{(i,j,u,v)\in B_L^+} \lambda_{ijuv}^{rs,t} y_{ijuv}. \tag{12}$$

Therefore, it is easy to decompose Model P1' into the following two subproblems, in which one only involves decision vector X, and the other is with respect to decision vector Y. For convenience, these two subproblems are denoted by Model P1'-X and Model P1'-Y in the following discussion, respectively.

$$\text{Model P1'-X}: \begin{cases} \min : L_X(X,\boldsymbol{\lambda}) = \sum_{(rs,t)\in W\times T'} \left(x_{rstv'}^{rs,t} + \sum_{(i,j,u,v)\in B_L^+} \lambda_{ijuv}^{rs,t} x_{ijuv}^{rs,t} \right) \\ s.t.\quad \text{Constraints (2) and (4).} \end{cases} \tag{13}$$

$$\text{Model P1'-Y}: \begin{cases} \min : L_Y(Y,\boldsymbol{\lambda}) = -\sum_{(rs,t)\in W\times T'} \sum_{(i,j,u,v)\in B_L^+} \lambda_{ijuv}^{rs,t} y_{ijuv} \\ s.t.\quad \text{Constraints (1) and (5).} \end{cases} \tag{14}$$

Proposition 1 *For a pre-given multiplier vector $\boldsymbol{\lambda}$, Model P1'-X can be further decomposed into a total of $|W \times T'|$ shortest path problems over different TDDPs.*

Proof. Note that, in the pre-constructed space-time network $(\mathcal{N},\mathcal{A})$, the traveling behavior of passengers of TDDPs have no couplings with each other in Model P1'-X. Therefore, the minimum of objective function (13) is equal to summing the minimum objective functions of $|W \times T'|$ subproblems. Namely, for any pre-given $(rs,t) \in W \times T'$, we need to solve the following problem.

$$\text{Model P1'-X}(rs,t): \begin{cases} \min : L_X(X,\boldsymbol{\lambda},rs,t) = x_{rstv'}^{rs,t} + \sum_{(i,j,u,v)\in B_L^+} \lambda_{ijuv}^{rs,t} x_{ijuv}^{rs,t} \\ s.t.\quad \text{Constraints (2) and (4).} \end{cases} \tag{15}$$

In the objective function of Model P1'-X(rs,t), the cost of super path (r,s,t,v') is 1, the cost of space-time arc $(i,j,u,v) \in \mathcal{A} \setminus B_L^+$ is 0, and the cost of space-time arc $(i,j,u,v) \in B_L^+$ is $\lambda_{ijuv}^{rs,t}$. Constraint (2) ensures that there exists one connected path for TDDP $(rs,t) \in W \times T'$ in the space-time network $(\mathcal{N},\mathcal{A})$, and the optimal objective value of Model P1'-X(rs,t) ensures the length of this path is shortest. Therefore, Model P1'-X(rs,t) is a shortest path problem. □

For the convenience of solving Model P1'-X, we here discuss the relationship between the shortest path of TDDP (rs,t) in space-time network $(\mathcal{N},\mathcal{A} \setminus A_{sup}^+)$ and the shortest path of TDDP (rs,t) in space-time network $(\mathcal{N},\mathcal{A})$. In general, three possible cases occur for TDDP (rs,t).

Case 1: If there exists a shortest path in space-time network $(\mathcal{N},\mathcal{A} \setminus A_{sup}^+)$, and it consists of non-last service trains, then this path is the shortest path in space-time network $(\mathcal{N},\mathcal{A})$. In this case, the optimal solution of Model P1'-X(rs,t) is shown as follows.

$$x_{rstv'}^{rs,t} = 0, \quad (r,s,t,v') \in A_{sup}^+;$$
$$x_{ijuv}^{rs,t} = 0, \quad \forall (i,j,u,v) \in B_L^+.$$

Thus, the corresponding objective value is 0, which is the least objective for Model P1'-X for a given $(rs,t) \in W \times T'$.

Case 2: If there exists a shortest path in space-time network $(\mathcal{N}, \mathcal{A} \setminus A_{sup}^+)$, and it involves at least one last service trains, then whether this path is the shortest path in space-time network $(\mathcal{N}, \mathcal{A})$ depends on the comparison between 1 and the value of $\sum_{(i,j,u,v)\in B_L^+(rs,t)} \lambda_{ijuv}^{rs,t}$, where notation $B_L^+(rs,t)$ is the set of space-time arcs of the last trains used by TDDP (rs,t).

If $\sum_{(i,j,u,v)\in B_L^+(rs,t)} \lambda_{ijuv}^{rs,t} \le 1$, then the shortest path in space-time network $(\mathcal{N}, \mathcal{A} \setminus A_{sup}^+)$ is the shortest path in space-time network $(\mathcal{N}, \mathcal{A})$, and the optimal solution of Model P1'-X(rs,t) is shown as follows.

$$x_{rstv'}^{rs,t} = 0, \quad (r,s,t,v') \in A_{sup}^+;$$
$$x_{ijuv}^{rs,t} = 1, \quad \forall (i,j,u,v) \in B_L^+(rs,t).$$

Then, the corresponding objective value is $\sum_{(i,j,u,v)\in B_L^+(rs,t)} \lambda_{ijuv}^{rs,t}$, which is the least objective for Model P1'-X for a given $(rs,t) \in W \times T'$.

If $\sum_{(i,j,u,v)\in B_L^+(rs,t)} \lambda_{ijuv}^{rs,t} > 1$, then the shortest path in space-time network $(\mathcal{N}, \mathcal{A} \setminus A_{sup}^+)$ is not the shortest path in space-time network $(\mathcal{N}, \mathcal{A})$, and the shortest path in space-time network $(\mathcal{N}, \mathcal{A})$ is the super path. Therefore, the optimal solution of Model P1'-X(rs,t) is shown as follows.

$$x_{rstv'}^{rs,t} = 1, \quad (r,s,t,v') \text{ is the super path of TDDP } (rs,t);$$
$$x_{ijuv}^{rs,t} = 0, \quad \forall (i,j,u,v) \in B_L^+.$$

Thus, the corresponding objective value is 1, which is the least objective for Model P1'-X for a given $(rs,t) \in W \times T'$.

Case 3: If there does not exist a shortest path in space-time network $(\mathcal{N}, \mathcal{A} \setminus A_{sup}^+)$, then the shortest path in space-time network $(\mathcal{N}, \mathcal{A})$ is the super path. In this case, the optimal solution of Model P1'-X(rs,t) is shown as follows.

$$x_{rstv'}^{rs,t} = 1, \quad (r,s,t,v') \text{ is the super path of TDDP } (rs,t);$$
$$x_{ijuv}^{rs,t} = 0, \quad \forall (i,j,u,v) \in B_L^+.$$

Here, the corresponding objective value is 1, which is the least objective for Model P1'-X for a given $(rs,t) \in W \times T'$.

Proposition 2 *For multiplier vector $\boldsymbol{\lambda}$ with nonnegative values, Model P1'-Y can be further decomposed into $|L|$ shortest path problems with non-positive weights.*

Proof. Note that, $\sum_{(i,j,u,v)\in B_L^+}$ can be equivalently rewritten as $\sum_{l\in L}\sum_{(i,j,u,v)\in B_l^+}$. Thus, the objective function of Model P1'-Y is equivalent to the following form.

$$L_Y(Y, \boldsymbol{\lambda}) = -\sum_{l\in L}\sum_{(i,j,u,v)\in B_l^+}\left[\left(\sum_{(rs,t)\in W\times T'} \lambda_{ijuv}^{rs,t}\right) y_{ijuv}\right] \tag{16}$$

Since the involved lines are independent of each other, the minimum value of (16) is equal to the sum of the minimum objectives of solving $|L|$ subproblems as follows.

$$\text{Model P1'-Y}(l): \begin{cases} \min: L_Y(Y, \boldsymbol{\lambda}, l) = -\sum_{(i,j,u,v)\in B_l^+}\left[\left(\sum_{(rs,t)\in W\times T'} \lambda_{ijuv}^{rs,t}\right) y_{ijuv}\right] \\ s.t.\ \text{ Constraints (1) and (5).} \end{cases} \tag{17}$$

Typically, for each pre-given $l \in L$, it is clear that Model P1'-Y is a shortest path problem with non-positive weights because of nonnegative Lagrangian multipliers. □

Remark 4.1 *Although the generalized link cost is nonpositive in Model P1'-Y for a given $l \in L$, the shortest path still exists in this model since there are no loops in the constructed space-time network.*

4.2 Algorithm description

We next design a heuristic algorithm based on Lagrangian relaxation according to the above decomposition process. As these two models have similar structures, we only present the detailed solution algorithm procedure for Model P1.

