B Online Supplemental Material

This online Appendix complements "Political Disagreement and Information in Elections," by Alonso and Câmara. Section B.1 provides conditions such that cutoffs v_1^A and v_2^A, defined by Corollary 1, are finite. Section B.2 emphasizes that the result in Proposition 1 fundamentally depends on the endogenous properties of optimal experiments. Section B.3 discusses the difference between political disagreement and polarization.

The remaining sections present extensions of the basic model. Section B.4 extends the model to heterogeneous prior beliefs. Section B.5 considers i.i.d. preference shocks. Section B.6 studies the role of post-election information. Section B.7 consider costly policy experiments. Section B.8 extends our model to the case of competition in information provision.

B.1 Finite cutoffs

In this section we provide conditions such that cutoffs v_1^A and v_2^A, defined by Corollary 1, are finite. We start with a technical result that extends Lemma A.1, presented in Appendix A.

Lemma B.1 *Define*

$$G\left(a, b, v^A\right) \equiv \frac{F\left(b+v^A\right)-F\left(a+v^A\right)}{f\left(a+v^A\right)}, \tag{20}$$

*where F and f satisfy **(A1)**. Fix any $a, b, c \in \mathbb{R}$. Then:*

(i) $G(a,b,v^A)$ is non-increasing in v^A, and it is strictly decreasing if f is strictly log-concave and $a \neq b$;

(ii) There exists a $v^{A\prime} < +\infty$ such that for any $v^A \geq v^{A\prime}(a,b)$ we have $G(a,b,v^A) \leq b-a$; and a $v^{A\prime\prime} < +\infty$ such that for any $v^A \leq v^{A\prime\prime}(a,b)$ we have $G(a,b,v^A) \geq b-a$;

(iii) If $b > a$ and $\lim_{v^B \to -\infty} \frac{f(v^B)}{F(v^B)} = \infty$, then there exists a $v^{A\prime} > -\infty$ such that for any $v^A \leq v^{A\prime}$ we have $\frac{F(b+v^A)-F(a+v^A)}{F(a+v^A)} \geq c$;

Proof of Lemma B.1:

Lemma A.1 proves part (i). To prove parts (ii) and (iii), we first recall some useful facts. Log-concavity of f and full support of F imply that F is also log-concave. Therefore, the ratio $\frac{f(v^B)}{F(v^B)}$ is everywhere decreasing. Log-concavity of f also implies that f is unimodal.

Full support then ensures the existence of a finite v^{A*} such that f is weakly increasing for $v^A \leq v^{A*}$ and weakly decreasing for all $v^A \geq v^{A*}$.

Part (ii): Fix any $a, b \in \mathbb{R}$, and define $v^{A\prime} = v^{A*} - \min\{a, b\}$. This implies that, for any $v^A \geq v^{A\prime}$, f is weakly decreasing in $\left[\min\left(a + v^A, b + v^A\right), \max\left(a + v^A, b + v^A\right)\right]$. We now show that for any $v^A \geq v^{A\prime}$, $G(a, b, v^A) \leq b - a$. The results hold trivially if $a = b$. Suppose $b > a$. Then, for any $v^A \geq v^{A\prime}$, we have that monotonicity of f implies that

$$G(a, b, v^A) = \frac{\int_{a+v^A}^{b+v^A} f(y)\, dy}{f(a + v^A)} \leq \frac{\int_{a+v^A}^{b+v^A} f(a + v^A)\, dy}{f(a + v^A)} = b - a.$$

Now suppose $b < a$. By the same argument, for any $v^A \geq v^{A\prime}$, we have

$$G(a, b, v^A) = \frac{\int_{a+v^A}^{b+v^A} f(y)\, dy}{f(a + v^A)} = \frac{-\int_{b+v^A}^{a+v^A} f(y)\, dy}{f(a + v^A)} \leq \frac{-\int_{b+v^A}^{a+v^A} f(a + v^A)\, dy}{f(a + v^A)} = b - a,$$

which concludes the proof.

Part (iii): Fix any $a, b, c \in \mathbb{R}$ such that $b > a$. Define $v^{A\prime} = v^{A*} - b$ so that for any $v^A \leq v^{A\prime}$, f is weakly increasing in $[a + v^A, b + v^A]$. This implies that for any $v^A \leq v^{A\prime}$ we have

$$\frac{F(b + v^A) - F(a + v^A)}{F(a + v^A)} = \frac{\int_{a+v^A}^{b+v^A} f(y)\, dy}{F(a + v^A)} \geq \frac{\int_{a+v^A}^{b+v^A} f(a + v^A)\, dy}{F(a + v^A)} = \frac{f(a + v^A)(b - a)}{F(a + v^A)}.$$

Assuming $\lim_{v^B \to -\infty} \frac{f(v^B)}{F(v^B)} = \infty$, the right hand side is unbounded for fixed a and b. As $\frac{f(v^B)}{F(v^B)}$ is decreasing, it follows that there is a $\hat{v^A} > -\infty$ such that $\frac{f(a+v^A)(b-a)}{F(a+v^A)} \geq c$ for all $v^A \leq \hat{v^A}$, concluding the proof. ■

Proposition B.1 *Let v_1^A and v_2^A be the cutoffs defined by Corollary 1. Then,*

(a) if political disagreement D is concave, then $v_1^A \leq v_2^A < +\infty$;

(b) if political disagreement is not maximized at the prior belief, and either D is locally convex at the prior belief or $\lim_{v^B \to -\infty} \frac{f(v^B)}{F(v^B)} = \infty$, then $v_2^A > -\infty$.

