Locomotory Advantages to Flapping Out of Phase

E. Kanso · P.K. Newton

Abstract The interactions between two fish swimming side by side are examined. An inviscid fluid model is employed to argue that flapping out of phase can lead to locomotory advantages for both fish. Two effects are examined: (a) the effect of the cooperation between the wakes of the two fish in increasing their forward swimming velocities and (b) the effect of the coupling between the lateral oscillations of the fish and the circulations around their bodies in enhancing their forward swimming motion. The models are stripped to their simplest level in order to highlight these effects.

Keywords Swimming · Dynamics · Vorticity

Introduction

Fish swimming and fish schooling are topics that have attracted tremendous interest recently with the potential of enabling novel engineering applications such as the design of biologically-inspired vehicles that move and steer by changes of shape, rather than by direct propulsion, in a coordinated way similar to that observed in natural fish schools. Significant success has been demonstrated in developing mathematical models that correctly capture the body-fluid interactions in individual fish; see, for example, the pioneering work of [1–3] and [4, 5] as well as the recent work [6–10]

E. Kanso (✉) · P.K. Newton
Aerospace and Mechanical Engineering,
University of Southern California, 854 Downey Way,
Los Angeles, CA 90089-1191, USA
e-mail: Kanso@usc.edu

for a reduced modeling approach. While these results are promising, they have yet to be scaled to produce physics-based models for fish schooling that properly encode the body-fluid interactions (see [10] for a step in this direction). Schooling is generally explained as an evolutionary behavior resulting from several factors such as protection from predators, profiting from a larger search pattern in the quest for food, and advantages for social and mating activities, [11]. While some hydrodynamic advantages for fish schooling have been put forward, see, for example, [12] and the more recent experimental work in [13], there is still much to learn about the role of the fluid in motion coordination and in enhancing the locomotory efficiency of the school.

In this note, we consider the simpler problem of two fish swimming side by side and argue that flapping out of phase has locomotory benefits arising from the effects of the wakes as well as from the interactions between the lateral motions of the fish and the circulations around their bodies (produced by flapping). Note that the problem considered is reminiscent to that of flapping flags. Michelin and Llewellyn Smith [14] reported that, for small separation distances, two flags flapping side by side passively coordinate their deformations to flap in phase and as the separation distance increases, they flap out of phase. Ristroph and Zhang [15], on the other hand, studied two flags in tandem (placed behind each other) and showed that for small separation distances, the two flags flap out of phase while for larger separations, they synchronize to flap in phase. Of course, flags are passive objects, whereas fish are not.

Each fish is assumed to swim due to lateral oscillatory motions (flapping) of its body and tail. This excludes the cetacean mammals and other fish species
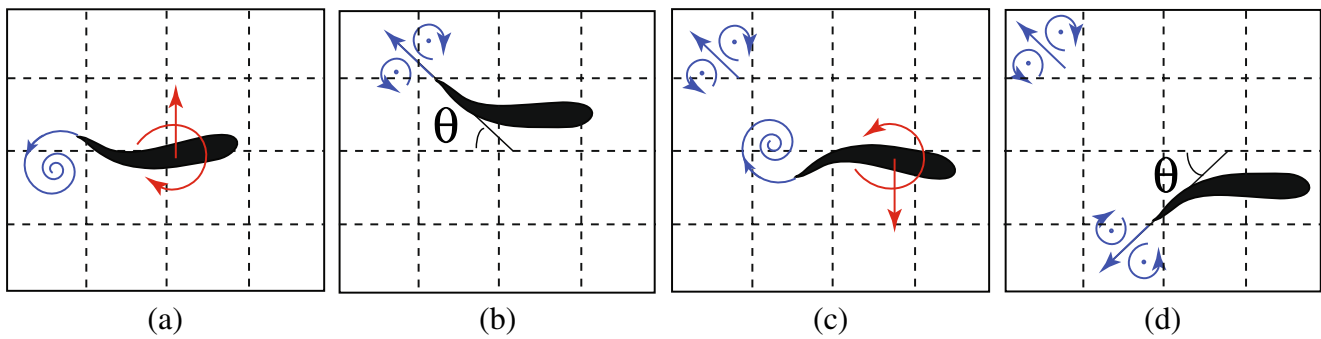
Fig. 1 Fish swimming due to lateral flapping. In (**a**), the fish—flapping clockwise—sheds a counter-clockwise vortex at its tail. The circulation around the fish body is represented by the circular red arrow. The straight red arrow shows the direction of the lateral motion of the fish. In (**b**), the fish reaches a maximum flapping amplitude θ at which instant the net circulation around its body is zero (actually, both the flapping and lateral velocities are instantaneously zero). At this instant a vortex is formed on the other side of the tail that pairs with the vortex shown in (**a**) to form a vortex dipole that propagates obliquely away from the fish. The fish continues to flap now in the counter-clockwise direction as shown in (**c**) and a clockwise vortex begins to form at its tail. When the fish reaches its maximum flapping amplitude θ again, shown in (**d**), a counter-clockwise vortex forms and pairs with the clockwise vortex to propagate away from the fish body as a dipole