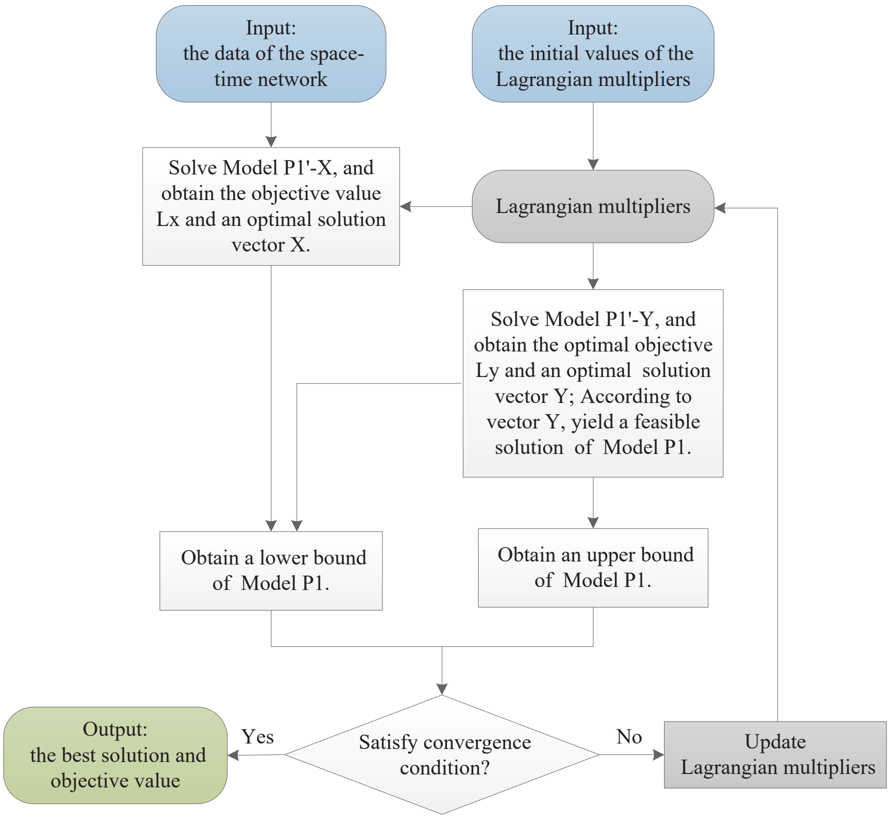

Figure 9: The flow diagram of the proposed algorithm for solving Model P1

For any given Lagrangian multipliers, the optimal objective function of the relaxed model is a lower bound to the original problem. In the searching process, we iteratively improve the lower and upper bounds to produce a near-optimal solution. To solve the dualized model, the necessary input data involves the initial space-time network (including all candidate space-time trajectories of the last trains) and the initial Lagrangian multipliers. In the space-time network, the shortest space-time path for any given TDDP (rs, t) can be found by a label setting/correcting algorithm. Also, according to the current Lagrangian multipliers, the optimal solution of Model P1'-Y can be obtained. Then, a lower bound of Model P1 can be calculated according to the aforementioned optimal solutions. On the other hand, we note that vector Y can yield a last train timetable that satisfies all system constraints. With these solutions, we then can update the upper bound of the proposed model. This type of algorithm can be found in Yang and Zhou (2014) for more information. The above process continues until the termination condition is satisfied, and the best solution is outputted as a near-optimal solution to the primal model.

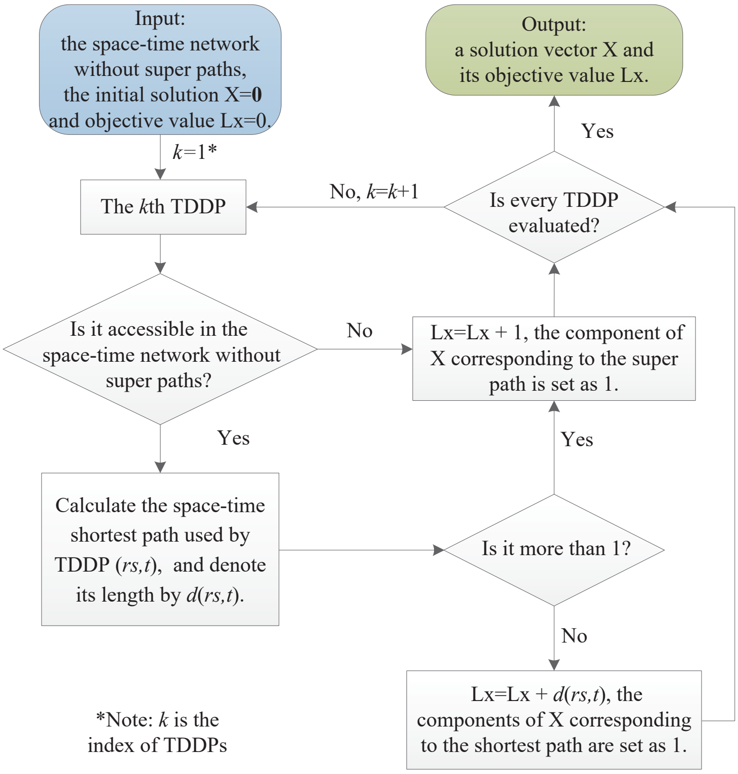

Figure 10: The flow diagram for solving subproblem P1'-X

Figure 9 gives a flow diagram of this algorithm, which describes the detailed procedure of the search process.

Next, we detail the solution procedure of Model P1'-X. As shown in the proof of Proposition 1, Model P1'-X obtains the optimal solution if and only if its subproblems P1'-X(rs, t) for each given $(rs, t) \in W \times T'$ independently generate their optimal solutions. In other words, every loop with respect to TDDPs yields some components of vector X and a part of the objective value of Model P1'-X. When the loops are terminated, a relaxed solution and its corresponding objective value can be computed. For TDDP (rs, t), we need to solve a space-time shortest path problem, in which the weight of space-time arc $(i, j, u, v) \in B_l^+$ is Lagrangian multiplier $\lambda_{ijuv}^{rs,t}$. According to the discussion on Model P1'-X(rs, t), we only find the shortest path in space-time network $(\mathcal{N}, \mathcal{A} \setminus A_{sup}^+)$, but not in space-time network $(\mathcal{N}, \mathcal{A})$. Figure 10 offers a detailed flow diagram for solving Model P1'-X. Note that when we use the flow diagram in Figure 10 to solve Model P2, the expression "Lx=Lx+1" must be modified as "Lx=Lx+q_{rs}^t". Of course, the Lagrangian multipliers of the different models must be set as different nonnegative numbers, since the economic significance under these models is different.

The proof of Proposition 2 provides some valuable information to solve P1'-Y. For a set of given candidate space-time arcs of the last train on metro line l, we first need to sum all Lagrangian multipliers corresponding to each candidate space-time arc, and then take its negative value as the weight of this candidate space-time arc. A shortest path is found from the dummy origin to the dummy destination in the constructed space-time network of the last train on metro line l (shown in Figure 6) as a trajectory of the last train on metro line l. When the above process is successively carried out over all $|L|$ lines, we obtain a solution Y of Model P1'-Y. Figure 11 gives a flow diagram for solving Model P1'-Y.