Proof of Proposition B.1:

To proof this proposition we will make use of the following fact. Since W is differentiable at the prior, we can apply the second part of Corollary 1 from Alonso and Câmara (2016a). In our setup, it implies that there is no value of persuasion if and only if

$$\langle \nabla W(p), q - p \rangle \geq W(q) - W(p), \qquad q \in \Delta(\Theta).$$

Since $\nabla W(p) = f\left(D(p) + v^A\right) \nabla D(p)$, and $f > 0$, we can rewrite the previous condition as

$$\langle \nabla D(p), q - p\rangle - G\left(D(p), D(q), v^A\right) \geq 0, \qquad q \in \Delta(\Theta), \tag{21}$$

Part (a): From Lemma B.1(ii), we know that for each q there exists a $v^{A\prime}(q) < \infty$ such that for any $v^A \geq v^{A\prime}(q)$ we have $G(D(p), D(q), v^A) \leq D(q) - D(p)$. Hence, the LHS of (21) is weakly greater than $\langle \nabla D(p), q - p\rangle - D(q) + D(p)$. Since D is concave, this term is weakly positive. Therefore, for any $v^A \geq \check{v}^A \equiv \sup v^{A\prime}(q)$ the LHS of (21) is positive for all $q \in \Delta(\Theta)$ and there is no value of persuasion.

Part (b): Suppose that there exists belief q^+ such that $D(q^+) > D(p)$. We will show that under the conditions of the proposition there exists a $v^A > -\infty$ such that persuasion is valuable by constructing an experiment that yields a payoff strictly higher than a completely uninformative experiment.

Suppose first that D is locally strictly convex at the prior. Local strict convexity guarantees the existence of q^+ and q^- with $D(q^+) > D(p)$ and $\lambda q^+ + (1-\lambda)q^- = p$ such that

$$\lambda D(q^+) + (1-\lambda) D(q^-) > D(p). \tag{22}$$

We now show that there exists v^A such that

$$\lambda F(D(q^+) + v^A) + (1-\lambda) F(D(q^-) + v^A) > F(D(p) + v^A), \tag{23}$$

so that this experiment outperforms a completely uninformative experiment. Lemma B.1(ii) then guarantees the existence of $v^{A\prime\prime}$ such that $G(a, b, v^A) \geq b - a$ for $v^A \leq v^{A\prime\prime}$. For any $v^A \leq v^{A\prime\prime}$ then we have

$$\begin{aligned}
& \lambda\left(F(D(q^+) + v^A) - F(D(p) + v^A)\right) + (1-\lambda)\left(F(D(q^-) + v^A) - F(D(p) + v^A)\right) \\
= \;& \lambda f(D(p) + v^A) G(D(p), D(q^+), v^A) + (1-\lambda) f(D(p) + v^A) G(D(p), D(q^-), v^A) \\
\geq \;& f(D(p) + v^A)\left(\lambda(D(q^+) - D(p)) + (1-\lambda)(D(q^-) - D(p)\right) > 0,
\end{aligned}$$

where the last inequality follows from (22). This establishes (23).

Suppose now that $\lim_{v^B \to -\infty} \frac{f(v^B)}{F(v^B)} = \infty$. Select a belief q^+ such that $D(q^+) > D(p)$ and $\lambda \in (0,1)$ and q^- such that $\lambda q^+ + (1-\lambda) q^- = p$. We now study the value to the IP that with probability λ induces posterior q^+, and with probability $1-\lambda$ induces posterior q^- in the majority group. If $D(q^-) \geq D(p)$ then the experiment outperforms an uninformative experiment for all v^A. Suppose $D(q^-) < D(p)$. Lemma B.1(iii), guarantees the existence of $v^{A\prime} > -\infty$ such that $\frac{F(D(q^+)+v^A)-F(D(p)+v^A)}{F(D(p)+v^A)} \geq c$ with $c = \frac{1-\lambda}{\lambda}$ holds for any $v^A \leq v^{A\prime}$. Then

$$\begin{aligned} & F(D(q^+)+v^A) - F(D(p)+v^A) + \frac{1-\lambda}{\lambda}\left(F(D(q^-)+v^A) - F(D(p)+v^A)\right) \\ \geq\; & F(D(q^+)+v^A) - F(D(p)+v^A) - \frac{1-\lambda}{\lambda} F(D(p)+v^A) \geq 0. \end{aligned}$$

Which shows that (23) holds for any $v^A \leq v^{A\prime}$. ■

B.2 Non-Optimal Experiments

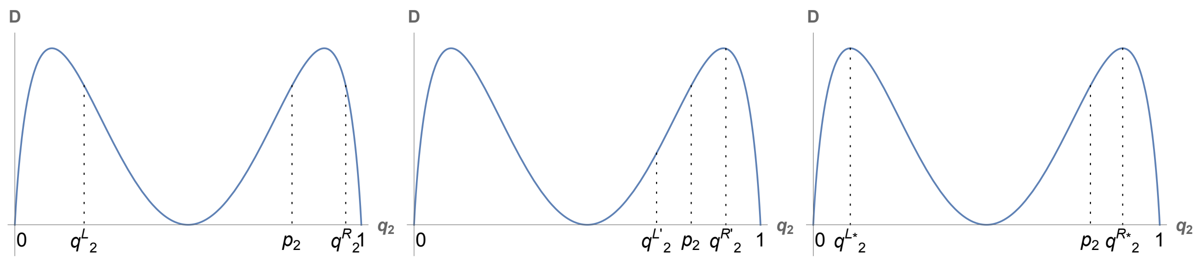
In this section we emphasize that the result in Proposition 1 is not a simple corollary of Lemma 1. While Lemma 1 follows from log-concavity of f and holds for *any* experiment π, the result in Proposition 1 is also rooted on the endogenous properties of optimal experiments.