that use paired fin motions for propulsion; see, e.g., [16] for a discussion of the various swimming modes. The fish sets the surrounding fluid into unsteady motion creating vortical structures, typically at its tail as shown in Fig. 1. The nonlinear coupling between the shed vortices and the fish flapping motion is responsible for the fish net locomotion or swimming. Unlike commonly employed models where the fish sheds one vortex with every half cycle of its flapping motion, we propose that the fish sheds a dipole (a vortex pair) in each half cycle that propagates away from the fish tail with a velocity oblique to the fish swimming velocity as shown in Fig. 1(b) and (d). Indeed, there is experimental and numerical evidence that fish and flapping airfoils shed a dipole every half cycle of their oscillation (see, e.g., [Personal Communication, 17, 18], and [19]). In this work, we are not attempting to model the gen-

eration of these dipole structures with the thin shear layer surrounding the fish body, but we insert them 'by hand', so-to-speak, in the surrounding inviscid fluid with the understanding that their genesis arose from the boundary layer. Their subsequent evolution and interactions with the fish body are our main concern. We start from the swimming mechanism shown in Fig. 1 and argue using a purely inviscid theory and other simplifying assumptions that, for two fish swimming side by side as depicted in Fig. 2, flapping out of phase leads to locomotory advantages.

Analysis

Consider two fish swimming side by side with identical yet out-of-phase flapping strokes as shown in Fig. 2.
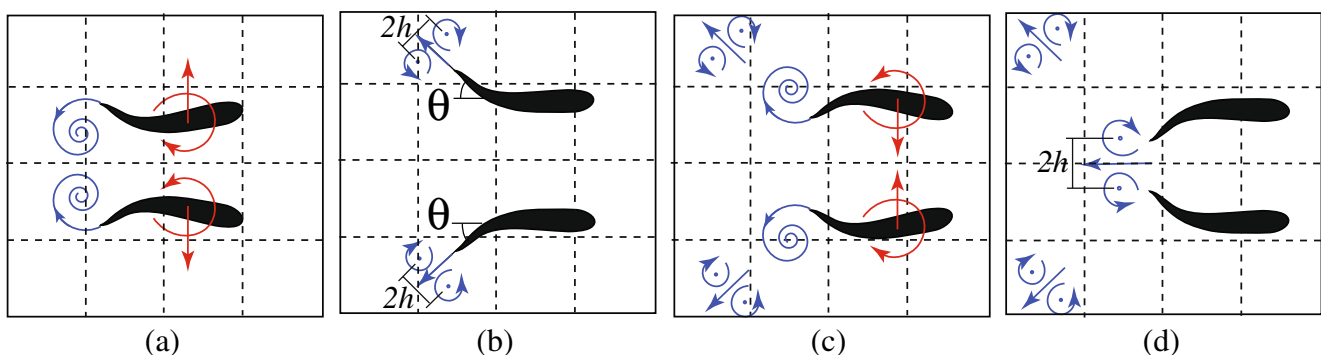

Fig. 2 Two fish swimming side by side and flapping out of phase. Each fish is using a flapping stroke similar to that shown in Fig. 1 with the exception that, during the 'closing' phase of the stroke, the wakes generated by the two fish merge into one dipole as shown in (**d**) in a way reminiscent to the wake generated by the clap phase in the famous Weis-Fog clap and fling mechanism

Each flapping stroke is assumed to generate a vortex structure similar to that detailed in Fig. 1. The difference here is that these vortex structures interact together and with the fish bodies in a non-trivial way that typically depends on the flapping motion, the proximity of the fish, and the viscosity in the fluid. Indeed, we propose that in the closing portion of the flapping stroke—when the lateral motion of the fish drives them towards each other as shown in Fig. 2(c)—the vortex structures generated by the two fish merge into one dipole shown in Fig. 2(d) that propagates away from the fish in the opposite direction to their swimming direction in a way reminiscent of the clap phase in the famous Weis-Fog clap and fling mechanism (see, e.g., [20] and references therein).