Vectors Y and X constitute a solution of Model P1' (a relaxed solution of Model P1), and this relaxed solution yields a lower bound to Model P1. Meanwhile, based on vector Y, we can construct a space-time

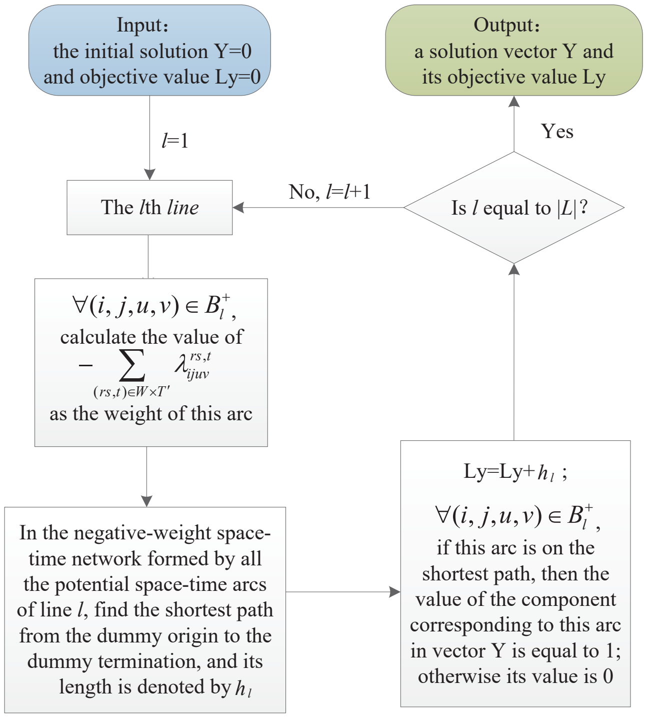

Figure 11: The flow diagram for solving subproblem P1'-Y

network involving a timetable of the last trains, which is used to load all the demands. Consequently, a feasible solution of Model P1 can be obtained, which yields an upper bound of Model P1. According to the specific analyses mentioned above, we give the pseudocode for solving Model P1, as shown in the Appendix.

5 Numerical Experiments

In this section, the validity of the proposed models and algorithms is evaluated on the basis of a series of numerical experiments on a small network and a large-scale Beijing metro network. All the experiments are implemented on a Lenovo ThinkCentre M8600t-D066 with 8 GB RAM and a 3.40 GHz Core i7-6700 CPU. The main procedure of the algorithm is coded in the environment of MATLAB R2016a.

5.1 An small-scale example

In this set of experiments, the proposed models and algorithms are evaluated on a small-scale metro network, as shown in Figure 12. This network involves four metro lines with a total of four transfer stations, in which 12 segments and 12 stations are included. Since we treat different directions of a metro line independently, a total of 8 last trains are considered in this small-scale network. Table 4 lists the link running time, headway, station dwelling time, transfer time and departure time of the last trains in the original timetable, where the time is assumed to start at timestamp 1. In this example, a total of 43 TDDPs and 8390 passengers over these TDDPs are given for Models P1 and P2, respectively. The relevant demand data are listed in Table 5.

Table 4: Relevant parameters of the small-scale metro network

Line: Direction	Link running time[1]	Headway	Dwell time	Transfer time	DTLT[2]
Line1: Up	$1 \xrightarrow{6} 2 \xrightarrow{6} 3 \xrightarrow{6} 4$	5	1	2	21
Down	$4 \xrightarrow{6} 3 \xrightarrow{6} 2 \xrightarrow{6} 1$	5	1	2	21
Line2: Up	$5 \xrightarrow{5} 6 \xrightarrow{6} 7 \xrightarrow{5} 8$	6	1	2	20
Down	$8 \xrightarrow{5} 7 \xrightarrow{6} 6 \xrightarrow{5} 5$	6	1	2	20
Line3: Up	$9 \xrightarrow{5} 2 \xrightarrow{4} 6 \xrightarrow{6} 10$	5	1	2	22
Down	$10 \xrightarrow{6} 6 \xrightarrow{4} 2 \xrightarrow{5} 9$	5	1	2	22
Line4: Up	$11 \xrightarrow{7} 3 \xrightarrow{5} 7 \xrightarrow{5} 12$	6	1	2	19
Down	$12 \xrightarrow{5} 7 \xrightarrow{5} 3 \xrightarrow{7} 11$	6	1	2	19

[1] Symbols $1 \xrightarrow{6} 2$ means that the running time is 6 time intervals from station 1 to station 2.
[2] DTLT refers to the departure time of last trains.

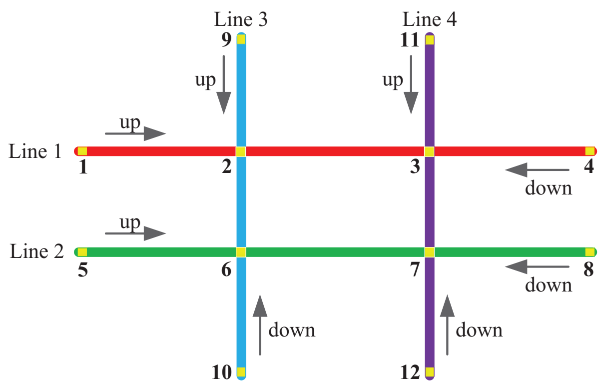

Figure 12: A small-scale metro network example

With the above pre-given parameters, we optimize the timetable of the last trains for this network by using different performance measures, i.e., space-time accessibility (STA) in Model P1 and flow-based accessibility (FBA) in Model P2. Note that STA is equal to the total TDDP amount minus the optimal objective of Model P1, and FBA is equal to the total passenger amount minus the optimal objective Model P2. In these two experiments, it takes 66 CPU seconds on average to run the proposed algorithm. We respectively obtain two optimal timetables of the last trains, which are listed in the last two columns of Table 6. The corresponding accessibility measures are also given as two bold numbers in the last two rows of Table 6, where the numbers in parentheses are the inaccessibility measures.

Table 5: The TDDPs and passenger amount on the small-scale metro network

TDDP			Passenger	TDDP			Passenger	TDDP			Passenger
O	D	t	amount	O	D	t	amount	O	D	t	amount
1	8	6	400	9	11	7	300	4	12	16	100
1	8	11	100	9	8	12	300	4	9	16	150
1	12	11	200	9	12	12	50	8	4	14	200
1	5	11	300	9	11	12	100	8	1	14	100
1	11	11	310	11	10	7	220	8	9	14	300
1	10	16	220	11	5	7	200	8	11	14	300
1	12	16	60	11	8	13	120	10	12	7	200
1	5	16	160	11	10	13	120	10	11	7	250
1	11	16	110	11	5	13	100	10	4	12	250
5	4	8	400	4	5	6	280	10	12	12	130
5	11	8	280	4	8	11	160	10	11	12	120
5	4	14	200	4	10	11	260	12	1	13	130
5	12	14	100	4	5	11	120	12	9	13	150
5	11	14	200	4	8	16	200	Total:			
9	12	7	280	4	10	16	160		43		8390

Table 6: The optimized timetables of last trains and the corresponding accessibility measures on the small-scale metro network

Line: Direction	Original DTLT	Optimized DTLT by Model P1	Optimized DTLT by Model P2
Line1: Up	21	23	23
Down	21	23	25
Line2: Up	20	24	22
Down	20	22	22
Line3: Up	22	26	26
Down	22	24	22
Line4: Up	19	23	19
Down	19	23	23
The measure of STA	30 (13)	**41 (2)**	40 (3)
The measure of FBA	6580 (1810)	8030 (360)	**8120 (270)**

As shown in Table 6, by comparing with the original timetable, the accessibility-based evaluation measures can be greatly improved in the optimal solutions provided by the algorithms of the two models. The space-time accessibility produced by Model P1 is improved by about 37% (from 30 to 41), and the inaccessible TDDP number decreases from 13 to 2. The flow-based accessibility produced by Model P2 is improved by about 23% (from 6580 to 8120), and the inaccessible passenger amount decreases from 1810 to 270.