To illustrate our point, we provide a simple example in which inequality (8) fails when we consider non-optimal experiments. Consider Example 3 from Section 2.4. Disagreement as a function of beliefs is depicted in Figure 8, assuming players have a common prior belief p_2. Consider two alternative experiments: π induces posterior beliefs q_2^L and q_2^R, while π' induces posterior beliefs $q_2^{L'}$ and $q_2^{R'}$ — see Figures 8(a) and (b). Note that experiment π is Blackwell more informative than π'. However, π does not change disagreement, while π' creates a lottery over disagreement. Suppose v^B is normally distributed. It then follows from our results that the IP strictly prefers the less informative (but riskier in terms of disagreement generated) experiment π' over π if v^A is sufficiently low, and the opposite is true if v^A is sufficiently high. Inequality (8) is then violated. Note that neither experiment is optimal — the optimal experiment for all values of v^A is illustrated in Figure 8(c).

B.3 Political Disagreement vs. Polarization

It is important to highlight that our notion of "political disagreement" is different from the notion of "polarization" present in many papers in the literature.[27] That is, an increase in

[27] Although there are different definitions of polarization in the literature, here we define polarization as the Euclidean distance between the policies supported by the candidates as in Dixit and Weibull (2007).



(a) More informative π (b) Less informative π' (c) Optimal experiment π^*

Figure 8: Example of non-optimal experiments violating inequality (8).

political disagreement does not imply an increase in polarization, and vice versa.

To illustrate this point, change the spatial policy model from Section 2.4 as follows: all voters share the same Euclidean policy payoff $u(x, \theta) = -|x - \theta|$, but have different prior beliefs. In this case, the optimal policy equals the expected median of the state, $x^{i*}(q^i) = M[\theta|q^i]$. Political disagreement then becomes

$$D^A(q^A, q^B) = -\sum_{\theta' \in \Theta} q^A_{\theta'} \Big[\big| M[\theta|q^A] - \theta' \big| - \big| M[\theta|q^B] - \theta' \big| \Big].$$

To see that information might strictly increase polarization and strictly decrease political disagreement, let $\Theta = \{-2, -1, +1, +2\}$ and consider priors $p^A = (.06, .8, .1, .04)$ and $p^B = (.04, .1, .8, .06)$. At the prior belief, candidate A prefers policy -1 while candidate B prefers $+1$. The original degree of political disagreement is $D^A(p^A, p^B) = 36/25$. Initial political disagreement is high because voters in group A are very confident that the state is -1 and not $+1$. Consider a binary experiment that simply reveals if the state is in the partition $\{-1, +1\}$ or $\{-2, +2\}$. When the experiment reveals that the state is in partition $\{-2, +2\}$, updated beliefs become $q^A = (.6, 0, 0, .4)$ and $q^B = (.4, 0, 0, .6)$. Candidates' preferred policies change to -2 and $+2$. Hence, the information results in more polarized policies. However, there is now a lower degree of political disagreement, $D^A(q^A, q^B) = 4/5$. Although policies are more polarized (farther away from each other), voters in group A now believe that there is a much higher chance that the opposing policy championed by candidate B might be the correct policy. In a nutshell, optimal policies are further apart, but voters suffer a smaller loss from appointing the rival candidate.

B.4 Extension: Belief Disagreement

In this section we extend our model to the case where voters have heterogeneous prior beliefs about the state.

Consider the general model from Section 2. Assume that voters in the same group share a common prior belief, but voters in opposite groups openly disagree over the likelihood of state θ. That is, voters in group i have a common prior belief $p^i = (p^i_1, \ldots, p^i_N)$ in the interior of the simplex $\Delta(\Theta)$, but prior beliefs differ across groups, $p^A \neq p^B$. To simplify presentation, assume that each party shares the beliefs of its affiliates. Preferences and prior beliefs are common knowledge — voters "agree to disagree." If we interpret θ as describing the mapping between policy x and outcomes, then different prior beliefs represent differences in voters' views of which outcomes are produced by the different government policies.

Given priors p^A and p^B, policy experiment π and signal realization s, let q^A and q^B be the respective posterior beliefs of each group. Since party filiation of candidates, prior beliefs and the experiment are common knowledge, voters can correctly infer the policy each candidate would implement if elected, $x^{i*}(q^i) \equiv \arg\max_{x \in X} \sum_{\theta \in \Theta} q^i_\theta u^i(x, \theta)$. So we can rewrite (1), the expected payoff of voter i if candidate j wins, as

$$\mathcal{U}^{ij}(q^A, q^B, v^A, v^B) = v^j + \sum_{\theta \in \Theta} q^i_\theta u^i(x^{*j}(q^j), \theta).$$

A voter from group i votes for the candidate from group A if and only if

$$\begin{aligned} \mathcal{U}^{iA}(q^A, q^B, v^A, v^B) &\geq \mathcal{U}^{iB}(q^A, q^B, v^A, v^B) \\ \iff \sum_{\theta \in \Theta} q^i_\theta \left[u^i(x^{*A}(q^A), \theta) - u^i(x^{*B}(q^B), \theta) \right] &\geq -(v^A - v^B). \end{aligned} \tag{24}$$

The LHS of (24) captures the degree of political disagreement. Disagreement from the point of view of voters in group A is

$$\mathcal{D}(q^A, q^B) \equiv \sum_{\theta \in \Theta} q^A_\theta \left[u^A(x^{*A}(q^A), \theta) - u^A(x^{*B}(q^B), \theta) \right]. \tag{25}$$

Since group A forms a majority, they are decisive: after a signal realization that induces posterior beliefs q^A and q^B, candidate A wins the election if and only if

$$\mathcal{D}(q^A, q^B) \geq -v^A + v^B.$$

Candidate A wins the election with a probability that increases in the degree of political disagreement — candidate A has a "policy advantage" because a majority of voters believe she has not only the "correct" preference, but also the "correct" belief, and hence she will implement the "correct" policy.