When the separation distance between the two fish is large compared to the dimensions of the fish, each fish can be modeled using a point vortex singularity (representing the 'bound' vorticity which compensates for the 'shed' vorticity to maintain conservation of circulation) with periodically-varying strength and lateral motion, thus tremendously simplifying the mathematical analysis. For concreteness, let (x, y) denote the inertial coordinates in the plane of motion. It is more convenient for what follows to introduce the complex coordinates $z = x + iy$ (where $i = \sqrt{-1}$) and the complex conjugate $\bar{z} = x - iy$. The fish are now modeled simply by their bound vorticity, i.e., as point vortices located at $z_{k, k=1,2}$ (say $z_1(0) = -z_2(0) = ia$ at time $t = 0$) with equal and opposite strengths $\Gamma_1 = -\Gamma_2 = \Gamma$ due to flapping out of phase. Let's ignore the fish wakes for the sake of elucidating the role of the coupling between the fish lateral oscillations and the circulations around their bodies in enhancing the forward swimming motion. That is, for now treat the fish as two isolated point vortices (no wakes) in an infinite fluid domain. The complex stream function $\omega(z)$ representing the two fish is given by

$$\omega(z) = \sum_{k=1,2} -\frac{i\Gamma_k}{2\pi} \ln(z - z_k). \tag{1}$$

The (conjugate of) complex fluid velocity $\bar{u} = u_x - iu_y$ at an arbitrary point z that does *not* coincide with any of the point vortices is given by $u_x - iu_y = \dfrac{d\omega}{dz}$, whereas the velocity induced at a point vortex is given by

$$\dot{\bar{z}}_k = \frac{d}{dz}\left[\omega(z) + \frac{i\Gamma_k}{2\pi}\ln(z - z_k)\right]_{z=z_k}. \tag{2}$$

More precisely, the vortex velocities are given by (see, e.g., [21])

$$\dot{\bar{z}}_1 = -\frac{i\Gamma_2}{2\pi}\frac{1}{z_1 - z_2}, \qquad \dot{\bar{z}}_2 = -\frac{i\Gamma_1}{2\pi}\frac{1}{z_2 - z_1}. \tag{3}$$

In the case when $\Gamma_2 = -\Gamma_1 = \Gamma$ is constant and there is no prescribed lateral motion, equation (3) affords an explicit solution where both vortices move along a straight line (in the positive x-direction for Γ negative). If Γ is periodic in time (with period T) the vortices move forward and backward in the x direction without exhibiting a net translation. That is, alone, the fact that the circulation around each fish is time dependent is inconsequential to the net locomotion of the system. It is the coupling between Γ being periodic in time and the lateral oscillations of the fish that results in locomotory benefits to both fish. Indeed, let $y_1(t) = a + Y(t)$ and $y_2(t) = -a - Y(t)$ oscillate following a (prescribed) periodic function of time $Y(t)$. Since the motion in the y-direction is prescribed, only the equations governing the change in x_1 and x_2 are relevant. From the symmetry of the problem, one has $x_1 = x_2$ which leaves the equation

$$\dot{x}_1 = \frac{\Gamma(t)}{2\pi}\frac{1}{(y_1(t) - y_2(t))} = \frac{1}{4\pi}\frac{\Gamma(t)}{a + Y(t)}, \tag{4}$$

whose solution is given by

$$x_1(t) = \frac{1}{4\pi}\int_0^t \frac{\Gamma(\tau)\,d\tau}{a + Y(\tau)}. \tag{5}$$

The net displacement in the x-direction can be interpreted as a geometric phase (see, e.g., [22]) over the *shape* space of periodic functions $\Gamma(t)$ and $Y(t)$. The special case $\Gamma(t) = \dot{Y}(t)$ corresponds to a degenerate subspace that results in no net translation in the x-direction. Indeed, in this case, the displacement after one period of oscillation of Γ and Y is zero,

$$x_1(T) = \frac{1}{4\pi}\int_{Y(0)}^{Y(T)} \frac{dY}{a + Y} = 0. \tag{6}$$

Consider now the class of sinusoidal functions which, without loss of generality, can be parameterized by

$$\Gamma(t) = \gamma \sin(wt + \alpha), \qquad Y(t) = b \sin(wt) \tag{7}$$

Substitute into equation (5) and use a standard trigonometric identity together with the result in equation (6) to get

$$x_1(T) = \frac{1}{4\pi}\int_0^T \frac{\gamma \cos\alpha \sin(wt)\,dt}{a + b \sin(wt)}. \tag{8}$$