In addition, it is worth noting that these two train timetables respectively produced by Models P1 and P2 are different from each other. To verify the difference of the proposed models and the effectiveness of algorithms, we also calculate all accessibility-based evaluation measures under these two optimal timetables, which are listed in the last two rows of Table 6. If the evaluation measure of the space-time accessibility is adopted, then the solution produced by Model P1 is superior than Model P2. If the evaluation measure of the flow-based accessibility is adopted, then the solution produced by Model P2 is superior than Model P1. To show the performance of the generated solutions, we here give Figure 13 to display the upper bounds, lower bounds and absolute gaps between them when the algorithms of the different models are set to run 300 iterations and applied on the small-scale metro network. Further discussion about the convergence of the algorithms will be conducted on a large-scale network in the next subsection.

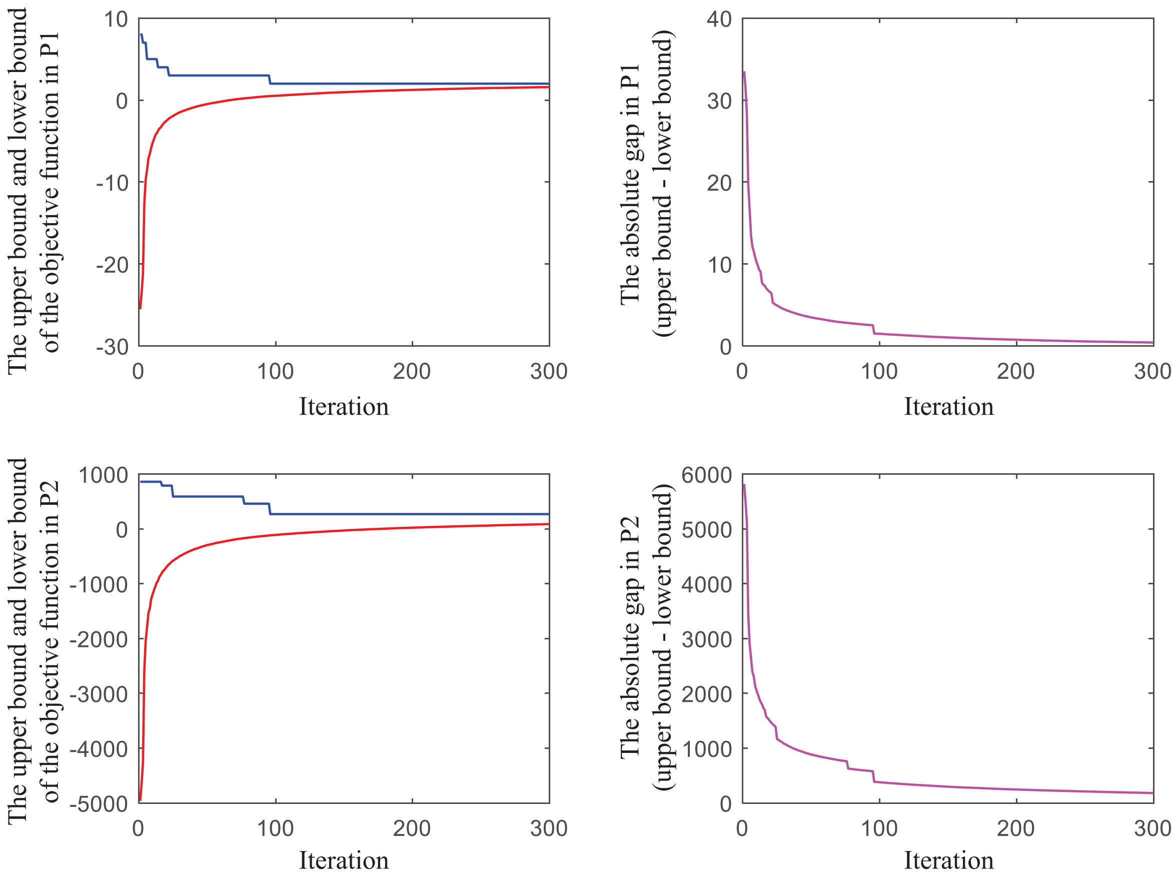

Figure 13: Upper bounds, lower bounds and gaps of the different models applied on the small-scale metro network

5.2 Numerical experiments in Beijing metro network

In the second set of experiments, the proposed models and algorithms are examined on a practical large-scale network: Beijing metro network, as shown in Figure 14. (`http://map.bjsubway.com`. Accessed: June 12, 2018.) By the end of 2018, Beijing has opened 21 lines with 316 stations (involving 55 transfer stations). Since this paper focuses only on successful transfers between the last trains on different lines, we delete some branch lines, and simplify the origins and destinations for computational convenience. Thus, we mainly consider 12 lines (No. 1, 2, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15) with 81 segments and 45 transfer stations. In practice, lines 1, 2, 4, 5, 6, 10, 13 and 15 simultaneously provide the last-train service for their up and down directions, while the last trains on lines 7, 8, 9 and 14 first serve their upper directions and then turn around for serving their down directions. Therefore, a total of 20 last trains are taken into consideration in this network.

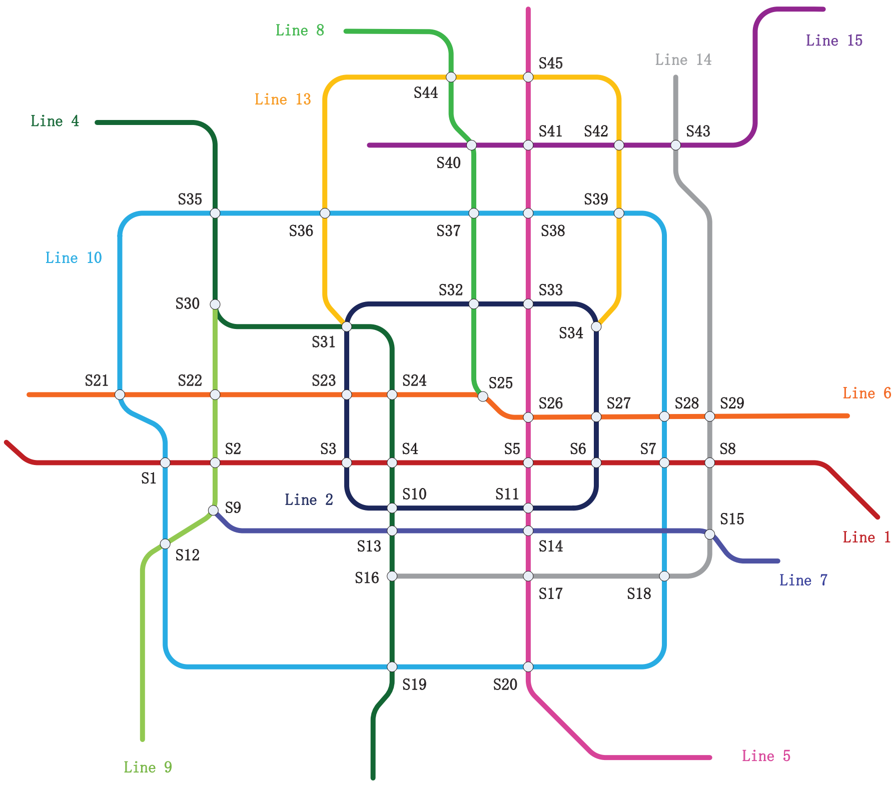

Figure 14: Simplified diagram of the Beijing subway network

In terms of parameter settings, Model P1 considers all potential OD pairs that need transfer, i.e., a total of 1782 OD pairs, and Model P2 takes into account 31288 passengers over these OD pairs (see the Appendix for more details). In the models, the considered time horizon is set as [22:00,00:50], which covers the departure and arrival times of all the last trains. Moreover, the time horizon is discretized into 340 timestamps, i.e., the discretized time interval $\delta = 30$s. The initial timetable of the last trains (i.e., the original DTLT in Table 7) is the actual timetable currently in use (`http://map.bjsubway.com`. Accessed: June 12, 2018). For each line, we suppose that there are three potential trajectories for its last train, and their time intervals are 5 minutes. Also, all Lagrangian multipliers are generated randomly.

In the solution process, the termination condition is set as follows: (1) when the optimality dual gap is less than 5%, the algorithm is terminated; (2) if the total number of iterations exceeds a pre-determined threshold (i.e., 300 iterations), the algorithm is terminated. With these settings, the algorithm outputs a solution after around 5040 CPU seconds. The optimal timetables of the last trains produced by the algorithms of Model P1 and Model P2, as well as the original timetable are listed in Table 7.