We now rewrite $\mathcal{D}$. Let $r_\theta \equiv \frac{p^B_\theta}{p^A_\theta}$ and $r \equiv \{r_\theta\}_{\theta\in\Theta}$ capture the likelihood ratio of prior beliefs. Alonso and Câmara (2016a, Proposition 1) show that independently of the experiment π and its realization s, we can rewrite q^B solely as a function of the belief of voters in group A,

$$q^B_\theta = \frac{q^A_\theta r_\theta}{\langle q^A, r\rangle}. \tag{26}$$

Therefore, we can express $\mathcal{D}(q^A, q^B)$ as a function of q^A only,

$$D(q^A) \equiv \mathcal{D}\left(q^A, q^A \frac{r}{\langle q^A, r\rangle}\right). \tag{27}$$

Since $v^B \sim F$, the majority candidate wins with probability

$$W(q^A; v^A) \equiv F(D(q^A) + v^A). \tag{28}$$

We can then replace (4) with (28) and show that all the results in Sections 3 and 4 and continue to hold.

B.4.1 Application: Budget Allocation

We conclude this section by presenting an application where the degree of political disagreement is endogenously given by the degree of belief disagreement, as measured by the relative entropy. Although any informative signal decreases the expected political disagreement, the IP still finds it optimal to implement a partially informative signal when the expected valence of candidate A is sufficiently low.

Consider the following budget allocation model. The government has one dollar to allocate among $N \geq 2$ different government projects. Let $x_n \geq 0$ represent the amount of money allocated to project n, such that the budget balances. Thus $X = \{x \in [0,1]^N | \sum_{n=1}^N x_n = 1\}$ and the vector $x = (x_1, \ldots, x_N) \in X$ represents a complete government budget.

There is uncertainty about the payoff derived from investing in each project. To simplify presentation, we consider the case where only one project is beneficial to voters — only one project can increase voters' payoff — while investment in any other project delivers a payoff of zero. Formally, there are N possible states, $\theta \in \Theta \equiv \{1, \ldots, N\}$, and citizens share a common payoff function: if the realized state is $\theta = n$ then voters receive a logarithmic payoff $\ln(x_n)$. In other words, $u(\theta, x) = \sum_{n=1}^{N} \mathbb{1}(n, \theta) \ln(x_n)$, where $\mathbb{1}(n, \theta) = 1$ if $\theta = n$, and $\mathbb{1}(n, \theta) = 0$ if $\theta \neq n$. If voter i has belief $q^i = (q_1^i, \ldots, q_N^i)$, then budget x delivers an expected policy payoff $\sum_{n=1}^{N} q_n^i \ln(x_n)$, where we apply the convention $0 \ln(0) = 0$. The logarithmic utility implies that each voter prefers the budget to be allocated proportionally to his own beliefs — the preferred budget x^{i*} of voter i is simply $x_n^{i*} = q_n^i$ for all n.

Political disagreement (25) becomes

$$\mathcal{D}^A(q^A, q^B) = \sum_{n=1}^{N} q_n^A \left[\ln(q_n^A) - \ln(q_n^B)\right] = \sum_{n=1}^{N} q_n^A \ln\left(\frac{q_n^A}{q_n^B}\right) \equiv D_{KL}(q^A || q^B).$$

The relative entropy $D_{KL}(q^A || q^B)$, or Kullback-Leibler distance[28] between probability distributions q^A and q^B, is a measure of the belief disagreement between the two groups. Therefore, in our electoral model, the degree of of *political disagreement* is given directly by the level of *belief disagreement* as measured by the relative entropy: from the point of view of the majority group, D_{KL} measures the difference in the expected payoff derived from the different policies favored by each group. Political disagreement is zero if and only if both groups share common beliefs, and it is increasing in the extent of belief disagreement between the groups. Figure 9(c) presents a binary-state example of disagreement as a function of the posterior belief of voter A, given a particular pair of prior beliefs.

It is a known fact that any informative experiment, on average, decreases the relative entropy. Consequently, information *always* decreases average political disagreement in this budget allocation model. Nevertheless, Lemma B.2 below shows that it is possible to design an experiment with at least one realization that strictly increases disagreement under the following condition:

Condition C1: *Interior prior beliefs* (p^A, p^B) *are such that* $r_\theta - \ln(r_\theta) \neq r_{\theta'} - \ln(r_{\theta'})$ *for at least one pair of states* $\theta, \theta' \in \Theta$.

[28] Although it is not formally a distance measure, the relative entropy is a measure of the inefficiency of assuming that the probability distribution is q^B when the true distribution is q^A. See Cover and Thomas (*Elements of Information Theory*, Second Edition, John Wiley & Sons, 2006, Chapter 2) for a discussion.