Clearly, $\alpha = 0$ is optimal in terms of distance traveled per one flapping cycle. This is in agreement with the physics of the problem where both the circulation

around each fish and the lateral displacement of the fish are zero simultaneously when the fish reaches its maximum flapping amplitude. Figure 3 shows the motion of one fish for $a = 1$, $b = 0.1$, $\gamma = -1$, $\alpha = 0$ and $w = 2\pi$ (the second fish undergoes a similar motion).

We now examine the effect of the wakes on the motion of the fish. To isolate this effect, we ignore the fish lateral oscillations and bound vorticity, the fish are now modeled as particles of equal mass m_f initially placed at $z_1 = -z_2 = ia$. Assume that two dipoles are shed instantaneously at the end of the 'opening' phase as shown in Fig. 2(b) and one dipole is shed instantaneously at the end of the 'closing' phase as in Fig. 2(d). Each dipole consists of two point vortices of equal and opposite strengths of intensity $\Gamma^d = -\int_0^{T/2} \Gamma \, dt$, which are initially separated by a distance $2h$ as shown in Fig. 2(b) and (d). The linear impulse (or momentum) of the fish-fluid system is given by (see, e.g., [8])

$$p_x = -\sum_i \Gamma_i^d y_i + m_f(v_1)_x + m_f(v_2)_x,$$

$$p_y = \sum_i \Gamma_i^d x_i + m_f(v_1)_y + m_f(v_2)_y. \quad (9)$$

From symmetry, the two fish have the same velocity in the x-direction $(v_1)_x = (v_2)_x = v_x$ and equal and opposite velocities in the y-direction $(v_1)_y = -(v_2)_y = v_y$. The linear momentum of the body-fluid system is assumed to remain constant during shedding which means that, at the time of the shedding t_{shed}, one has

$$p_x\big|_{t_{shed}^+} = p_x\big|_{t_{shed}^-}, \qquad p_y\big|_{t_{shed}^+} = p_y\big|_{t_{shed}^-}. \quad (10)$$

The conservation of momentum gives rise to a jump in the translational velocity of the fish at each shedding event: $[\![v_x]\!] = v_x\big|_{t_{shed}^+} - v_x\big|_{t_{shed}^-}$ and $[\![v_y]\!] = v_y\big|_{t_{shed}^+} - v_y\big|_{t_{shed}^-}$. By virtue of equations (9) and (10) and from symmetry, the jump in v_y at the end of both the opening and closing phases is zero ($[\![v_y]\!] = 0$) while the jump in the x-direction is given by

$$[\![v_x]\!]_{opening} = \frac{2\Gamma^d h \cos\theta}{m_f}, \qquad [\![v_x]\!]_{closing} = \frac{2\Gamma^d h}{m_f}. \quad (11)$$

The zero jump in the y-component of the velocity reflects the fact that the wakes induce no lateral motion on the fish. Meanwhile, the jump in the x-component of the velocity is larger in the closing phase reflecting the fact that the merging of the wakes during the closing phase into one dipole structure maximizes the velocity imparted to the fish in the direction of forward swimming.
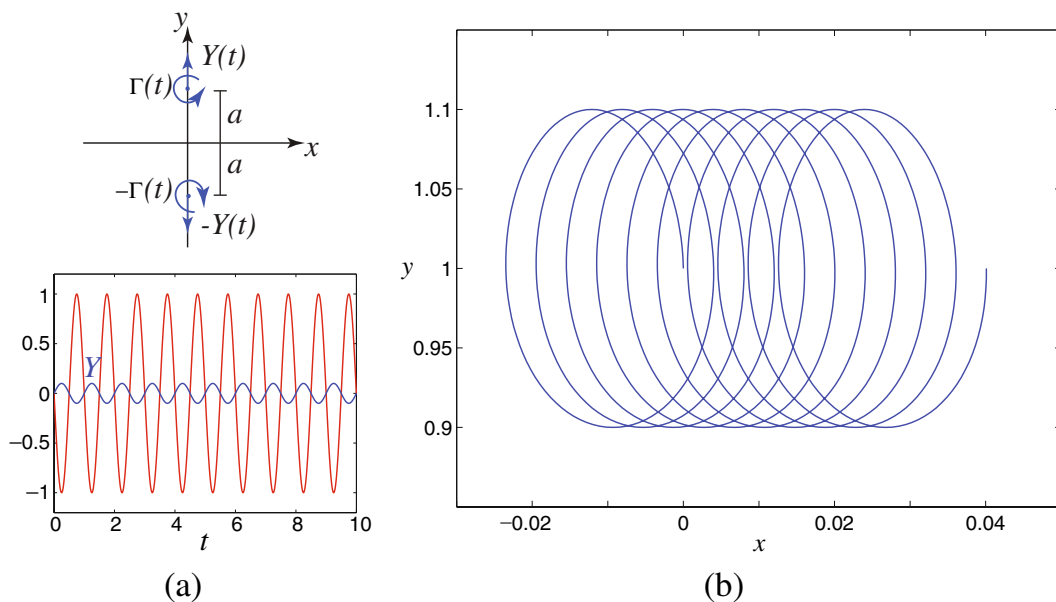