Table 7: The optimized timetables of last trains and the corresponding accessibility measures on the Beijing metro network

Line: Direction	Original DTLT	Optimal DTLT of Model P1	Optimal DTLT of Model P2
Line 1: Up	23:45	23:50	23:50
Down	23:33	23:33	23:38
Line 2: Up	23:00	23:10	23:05
Down	23:00	23:05	23:05
Line 4: Up	23:10	23:15	23:10
Down	22:55	23:00	23:05
Line 5: Up	23:09	23:19	23:19
Down	22:52	22:57	23:02
Line 6: Up	23:19	23:29	23:29
Down	23:22	23:32	23:22
Line 7: Circle	22:39	22:44	22:49
Line 8: Circle	22:14	22:19	22:24
Line 9: Circle	22:48	22:58	22:58
Line 10: Up	22:31	22:41	22:41
Down	22:23	22:28	22:33
Line 13: Up	22:42	22:42	22:47
Down	22:42	22:52	22:42
Line 14: Up	22:40	22:50	22:50
Down	22:34	22:44	22:39
Line 15: Circle	22:46	22:56	22:56
The measure of STA	1606 (176)	**1706 (76)**	1709 (73)
The measure of FBA	28080 (3208)	29898 (1390)	**29951 (1337)**

Next, we analyze the performance of the different optimal solutions. Without considering our models, a total of 1606 OD pairs are accessible and 176 OD pairs are inaccessible in the original timetables of the last trains. In comparison, after optimizing by Model P1, these two numbers are 1706 and 76, respectively, which implies that the space-time accessibility has improved (1706-1606)/1606 = 6.23% in the considered network. On the other hand, in the original timetable, the total number of accessible and inaccessible passengers are 28080 and 3208, and these two numbers respectively become 29951 and 1337 when the timetables of the last trains are optimized by Model P2, corresponding to the improvement of flow-based accessibility by (29951-28080)/28080 = 6.66%. Additionally, we also calculate all accessibility-based evaluation measures under these two optimal timetables, which are also listed in the last two rows of Table 7. Only from these computational results, the solution produced by Model P2 is superior than the solution produced by Model P1 in terms of both accessibility measures, but the latter is very close to the former. Above phenomenon is possibly caused by two reasons. On one hand, it becomes very hard to find the exact optimal solution when the scale of network is large (i.e., the returned solutions are only near-optimal). On the other hand, in the stage of data preparation, we set more passenger demands on the accessible OD pairs in this set of experiments, which leads to the consistence of these two objectives.

In the results presented in Table 6 and 7, the optimal DTLTs in Models P1 and P2 are always later than the original DTLT. Indeed, this phenomenon is due to the structure of input data in our experiments.

Specifically, the favorite departure times of passengers are relatively late in this example. Then, all the potential departure times of last trains are set to be later than the original DTLT. For instance, if the original DTLT is 11:00PM, the set of potential departure times might be set as {11:00PM, 11:05PM, 11:10PM}. With this data structure, the optimal DTLT is always later than the original DTLT. It is obvious that the optimization of last-train timetable is associated with both the favorite departure time of passengers and potential departure time of last trains.

Finally, we discuss the convergence and steadiness of the algorithms in the solution process. Specifically, we implement the algorithm with 300 iterations, and record the upper bounds, lower bounds and duality gaps of these two models, as shown in Figure 15. It is easy to see from each instance that since all the initial Lagrangian multipliers are generated randomly, the relative gaps at the beginning of the iterations are relatively large. However, it reduces rather quickly. When the number of iterations is equal to 300, the relative gaps of these two models drop to an acceptable level, respectively. In addition, to show the robustness and steadiness of the proposed algorithms, each algorithm is executed many times with different initial Lagrangian multipliers generated randomly. The computational results show that the solution algorithm usually can generate very similar or even identical solutions for each model in different implementations, demonstrating the robustness of the proposed algorithms.

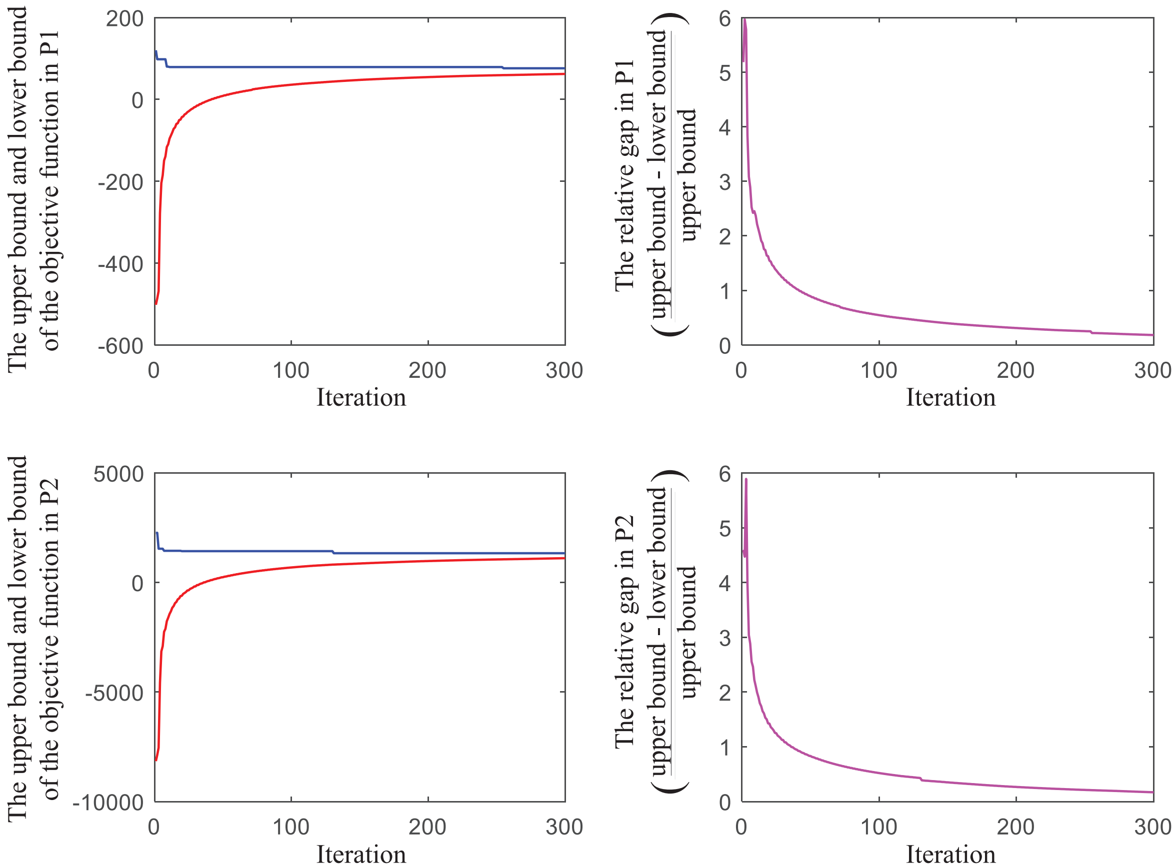

Figure 15: Upper bounds, lower bounds and gaps of the different models applied on the Beijing metro network

6 Conclusions and future research

Transfer activities between the last trains have become more and more important for night operations in the interlaced metro network. Aiming to improve the accessibility of the metro networks for the night operation period, this study investigated the timetabling problem of the last trains from the perspective of network design. With the space-time network representation, two 0-1 integer linear programming models were formulated with respect to different measures of accessibility, including space-time accessibility and flow-based accessibility. The variation of performance measures in two models reflects the influence of TDDP weights on the timetable. To solve the proposed models, the involved hard constraints are dualized by introducing a set of Lagrangian multipliers, and the relaxed models are decomposed into a series of classical and tractable shortest path problems in the space-time network. Then, the subgradient algorithm is used to find the near-optimal solutions to the original problem. Finally, a small-scale numerical example and a large-scale numerical example for the Beijing metro network were tested to verify the performance of the proposed approaches.