Lemma B.2 *If condition **(C1)** holds, then for any $\delta \in (0,1)$ the IP can design an experiment π with realization s^+ such that: (i) s^+ strictly increases political disagreement, and (ii) s^+ occurs with probability δ.*

Proof of Lemma B.2: Suppose condition **(C1)** holds for priors p^A and p^B in the interior of the simplex $\Delta(\Theta)$. Then there are states $\theta_H, \theta_L \in \Theta$ such that $r_{\theta_H} + \ln(r_{\theta_H}) > r_{\theta_L} + \ln(r_{\theta_L})$. Fix any probability $\delta \in (0,1)$. For a sufficiently small ϵ, construct an experiment π with realization space $\{s^+, s^-\}$ as follows:

$$\begin{aligned} Pr[s = s^+|\theta_H] &= \delta\left(1 + \frac{\epsilon}{p^A_{\theta_H}}\right), \\ Pr[s = s^+|\theta_L] &= \delta\left(1 - \frac{\epsilon}{p^A_{\theta_L}}\right), \end{aligned}$$

and $Pr[s = s^+|\theta] = \delta$ for all other $\theta \in \Theta$. Note that signal s^+ occurs with probability δ and results in a posterior q^+ such that: $q^+_{\theta_H} = p^A_{\theta_H} + \epsilon$, $q^+_{\theta_L} = p^A_{\theta_L} - \epsilon$, and $q^+_{\theta} = p^A_{\theta}$ for all other states. Moreover, using the notation $\ln(r) = \{\ln(r_\theta)\}_{\theta\in\Theta}$, we have

$$\begin{aligned} D(q^+) &= \ln(\langle q^+, r\rangle) - \langle q^+, \ln(r)\rangle \\ &= \ln(\langle p^A, r\rangle + \epsilon(r_{\theta_H} - r_{\theta_L})) - \langle p^A, \ln(r)\rangle - \epsilon(\ln(r_{\theta_H}) - \ln(r_{\theta_L})) \\ &= \ln(1 + \epsilon(r_{\theta_H} - r_{\theta_L})) - \epsilon(\ln(r_{\theta_H}) - \ln(r_{\theta_L})) + D(p^A). \end{aligned}$$

First note that if $\epsilon = 0$, then $D(q^+) = D(p^A)$. Second, $\left.\frac{\partial D(q^+)}{\partial \epsilon}\right|_{\epsilon=0} = r_{\theta_H} - r_{\theta_L} - \ln(r_{\theta_H}) + \ln(r_{\theta_L}) > 0$. Therefore, $D(q^+) > D(p^A)$ for any $\epsilon > 0$ sufficiently small. Consequently, signal s^+ occurs with probability δ and strictly increases disagreement, concluding the proof. ■

Condition **(C1)** is violated if priors are common, in which case $r_\theta - \ln(r_\theta) = 1$ for all states. Nevertheless, condition **(C1)** holds generically, where genericity is interpreted over the space of pairs of prior beliefs.

The policy advantage of candidate A derives solely from the belief disagreement among voters, thus full information disclosure is never optimal, $v^A_1 = -\infty$. Moreover, any information disclosure always decreases the expected disagreement. Does the IP ever benefit from disclosing some information? The answer is yes if the incumbent politician is sufficiently incompetent.

Proposition B.2 *In the budget allocation model, if* $\lim_{v^B \to -\infty} \frac{f(v^B)}{F(v^B)} = \infty$ *and condition* ***(C1)*** *holds, then there exists a finite cutoff* v_2^A *such that a partially informative is optimal iff* $v^A < v_2^A$*.*

Proof of Proposition B.2: We will show that v_1^A and v_2^A from Corollary 1 are such that $v_1^A = -\infty < v_2^A < \infty$, which implies that partial information disclosure is optimal if and only if $v^A < v_2^A$. Condition **(C1)** implies that political disagreement is not maximized at the prior belief (Lemma B.2). Proposition B.1(b) shows that if political disagreement can be increased and $\lim_{v^B \to -\infty} \frac{f(v^B)}{F(v^B)} = \infty$, then $v_2^A > -\infty$. Proposition B.1(a) also shows that if D is concave (which is the case here) then $v_2^A < \infty$. Therefore v_2^A is finite. Full information disclosure is never optimal since it decreases disagreement to zero with certainty, thus $v_1^A = -\infty$. ■

Loosely speaking, condition $\lim_{v^B \to -\infty} \frac{f(v^B)}{F(v^B)} = \infty$ (which holds, for instance, in F is a Normal Distribution) implies that the IP's payoff W becomes a "very convex" function of beliefs when v^A is low. Consequently, although information always decreases average *political disagreement*, information can increase average *victory probability* if the majority candidate is sufficiently incompetent.

Figure 9 illustrates political disagreement D in our budget allocation model, in an example with a binary state $\Theta = \{0, +1\}$, with $q_2 = Pr(\theta = 1)$. Using this disagreement, Figures 10 and 11 illustrate the IP's payoff. Figure 10 illustrates how increasing v^A can change the overall curvature of W, assuming F follows a Normal Distribution. Figure 11 depicts the concave closure of W for different values of v^A. Consider the case in which v^A is low, given by Figure 11(a). The concave closure $\widetilde{W}$ is a straight line in the set of beliefs $q_2 \leq \bar{q}_L$, it is W itself in $q_2 \in [\bar{q}_L, \bar{q}_R]$, and it is a straight line in the set $q_2 \geq \bar{q}_R$. Consequently, if the prior belief is in the set $p_2 \leq \bar{q}$, then the optimal experiment induces posterior beliefs $q_2 = 0$ and $q_2 = \bar{q}_L$. If the prior belief is sufficiently close to the maximum feasible disagreement, $p_2 \in [\bar{q}_L, \bar{q}_R]$, then no experimentation is optimal. If the prior belief is in the set $p_2 \geq \bar{q}_R$, then the optimal experiment induces posterior beliefs $q_2 = \bar{q}_R$ and $q_2 = 1$. Optimal experiments become less informative as we increase v^A. In Figure 11(c), no information disclosure is optimal for all prior beliefs. Finally, note that a fully informative experiment is never optimal, since it minimizes disagreement with certainty.