Fig. 3 Fish motion produced by the coupling between their bound vorticity and their lateral oscillations during out-of-phase flapping. (**a**) shows the initial conditions (*top*) and the prescribed bound vorticity $\Gamma(t)$ and lateral oscillations $Y(t)$ of the top fish while (**b**) shows the response in the (x, y)-plane where $x(t)$ is given by equation (8). The parameter values are set to $a = 1$, $b = 0.1$, $\gamma = -1$, $\alpha = 0$ and $w = 2\pi$. The integration time is 10. It is clear from (**b**) that there is a net, though small, displacement in the x direction. Note that the displacement is the same over each period ($x(T = 1) \approx 0.004$) and it could be cast in terms of geometric phases over a *shape* space parameterized by the periodically-varying bound vorticity $\Gamma(t)$ and the lateral oscillations $Y(t)$

The net locomotion gained by the two fish over one-period of their flapping motion due solely to the jump in velocities at the shedding events is given by

$$x_1(T) = [\![v_x]\!]_{opening} \frac{T}{2} + [\![v_x]\!]_{closing} \frac{T}{2} = \frac{\Gamma^d h T(1+\cos\theta)}{m_f}.$$

(12)

A direct comparison of this displacement with that obtained in equation (8) due to the geometric phase effect shows that the wake effect is dominant. Indeed, consistently with the parameter values used in Fig. 3, let $T = 1$, $\Gamma^d = -\int_0^{1/2} \Gamma dt = 1/\pi$, $h = 0.1$, and estimate the mass of the fish using a circular shape of radius $R_f = 0.2$ to get $m_f = 2\rho_f \pi R_f^2 = 2(0.2)^2 \pi$ (note that the fluid density is normalized to $\rho_f = 1$ throughout and the factor 2 in m_f is due to the fact that we account for both the actual mass of the fish and added mass due to presence of the fluid). Also, assume that the maximum flapping angle is equal to $\theta = \pi/4$. To this end, the net displacement in equation (12) ($x_1(1) \approx$ 0.200) is roughly 50 times that due to the geometric effect ($x_1(1) \approx 0.004$) shown in Fig. 3.

We conclude this section by noting that when the two fish flap in phase, one can no longer argue that the wakes merge together in the closing phase to produce one dipole as shown in Fig. 2(d), and the shed dipoles, both in the opening and closing phases, induce a lateral motion (in the y-direction) on the fish while their effect on the forward motion (in the x-direction) is diminished in the closing phase. We leave the details of such analysis as an exercise to the interested reader.

Conclusions

The main goal of this note is to suggest a simple mechanism by which a pair of fish can gain locomotory advantages using out-of-phase flapping. It also suggests that through clever body maneuvers designed to produce advantageous wakes, a school of fish, viewed as a system of oscillators coupled via fluid interactions, could in principle gain energetic advantages by working together. Of course, the question of how the arguments presented here in favor of flapping out of phase scale to large fish schools remains open. In fact, Weihs put forward several hydrodynamic advantages when a large fish school flaps in phase [12] while Childress favored out-of-phase flapping, arguing that flapping in phase produces a net lateral flow which would require fish within the school to develop a lateral force in order

to maintain position [1]. Both Weihs and Childress's arguments are based on modeling the wake of each fish as an inverse (thrust) von Kármán street whereas the wake model proposed and employed here consists of dipole shedding with every half-period of the flapping stroke.

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