In this study, the proposed approach can effectively deal with the time-dependent/dynamic passenger demands in the metro network with the maximal accessibility. Practically, if the timetable of last trains is optimized, all the involved passengers can find their paths in the space-time network by using label setting/correcting algorithm once the favorite departure time is determined. This can be calculated in the path guidance system (or navigation APP) released by the metro company, which can improve the service level of the night operation process. With this function, each passenger will know the accessibility/inaccessibility of his/her travel demand, and correspondingly, the detailed transfer process can be generated if the demand is accessible. In this sense, our proposed method has its promising application potentials in the real-world operations.

Further researches can focus on the following aspects. (1) This paper only considers the collaborative optimization of the last trains in metro systems. When the multi-mode public transportation is taken into consideration (e.g., trains, bus, etc.), an interesting topic can be investigated for timetable optimization of the last services with other different modes. (2) For simplicity, all the parameters are assumed to be fixed quantities in the current version, corresponding to deterministic optimization models. As a lot of uncertainties still exist in the real operations, e.g., uncertain demand distribution and uncertainty transfer time, how to effectively handle these uncertainties and then model the problem should be practically significant. (3) The space-time network presentation method in this paper can also be generalized to the studies of timetable collaboration of high-speed railways or intercity railways, e.g., adding trains in the peak period. Also, it is feasible to model this problem by minimizing the total travel time or using multi-objective optimization methods under the current space-time network.

Acknowledgement

The research of this paper was supported by the National Natural Science Foundation of China (Nos. 71825004, 71621001, 71801093).

References

An K., Lo H.K., 2016. Two-phase stochastic program for transit network design under demand uncertainty. Transportation Research Part B 84, 157-181.

Antunes A., Seco A., Pinto N., 2002. Inter-urban road network planning: An accessibility-maximization approach. Journal of Decision Systems 11 (3-4), 283-296.

Bhat C.R., Bricka S., La Mondia J., Kapur A., Guo J.Y., Sen S., 2006. Metropolitan area transit accessibility analysis tool. Handbooks.

Canca D., Barrena E., Algaba E., Zarzo A., 2014. Design and analysis of demand-adapted railway timetables. Journal of Advanced Transportation 48 (2), 119-137.

Cancela H., Mauttone A., Urquhart M.E., 2015. Mathematical programming formulations for transit network design. Transportation Research Part B 77, 17-37.

Chen J., Ni J., Xi C., Li S., Wang J., 2017. Determining intra-urban spatial accessibility disparities in multimodal public transport networks. Journal of Transport Geography 65, 123-133.

Chen Y., Mao B., Bai Y., Ho T.K., Li Z., 2019. Timetable synchronization of last trains for urban rail networks with maximum accessibility. Transportation Research Part C 99, 110-129.

D'Acierno L., Botte M., Placido A., Caropreso C., Montella B., 2017. Methodology for determining dwell times consistent with passenger flows in the case of metro services. Urban Rail Transit 3 (2), 73-89.

Di Z., Yang L., Qi J., Gao Z., 2018. Transportation network design for maximizing flow-based accessibility. Transportation Research Part B 110, 209-238.

Dou X., Meng Q., Guo X., 2015. Bus schedule coordination for the last train service in an intermodal bus-and-train transport network. Transportation Research Part C 60, 360-376.

Fan W., Mei Y, Gu W., 2018. Optimal design of intersecting bimodal transit networks in a grid city. Transportation Research Part B 111, 203-226.

Farahani R.Z., Miandoabchi E., Szeto W.Y., Rashidi H., 2013. A review of urban transportation networkdesign problems. European Journal of Operational Research 229 (2), 281-302.

Friesz T.L., 1985. Transportation network equilibrium, design and aggregation: Key developments and research opportunities. Transportation Research Part A 19 (5-6), 413-427.

Fu L., Xin Y., 2007. A new performance index for evaluating transit quality of service. Journal of Public Transportation 10(3), 47-69.

Gao Z., Wu J., Sun H., 2005. Solution algorithm for the bi-level discrete network design problem. Transportation Research Part B 39(6), 479-495.

Geurs K.T., Paix L.L., Weperen S.V., 2016. A multi-modal network approach to model public transport accessibility impacts of bicycle-train integration policies. European Transport Research Review 8, 25.

Gleason J.M. 1975. A set covering approach to bus stop location. Omega 3(5), 605-608.

Guo X., Sun H., Wu J., Jin J., Zhou J., Gao Z., 2017. Multiperiod-based timetable optimization for metro transit networks. Transportation Research Part B 96, 46-67.

Handy S., Niemeier D., 1997. Measuring accessibility: An exploration of issues and alternatives. Environment and Planning A 29, 1175-1194.

Hillman R., Pool G., 1997. GIS-based innovations for modeling public transport accessibility. Traffic Engineering and Control 38(10), 554-559.

Huang Y., Yang L., Tao T., Gao Z., Fang C., 2017. Joint train scheduling optimization with service quality and energy efficiency in urban rail transit networks. Energy 138, 1124-1147.

Kang L., Meng Q., 2017. Two-phase decomposition method for the last train departure time choice in subway networks. Transportation Research Part B 104, 568-582.

Kang L., Wu J., Sun H., Zhu X., Gao Z., 2015. A case study on the coordination of last trains for the beijing subway network. Transportation Research Part B 72, 112-127.

Kang L., Wu J., Sun H., Zhu X., Wang B., 2015. A practical model for last train rescheduling with train delay in urban railway transit networks. Omega 50, 29-42.

Kang L., Zhu X., 2017. Strategic timetable scheduling for last trains in urban railway transit networks. Applied Mathematical Modelling 45, 209-225.

Kerrigan M., Bull D., 1992. Measuring accessibility: A public transport accessibility index. Environmental Issues: Selected Proceedings of the Seminar B PTRC Summer Annual Meeting, PTRC Education and Research Services, 245-256.

Krasemann J.T., 2012. Design of an effective algorithm for fast response to the rescheduling of railway traffic during disturbances. Transportation Research Part C 20(1), 62-78.

Lee J., Miller H.J., 2018. Measuring the impacts of new public transit services on space-time accessibility: An analysis of transit system redesign and new bus rapid transit in Columbus, Ohio, USA. Applied Geography 93, 47-63.

Li L., Ren H., Zhao S., Duan Z., Zhang Y., Zhang A., 2017. Two dimensional accessibility analysis of metro stations in Xi'an, China. Transportation Research Part A 106, 414-426.

Liu J., Zhou X., 2016. Capacitated transit service network design with boundedly rational agents. Transportation Research Part B 93, 225-250.

Mavoa S., Witten K., Mccreanor T., O'Sullivan D., 2012. GIS based destination accessibility via public transit and walking in auckland, New Zealand. Journal of Transport Geography, 20(1), 15-22.

Moniruzzaman M., Paez A., 2012. Accessibility to transit, by transit, and mode share: Application of a logistic model with spatial filters. Journal of Transport Geography 24(3), 198-205.

Murawski L., Church R.L., 2009. Improving accessibility to rural health services: The maximal covering network improvement problem. Socio-Economic Planning Sciences 43 (2), 102-110.

Murray A.T., Davis R., Stimson R.J., Ferreira L., 1998. Public transportation access. Transportation Research Part D 3(5), 319-328.

Polzin S.E., Pendyala R.M., Navari S., 2002. Development of time-of-day-based transit accessibility analysis tool. Transportation Research Record 1799, 35-41.

Santos B., Antunes A., Miller, E.J., 2008. Integrating equity objectives in a road network design model. Transportation Research Record, 2089(2089), 35-42.

Schoon J., Mcdonald M., Lee A., 1999. Accessibility indices: Pilot study and potential use in strategic planning. Transportation Research Record, 1685(1), 29-38.

Shang P., Li R., Liu Z., Yang L., Wang Y., 2018. Equity-oriented skip-stopping schedule optimization in an oversaturated urban rail transit network. Transportation Research Part C 89, 321-343.

Tong L., Zhou X., Miller H.J., 2015. Transportation network design for maximizing space-time accessibility. Transportation Research Part B 81, 555-576.