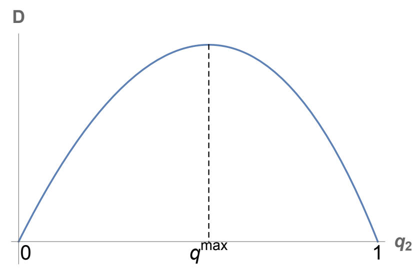
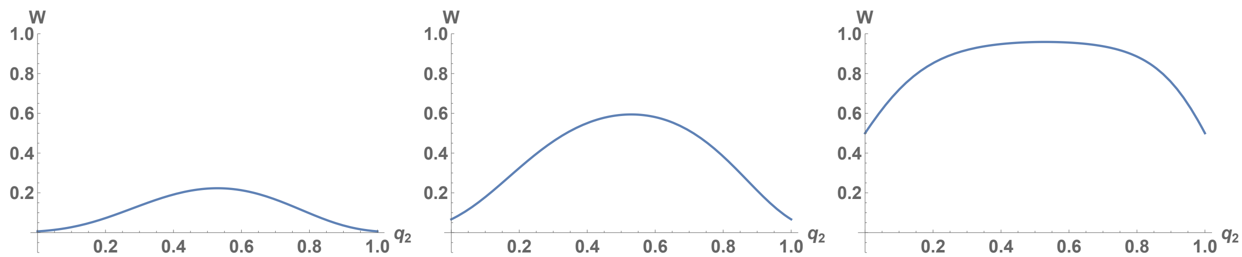


Figure 9: Budget Allocation Model: Political disagreement as a function of belief, with $\Theta = \{\theta_1, \theta_2\}$, $q_2 = Pr(\theta = \theta_2)$.



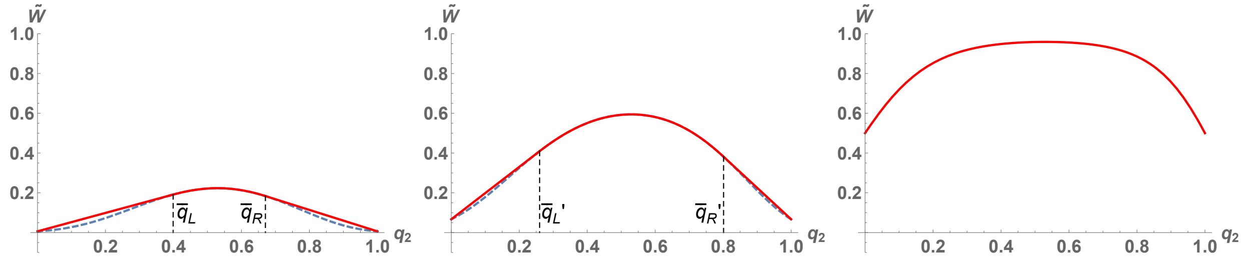
(a) Low Values of v^A (b) Intermediate Values of v^A (c) High Values of v^A

Figure 10: Effects of v^A on victory probability W, using disagreement D from Figure 9.

B.5 Independent Shocks

In the basic model we assume that all voters receive the same valence shock. Now suppose the opposite: there is no aggregate uncertainty over valence, only individual uncertainty. To this end, suppose there is a measure one of voters, where fraction $\alpha \in (1/2, 1)$ of voters form group A, and $(1 - \alpha)$ form group B. Each voter draws an i.i.d. shock from F. Since shocks are i.i.d., there is no longer a representative voter and we must take into account the behavior of voters from group B.

From (3), let $D^A(q) \equiv \sum_{\theta \in \Theta} q_\theta \left[u^A(x^{*A}(q), \theta) - u^A(x^{*B}(q), \theta) \right]$ be the political disagreement from the point of view of voters in group A. Similarly define the political disagreement from the point of view of voters in B, $D^B(q) \equiv \sum_{\theta \in \Theta} q_\theta \left[u^B(x^{*B}(q), \theta) - u^B(x^{*A}(q), \theta) \right]$. Given posterior belief q, a fraction $F(D^A(q) + v^A)$ of voters from group A vote for candidate A, while fraction $F(-D^B(q) + v^A)$ of voters from group B vote for candidate A. Candidate A wins if he receives at least half of the votes and loses otherwise. The expected payoff of



(a) Low Values of v^A (b) Intermediate Values of v^A (c) High Values of v^A

Figure 11: Concave closure of W from Figure 10.

the IP supporting candidate A is

$$W(q; v^A) = \begin{cases} 1 & \text{if} \quad \alpha F(D^A(q) + v^A) + (1-\alpha)F(-D^B(q) + v^A) \geq 0.5 \\ 0 & \text{if} \quad \alpha F(D^A(q) + v^A) + (1-\alpha)F(-D^B(q) + v^A) < 0.5 \end{cases}$$

The IP faces a new trade off. Increasing disagreement $D^A(q)$ increases the number of majority voters supporting candidate A, but increasing disagreement $D^B(q)$ decreases the number of minority voters supporting A. Nevertheless, one can show that Corollary 1 continues to hold: There are cutoffs $v_1^{A\prime}$ and $v_2^{A\prime}$ such that full information disclosure is optimal if $v^A < v_1^{A\prime}$, a partially informative experiment is optimal if $v_1^{A\prime} < v^A < v_2^{A\prime}$, and no information disclosure is optimal if $v_2^{A\prime} < v^A$.[29] Moreover, since there is no aggregate uncertainty, one can use the techniques in Alonso and Câmara (2016b) to compute the optimal experiment.