Tong, L., Pan Y., Shang P., Guo J., Xian K., Zhou X., 2019. Open-source public transportation mobility simulation engine DTALite-S: A discretized space-time network-based modeling framework for bridging multi-agent simulation and optimization. Urban Rail Transit 5(1), 1-16.

Wang S., Meng Q., Yang H., 2013. Global optimization methods for the discrete network design problem. Transportation Research Part B 50 (4), 42-60.

Xu R.H., Zhang M., Jiang Z.B., 2008. Study on departure time domain of the first and last trains of urban mass transit network based on operation coordination. Journal of the China Railway Society 30, 7-11.

Yang H., Bell M.G.H., 1998. Models and algorithms for road network design: A review and some new developments. Transport Reviews 18 (3), 257-278.

Yang L., Zhang Y., S. Li, Gao Y., 2016. A two-stage stochastic optimization model for the transfer activity choice in metro networks. Transportation Research Part B 83, 271-297.

Yang L., Zhou X., 2014. Constraint reformulation and Lagrangian relaxation-based solution algorithm for a least expected time path problem. Transportation Research Part B 59, 22-44.

Yang L., Zhou X., Gao Z., 2014. Credibility-based rescheduling model in a double-track railway network: A fuzzy reliable optimization approach. Omega 48(10), 75-93.

Yang S., Yang K., Gao Z., Yang L., Shi J., 2017. Last-train timetabling under transfer demand uncertainty: Mean-variance model and heuristic solution. Journal of Advanced Transportation 2017, 5095021. (DOI: 10.1155/2017/5095021)

Yin J., Yang L., Tang T., Gao Z., Ran B., 2017. Dynamic passenger demand oriented metro train scheduling with energy-efficiency and waiting time minimization: Mixed-integer linear programming approaches. Transportation Research Part B 97, 182-213.

Zhou X., Tong L., Mahmoudi M., Zhuge L., Yao Y., Zhang Y., Shang P., Liu J., Shi T., 2018. Open-source VRPLite Package for Vehicle Routing with Pickup and Delivery: A Path Finding Engine for Scheduled Transportation Systems. Urban Rail Transit 4(2), 68-85.

Zhou Y., Wang Y., Yang H., Yan X., 2019. Last train scheduling for maximizing passenger destination reachability in urban rail transit networks, Transportation Research Part B 129, 79-95.

Appendix

Algorithm 1 Pseudocode for solving Model P1

Input the basic information of the network, including demand set $W \times T'$ and the space-time network $(\mathcal{N}, \mathcal{A} \setminus A^{+}_{sup})$; initialize Lagrangian multipliers $\boldsymbol{\lambda}$ (generated randomly in this paper), solution $X = \mathbf{0}$, $Y = \mathbf{0}$, upper bound $\overline{Z}$ and lower bound $\underline{Z}$.
for each iteration $(n = 1; n \leq n_{max}; n++)$ **do**
  Lx=0;
  for each TDDP $(k = 1; k \leq |W \times T'|; k++)$ **do**
    Find the space-time shortest path of this TDDP in $(\mathcal{N}, \mathcal{A} \setminus A^{+}_{sup})$, and denote its length by $d(rs, t)$;
    if $d(rs,t) = \infty$ **then** let $x^{rs,t}_{rstv'} = 1$, where (r, s, t, v') is the super path of TDDP (rs, t), and let Lx $\leftarrow$ Lx+1;
    else
      let Lx $\leftarrow$ Lx + min$\{1, d(rs,t)\}$;
      if $d(rs,t) \leq 1$ **then** let $x^{rs,t}_{ijuv} = 1$, where $(i,j,u,v) \in B^{+}_{L}(rs,t)$, namely, (i,j,u,v) is on the space-time shortest path of TDDP (rs,t);
      else
        let $x^{rs,t}_{rstv'} = 1$, where (r, s, t, v') is the super path of TDDP (rs, t).
      end if
    end if
  end for
  Obtain X and Lx.
  Ly=0;
  for each line $(l = 1; l \leq |L|; l++)$ **do**
    for each candidate arc $(i,j,u,v) \in B^{+}_{l}$ **do**
      Set $w^{ijuv}_{l} = -\sum_{(rs,t) \in W \times T'} \lambda^{rs,t}_{ijuv}$ as weight of the space-time arc (i,j,u,v);
    end for
    Find the shortest path from dummy origin O'_l to dummy destination D'_l in the space-time network of the last train on line l, and denote its length by h_l, let Ly $\leftarrow$ Ly+h_l;
    for each candidate arc $(i,j,u,v) \in B^{+}_{l}$ **do**
      if space-time arc (i,j,u,v) is on the shortest path **then** let $y_{ijuv} = 1$;
      end if
    end for
  end for
  Obtain Y and Ly.
  A lower bound can be generated: $\underline{Z}$ =Lx+Ly.
  According to solution Y, a new timetable of the last trains can be generated.
  According to the new timetable, an upper bound $\overline{Z}$ can be calculated.
  Calculate the relative gap value $\frac{|\overline{Z} - \underline{Z}|}{\overline{Z}} \times 100\%$.
  According to the current X, Y, $\boldsymbol{\lambda}$, the value of $\boldsymbol{\lambda}$ can be updated by using the subgradient method, i.e., $\lambda^{rs,t}_{ijuv} \leftarrow \lambda^{rs,t}_{ijuv} + \beta^{n} \cdot \left(x^{rs,t}_{ijuv} - y_{ijuv}\right)$, where β^{n} represents the step length factor at iteration n.
end for

Table 8: Demand data of the Beijing metro network

O(t)*\D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1(136)	-	-	-	-	-	-	-	-	27	16	23	28	17	27	13	5	21	6	26	16	9
2(136)	-	-	-	-	-	-	-	-	16	28	23	12	8	26	22	21	25	29	15	24	13
3(136)	-	-	-	-	-	-	-	-	18	15	9	20	24	11	6	24	13	19	24	27	26
4(136)	-	-	-	-	-	-	-	-	25	9	9	27	11	20	12	26	14	19	7	24	11
5(136)	-	-	-	-	-	-	-	-	15	10	17	6	15	28	21	18	27	11	6	13	29
6(136)	-	-	-	-	-	-	-	-	29	18	8	27	26	18	23	18	10	23	25	16	10
7(136)	-	-	-	-	-	-	-	-	17	19	12	29	27	12	6	6	6	13	25	11	28
8(136)	-	-	-	-	-	-	-	-	11	13	8	18	7	20	21	28	29	8	19	15	12
9(80)	15	25	9	26	20	19	25	17	-	10	26	28	-	-	-	8	16	29	12	30	7
10(106)	29	26	6	29	25	25	6	21	8	-	-	21	23	6	29	13	9	22	29	22	29
11(106)	24	22	15	15	17	29	26	17	17	-	-	30	6	27	16	26	10	17	18	17	30
12(95)	17	24	23	23	18	10	7	6	21	24	28	-	19	28	13	11	9	10	24	5	5
13(80)	15	29	12	20	7	12	23	14	-	23	15	11	-	-	-	17	27	7	15	6	12
14(80)	13	18	28	15	22	28	17	11	-	26	15	22	-	-	-	30	9	28	20	19	9
15(80)	21	22	26	9	26	13	25	30	-	30	14	23	-	-	-	21	28	13	9	16	18
16(70)	11	13	18	27	26	25	23	13	18	30	26	13	19	9	15	-	-	-	26	11	25
17(70)	8	13	19	22	6	7	26	9	18	14	27	27	22	9	13	-	-	-	13	10	28
18(70)	15	15	28	30	27	19	8	27	21	13	17	6	7	16	26	-	-	-	24	14	28
19(108)	22	11	11	24	15	10	28	24	22	29	29	19	16	25	25	22	19	27	-	16	29
20(100)	29	15	11	29	9	23	7	9	17	12	7	23	12	8	16	27	16	11	8	-	11
21(120)	19	18	13	18	10	6	22	7	16	14	30	23	14	30	15	17	27	12	27	17	-
22(120)	22	7	6	6	24	18	11	30	23	7	21	13	24	12	14	27	13	21	27	29	-
23(106)	13	14	18	15	21	17	7	16	27	-	-	27	27	8	30	5	29	6	11	11	27
24(120)	11	23	10	20	29	10	23	8	14	27	14	26	23	9	28	9	19	9	20	9	-
25(120)	10	16	17	11	16	26	15	24	25	23	18	19	18	8	14	19	5	18	24	7	-
26(120)	12	9	23	23	15	18	26	25	8	23	24	27	20	19	19	13	18	26	14	7	-
27(106)	12	27	11	17	28	23	27	13	8	-	-	16	27	6	7	13	21	23	17	17	26
28(120)	18	10	19	12	27	16	19	24	28	11	12	27	20	8	25	24	23	16	27	18	-
29(120)	9	9	16	26	23	23	18	25	17	12	15	25	28	18	7	22	19	17	20	10	-
30(95)	26	5	27	26	24	18	26	9	11	23	9	-	18	17	20	21	19	8	7	25	14
31(106)	26	28	14	11	17	16	15	26	9	-	-	23	24	16	20	16	26	18	14	23	14
32(106)	8	25	8	15	19	10	30	22	21	-	-	15	27	19	24	23	14	15	23	9	6
33(106)	22	21	19	26	22	30	11	7	22	-	-	25	12	11	28	28	18	15	27	11	25
34(106)	7	15	9	28	30	6	30	27	30	-	-	23	30	16	10	15	19	9	21	15	30
35(108)	11	23	12	19	25	25	10	6	16	19	13	28	25	7	21	10	10	17	-	13	19
36(80)	17	29	7	14	10	9	13	21	16	21	13	6	19	27	24	24	17	17	11	17	7
37(28)	13	11	8	25	13	17	26	12	25	10	12	18	20	23	22	7	16	23	7	28	18
38(100)	29	6	28	11	18	22	27	7	27	24	21	11	21	27	5	7	27	11	12	-	19
39(80)	23	12	22	26	9	17	8	21	9	18	8	24	11	7	25	9	22	14	7	14	6
40(90)	9	16	23	19	5	28	26	30	22	8	10	17	17	8	21	12	21	29	18	22	6
41(90)	13	23	15	29	8	15	12	18	14	8	8	7	14	17	18	20	19	17	17	9	29
42(90)	24	12	8	15	21	13	28	16	10	14	16	20	24	5	12	28	18	23	25	19	27
43(90)	27	26	30	11	23	26	8	9	7	16	25	9	29	23	30	13	26	21	23	11	25
44(28)	7	7	9	17	26	28	21	8	19	21	12	20	10	21	30	17	12	12	28	17	8
45(100)	8	12	25	9	8	27	22	8	11	8	19	16	28	7	7	29	10	27	20	-	6

*The numbers in brackets are the departure time of passengers.

Continue

22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
10	28	13	17	9	25	15	20	14	20	6	14	17	17	7	16	5	11	29	19	7	7	6	14
30	17	8	6	8	16	27	14	22	24	11	21	15	24	20	28	26	6	20	10	14	9	27	16
12	15	18	26	19	7	22	16	30	11	9	24	24	11	15	24	19	25	15	19	10	7	26	21
22	6	16	5	25	30	28	26	30	12	11	15	6	27	29	26	15	18	26	11	15	25	21	29
21	16	15	30	28	25	17	27	19	20	25	23	9	13	25	27	21	29	24	18	29	6	24	12
29	16	16	6	18	16	16	24	12	20	11	27	25	9	7	14	6	24	28	20	21	6	18	25
24	26	22	8	9	25	7	23	13	11	25	12	16	11	20	22	28	7	28	13	17	20	8	6
19	21	12	27	15	5	11	29	25	22	8	17	29	6	12	12	13	5	20	18	28	12	8	7
10	9	13	18	15	25	20	15	12	11	11	10	26	5	12	29	7	29	9	20	15	27	15	18
19	-	11	16	7	-	6	27	10	-	-	-	-	12	22	12	16	24	23	20	7	17	29	10
28	-	19	26	21	-	17	5	11	-	-	-	-	24	16	20	18	13	19	26	10	27	27	8
24	10	8	9	25	16	22	23	-	10	9	8	19	7	29	27	6	8	17	18	23	21	10	12
27	28	10	27	29	6	9	21	11	24	29	18	21	11	16	24	30	13	21	10	27	9	23	14
17	15	24	26	26	10	15	17	19	19	10	14	22	8	6	17	9	18	19	27	9	24	10	19
29	16	8	20	16	12	7	21	7	15	16	11	6	18	16	12	23	21	11	23	19	21	19	10
27	28	18	19	13	19	28	29	10	18	27	18	21	21	12	17	7	16	29	8	10	21	24	5
14	28	25	22	29	8	9	21	25	15	15	10	28	5	11	14	16	11	19	27	26	12	18	12
10	9	7	12	8	5	25	11	21	14	26	21	19	26	15	17	14	25	12	25	18	15	17	19
15	26	11	12	16	16	8	18	7	8	28	25	8	-	20	8	24	21	20	24	6	19	20	14
27	11	20	9	10	23	13	6	13	21	28	6	21	13	23	18	0	20	26	18	9	28	24	-
-	26	-	-	-	30	-	-	7	13	18	7	15	19	20	10	21	18	30	6	9	17	6	21
-	23	-	-	-	16	-	-	11	20	24	15	26	26	24	17	11	14	27	17	17	30	13	28
16	-	17	17	17	-	7	27	5	-	-	-	-	26	21	14	25	8	11	23	20	11	20	27
-	22	-	-	-	19	-	-	10	18	20	25	13	30	22	17	13	19	6	24	6	18	27	5
-	26	-	-	-	10	-	-	10	25	29	13	8	29	28	6	11	13	28	12	27	12	24	25
-	24	-	-	-	27	-	-	25	22	17	19	29	14	22	30	11	15	26	29	7	26	23	12
14	-	18	7	7	-	8	27	22	-	-	-	-	24	24	22	28	15	21	22	17	7	29	8
-	14	-	-	-	14	-	-	13	27	20	5	23	8	22	7	22	20	13	19	26	28	12	18
-	8	-	-	-	28	-	-	10	20	11	24	6	29	10	11	15	27	22	30	16	20	18	14
10	24	21	25	17	21	8	28	-	12	15	8	26	6	28	9	27	14	17	24	16	7	8	5
10	-	16	7	22	-	26	20	11	-	-	-	-	28	7	28	19	6	24	7	23	24	5	26
25	-	7	14	18	-	25	8	23	-	-	-	-	22	7	15	13	6	26	26	8	6	9	11
9	-	14	20	28	-	9	28	9	-	-	-	-	17	24	24	23	15	21	22	24	20	22	11
19	-	13	5	28	-	17	13	26	-	-	-	-	14	27	29	9	26	19	20	6	20	6	5
8	9	29	23	16	19	15	12	9	28	5	19	15	-	24	25	11	8	28	15	26	14	13	21
12	5	22	10	10	8	13	18	25	10	24	26	9	28	-	24	24	-	29	13	27	11	21	16
30	10	21	5	18	28	6	6	22	15	10	22	5	8	11	-	29	13	8	19	15	18	-	29
7	15	6	6	18	26	15	11	21	18	28	12	21	14	19	17	-	21	26	29	25	21	11	-
9	25	25	14	13	29	25	12	17	18	8	7	21	27	-	24	28	-	16	5	19	19	28	12
19	24	10	9	5	13	17	9	16	8	28	10	11	12	28	29	17	12	-	-	-	-	25	17
7	27	9	6	25	29	26	21	20	10	22	10	30	27	29	14	24	8	-	-	-	-	26	12
7	14	24	8	22	20	27	8	12	27	29	17	21	17	9	28	16	14	-	-	-	-	14	6
17	25	29	19	17	26	13	19	18	13	18	7	29	20	6	15	12	11	-	-	-	-	16	17
20	15	25	23	16	24	11	6	26	14	14	20	10	9	25	-	24	22	24	28	19	10	-	6
13	5	26	9	29	15	27	22	9	10	26	19	28	28	10	24	-	6	21	17	8	21	26	-