B.6 The Role of Post-election Information

Our basic model posits that informative experiments of the underlying state will only be available to voters and politicians before voters are called to a vote. This can be the case if there is ample time prior to an election to evaluate alternative policies, while the elected politician would need to quickly implement the chosen policy once in office. In other cases, however, politicians may have access to additional information after the election, for instance as uncertainty naturally resolves. The policy-motivated politician would then take this new information into account when selecting his policy.

[29] Cutoff $v_2^{A\prime}$ is the minimum v^A such that the majority candidate wins with probability one without an experiment, $W(p; v_2^{A\prime}) = 0.5$. Cutoff $v_1^{A\prime}$ is the supremum v^A such that the majority candidate loses with probability one for every posterior belief, $v_1^{A\prime} = \sup\{v^A \in \mathbb{R} | W(q; v^A) < 0.5 \text{ for all } q \in \Delta(\Theta)\}$. In this case any experiment is optimal, including full information disclosure.

Suppose that after the election, but before choosing a policy, the elected politician has access to additional information. Formally, the elected politician observes a signal τ that is correlated with the underlying state θ. Let Z be the set of possible signal realizations, with $z \in Z$. If at the time of the election players have belief q, then it is straightforward to compute updated belief $q(z)$ obtained through Bayes' rule after observing realization z of signal τ. It is also straightforward to compute policy $x^{*i}(q(z))$ that would be chosen by candidate i. Consequently, at the time of the election, policy disagreement (3) from the point of view of voters in majority group A becomes

$$D(q) \equiv E_\tau[u^A(x^{*A}(q(z)), \theta)|q] - E_\tau[u^A(x^{*B}(q(z)), \theta)|q], \tag{29}$$

where $E_\tau[\cdot|q]$ denotes the expectation over the distribution of posterior beliefs that is obtained through Bayes' rule from belief q and the signal τ. The results of Lemma 1, Proposition 1 and Corollary 1 are directly applicable to this definition of the degree of political disagreement. In essence, even if the elected politician has access to better information after the election, it is still the case that improving the majority's valence advantage makes it less likely for the IP to engage in persuasion.

However, access to better information will affect our results in Sections 6.1 and B.4.1. In these sections, voters have the same underlying preferences, hence they agree on the optimal policy if they know the true state. In this case, if the post-election signal τ is perfectly informative of the state, then the IP cannot benefit from disclosing information prior to the election. This is so because expected political disagreement is zero as the same policy will be implemented regardless of the identity of winner. Consequently, Proposition 4 and B.2 no longer hold. However, if the post-election signal is not fully informative, then one can redefine conditions on Proposition 4 and B.2 such that the IP can benefit from disclosing some information prior to the election.

B.7 Costly Policy Experiments

Our basic model assumes that experiments are costless. If every experiment is equally costly, then, whenever the IP decides to implement an experiment, it implements the optimal experiment we describe. The only change is that the IP implements an experiment only if the value of persuasion is higher than the fixed cost of implementation.

What if different experiments have different costs? Following Gentzkow and Kamenica[30], suppose that the cost of an experiment is given by the expected relative entropy of the beliefs that it induces. Consequently, more-informative experiments are more costly. In this case, one can show that our results from Section 3 continue to hold. However, perfectly revealing a state is infinitely costly. Therefore, full information disclosure and upper-censoring experiments are never optimal.

B.8 Competition

In this paper, we have focused on the case in which the incumbent has the monopoly over the information that reaches voters. What happens if, after the incumbent has released the result of the experiment, the challenger can further uncover some information? That is, what happens if the challenger can launch her own public investigation? We now provide some results on this "competition-in-persuasion" game.

The timing of this extended game is as follows. The incumbent party implements an experiment π, and its outcome becomes public. The opposing party then chooses an experiment, and its result becomes public.[31] The valence of the challenging candidate is realized and becomes public information. The election takes place.

First, consider the case in which, mirroring the incumbent's ability to experiment, the challenger has access to *every* experiment that is correlated with the state. The following result holds, independently of the shape of disagreement function D and valence distribution F.

Proposition B.3 *Suppose that F and D are continuous. If the challenger has access to every experiment that is correlated with the state, then there is always a subgame perfect equilibrium in which the incumbent selects a fully informative experiment.*

Proof of Proposition B.3: The proof has two steps.

Step 1) We first show that parties have opposing preferences over experiments. Fix any

[30]Gentzkow, M., and E. Kamenica (2014): "Costly Persuasion," *American Economic Review P&P*, 104(5), pp, 457-462.

[31]We believe that the most natural assumption for our model is to have the incumbent playing first. See Gentzkow and Kamenica (2016) for a model in which players choose experiments simultaneously.

$q \in \Delta(\Theta)$ and suppose that party A prefers experiment π over experiment π',

$$E_{\pi}[F(D(\tilde{q}) + v)] \geq E_{\pi'}[F(D(\tilde{q}) + v)]. \tag{30}$$

Then, trivially, party B prefers experiment π' over experiment π,

$$1 - E_{\pi'}[F(D(\tilde{q}) + v)] \geq 1 - E_{\pi}[F(D(\tilde{q}) + v)]. \tag{31}$$

Step 2) We now show that there is no profitable deviation for party A if it offers a fully informative experiment. Suppose that party A deviates to experiment π_A with realizations space S_A, and that for each $s_A \in S_A$, the challenger implements experiment $\pi_B(s_A)$, with realization space $S_B(s_A)$. If $\pi_B(s_A)$ is fully revealing for each $s_A \in S_A$, then voters always learn the state, and party A cannot gain by deviating to π_A. Consider a realization $\hat{s}_A$ that leads to a non-degenerate belief $\hat{q}$, and the best response of the challenger is an experiment $\pi^*_B(\hat{s}_A)$ that is not fully informative of the state. This implies that, given $\hat{q}$, the challenger prefers the partially informative experiment $\pi^*_B(\hat{s}_A)$ over a fully informative experiment, and, correspondingly, party A cannot be worse off with a fully informative experiment rather than $\pi^*_B(\hat{s}_A)$. Therefore, party A cannot gain from switching to experiment π_A, and providing a fully informative experiment is a subgame perfect equilibrium. ■

The intuition behind the result is straightforward. As any experimental outcome that increases the incumbent's victory probability must reduce the challenger's chances by the same amount, parties have opposite preferences over any given pair of experiments. That is, for any belief $q \in \Delta(\Theta)$, if party $\mathcal{A}$ prefers π over π', then party $\mathcal{B}$ must prefer π' over π. In particular, an optimal experiment by party $\mathcal{B}$ at belief q must necessarily minimize the victory probability of party $\mathcal{A}$. Consider, then, an incumbent's deviation from a fully informative experiment and a non-degenerate belief q induced by an outcome of this new experiment. The challenger's optimal response minimizes the incumbent's expected victory probability, so that, at belief q, the incumbent would instead (weakly) prefer a fully informative experiment. Therefore, the incumbent cannot gain by deviating to a less informative experiment.

The challenger's ability to carry out her own arbitrary investigation leads the incumbent to be fully transparent. In practice, however, the incumbent typically has access to a richer set of experiments than the challenger does since the incumbent directly controls the government. How do constraints in the challenger's experimentation alter Proposition B.3? To provide some insights into this question, suppose that, after the results of the incumbent's experiment become public, the challenger chooses whether or not to launch an investigation. If the challenger chooses not to investigate, then no further information about θ reaches the players. If the challenger launches the investigation, then with probability $\alpha \in (0,1)$ the investigation reveals the true state to all players, and with probability $(1-\alpha)$ the investigation is unsuccessful and no further information reaches the players. This investigation represents a full scrutiny of the experiment results.[32] We denote this fully informative experiment $\pi_{FI}(\alpha)$, where α is an exogenous technology parameter. Then we have the following result.

Proposition B.4 *Suppose that F and D are continuous, and the challenger has access to experiment $\pi_{FI}(\alpha)$, for some $\alpha \in [0,1)$. Then, $\pi^*(\alpha)$ is an equilibrium experiment by the incumbent if, and only if, $\pi^*(\alpha)$ is an optimal experiment when the challenger cannot experiment.*

Proof of Proposition B.4: The proof has two steps.

Step 1) We start with a preliminary result that shows equivalence between two auxiliary games. In the first game, the challenger cannot experiment — that is our benchmark model. In the second game, the challenger is not strategic: the challenger always launches investigation $\pi_{FI}(\alpha)$, independently of the choice of the incumbent. Then, it is immediate that experiment π_A is optimal for the incumbent in the first game if and only if it is optimal in the second game.

Step 2) Now, consider the extended game, in which the challenger is strategic. We now show that the challenger launches an investigation whenever the incumbent's experiment does not perfectly reveal the state. By contradiction, suppose that, in equilibrium, party

[32] For example, using the Freedom of Information Act, the challenger can file a request to force the incumbent to disclose all the information about the policy experiment. The IP can take actions to avoid disclosure, so the challenger has to go to court. Thus, α represents the probability that the court will actually force the IP to disclose the information. For instance, in the case of the launch of the ObamaCare website HealthCare.org, the IP repeatedly denied requests to further disclose information about enrollment numbers and security measures. Members of the opposing party then had to sue the government, in order to try to obtain the information.

A implements experiment π_A^*, and there is a realization $\hat{s}_A$ that leads to a non-degenerate belief $\hat{q}$, such that the challenger strictly prefers to not investigate. In this case, given $\hat{q}$, the challenger considers no further information disclosure strictly better than full information disclosure with probability α. Therefore, it has to be the case that, given $\hat{s}_A$, the incumbent would strictly benefit from further disclosing information. This contradicts the optimality of π_A^*.

Therefore, in any equilibrium, the challenger engages in further investigation if realization s_A does not fully disclose the state. Therefore, voters' posterior beliefs are equivalent to the posteriors in an auxiliary game in which they learn the state with probability α, independently of the choice of the incumbent, and, with probability 1-α, can rely only on the outcome of π_A^*. From Step 1, we know that π_A^* is optimal in this auxiliary game if and only if it is optimal in the benchmark game, when the challenger has no access to experiments. ■

This result follows again from the parties' opposite preferences: the challenger benefits from further disclosing information whenever the incumbent benefits from not further disclosing information. Consequently, the challenger experiments after every non-degenerate belief q induced by the incumbent's optimal experiment. The incumbent is effectively facing an environment in which voters will learn the state with probability α, while with probability $1 - \alpha$ voters have no alternative source of information. This implies that any experiment that is optimal when the challenger cannot experiment must also be optimal in this case.

Therefore, in equilibrium, voters have access to more information if the challenger has easier access to the government's information (a higher α). Understanding how different informational constraints of incumbent and challenger affect equilibrium outcome is an interesting question, but beyond the scope of this paper, so we leave it for future research.