Competence and Ideology*

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Abstract

We develop a dynamic repeated election model in which citizen-candidates are distinguished by both their ideology and valence. Voters observe an incumbent's valence and policy choices, but only know the challenger's party. Our model provides a rich set of novel results. In contrast to existing predictions from static models, we prove that dynamic considerations make higher valence incumbents more likely to compromise and win re-election, even though they compromise to more extreme policies. Consequently, we find that the correlation between valence and extremist policies rises with office-holder seniority. This result may help explain previous empirical findings. Despite this result, we establish that the whole electorate gains from improvements in the distribution of valences. In contrast, fixing average valence, the greater dispersion in valence associated with a high-valence political elite always benefits the median voter, but can harm a majority of voters when voters are sufficiently risk averse. We then consider interest groups or activists who search for candidates with better skills. We derive a complete theoretical explanation for the intuitive conjectures that policies are more extreme when interest groups and activists have more extreme ideologies, and that such extremism reduces the welfare of all voters.

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1 Introduction

While much of the literature on voting models has studied how politician and voter ideologies affect policy choice and electoral outcomes, politicians are also distinguished by fundamental characteristics—competence, character, or organizational efficiency—that all voters value independently of their ideology. Since Stokes (1963), several papers have explored this so-called *valence* dimension. Most of these models consider a single election, and assume that voters know either the valences of all candidates, or none. This paper develops a dynamic citizen-candidate repeated election model in which candidates are distinguished by both their ideology and their valence, and in every period the current incumbent runs election against a challenger drawn from the opposite party. Our model incorporates the feature that the electorate knows more about an incumbent than an untried opponent drawn from the opposing party. Having observed the incumbent while in office, the electorate can assess his valence and can forecast his future policy choices. The electorate is less well informed on the valence of the challenger and cannot precisely forecast his future policies.

Our analysis provides a rich set of results. In a general setting, we first provide conditions for the existence of a unique symmetric stationary equilibrium, thereby extending the analyses of Duggan (2000) and Bernhardt et al. (2009) to allow for stochastic heterogeneity in candidate valences. We then characterize how incentives to compromise vary with incumbent valence, proving that higher valence incumbents are more likely to compromise and win re-election, even though they compromise to more extreme policies. We find that the correlation between valence and extremist policies rises with office-holders seniority. In particular, our model can generate a negative correlation between valence and extremism in newly-elected officials, but a positive correlation between valence and extremism in re-elected officials. Later we explain how these results contrast with previous findings in the literature and how they may reconcile previous conflicting empirical findings.

We then study the welfare properties of valence. If the distribution of politician valences improves, all voters directly benefit from the higher office-holders' valence, but could be harmed because high valence incumbents can win with more extreme policies. Further, one suspects that because voters trade off valence and expected policy differently depending on their ideology, a unanimous ranking of valence will not exist. Despite these conflicting effects, we establish that the *whole* electorate benefits from any first order stochastic improvement in the distribution of valences. We then consider changes in the degree of valence heterogeneity among politicians. We find that fixing average valence, the median voter always benefits from the greater dispersion in valence associated with a high-valence political elite. In sharp contrast, greater dispersion in valence can harm a majority of voters—those with more extreme ideologies—when voters are sufficiently risk averse. These results highlight important welfare implications of selection and training of politicians, via the actions of parties and interest groups.

To address this issue, we extend our model to allow for the endogenous determination of the valence of challenging candidates. We suppose that interest groups or activist groups may engage in costly search to identify candidates with better skills. We find that interest groups with more extreme ideologies care less about identifying high-skilled candidates, searching less than more moderate interest groups, thereby reducing the welfare of all voters. We then provide conditions under which more extreme interest groups lead to more extreme expected policy outcomes. Thus, we provide a complete theoretical explanation for the idea that more extreme interest groups give rise to more extreme policies and that such extremism harms all voters. As we later explain, alternative explanations may be hard to derive.

The details of our model are as follows. The ideology and valence of citizen-candidates are independently distributed. Citizens are divided into two parties: a left-wing party comprised of citizens with ideologies to the left of zero, and a right-wing party that consists of citizens with ideologies to the right. Valence is initially private information, but an office holder's performance reveals her valence to the electorate. Each period, the office-holder chooses and implements a policy that is observed by voters. Voters evaluate the implemented policy with a symmetric and concave loss function, that is maximized if the policy coincides with their ideology. At the end of each period, the office holder retires with some exogenous probability; otherwise she decides whether to run for re-election or not. When an incumbent runs for re-election, the challenger is randomly drawn from the opposing party. Otherwise, two challengers, one randomly drawn from each party, run for election. Citizens are forward-looking: they vote for the candidate who provides the higher expected discounted utility if elected.

Our analysis focuses on symmetric stationary equilibria. We provide sufficient conditions under which there exists a unique equilibrium. The median voter is decisive and the equilibrium is completely summarized by thresholds that divide office-holders in three groups: centrists who adopt their preferred platforms and are able to win re-election, moderates who compromise to be able to win re-election, and extremists who adopt their extreme platforms and then are not able to win re-election—as in Duggan (2000). Here, however, *thresholds vary with a politician's valence.* Specifically, (i) the decisive median voter is willing to trade-off valence for policy, so that incumbents with higher valence can win re-election by adopting more extreme policies than lower valence incumbents, and (ii) both the compromise set and probability of winning re-election strictly increase in valence. Higher valence politicians have greater incentives to compromise because they can win re-election with more extreme policies and, more importantly, they incur higher costs from being replaced by an untried candidate. These higher costs reflect that the ideology of the high valence office holder who is indifferent to compromising is further from the likely policy choices of the challenger, and the new office-holder could have a lower valence.

Hence, we find opposing effects on the correlation between valence and policy extremism: higher valence incumbents can win re-election by adopting more extreme policies, inducing a *positive* correlation; but they are also more willing to compromise, inducing a *negative* correlation. We derive conditions under which, in equilibrium, the second effect dominates for first-term office holders: the expected degree of extremism in the policy choices of newly-elected representatives *falls* with valence. This negative correlation is driven by the extreme platform choices taken by lemons—newly-elected representatives with both low valence and extreme ideologies—who adopt extreme losing positions that reflect their underlying ideological preferences.

In contrast, the first effect dominates for the stationary distribution of a large congress as long as incumbents are likely to run for re-election. The comparison between newly-elected and re-elected office-holders also delivers the empirical implication that the correlation between valence and extremist policies *increases* with office-holders seniority, and indeed the sign of the correlation reverses for more senior incumbents. These results show how important it is to consider the implications of incentives in a dynamic framework, when investigating the correlation between valence and extremism. Stone and Simas (2010) observe that there are inconsistent empirical results in the literature examining the relationship between valence and policy. In the next section we suggest how empirical investigations could account for the dynamic considerations that we identify.

Our results on the correlations between valence and probability of winning the election, and between valence and extremism contrast with the simpler, albeit conflicting, predictions of existing static models. Models where voters know all candidate valences (Ansolabehere and Snyder (2000), Aragones and Palfrey (2002), Groseclose (2001)) predict a *negative* correlation between valence and extremism, and a *positive* correlation between valence and the probability of winning the election. The models where voters know no candidate's valence (Kartik and McAfee (2007), Callander and Wilkie (2007)) suppose an *exogenous* cost of compromising for candidates with character/valence, while candidates without character can costlessly locate moderately; thereby generating a *positive* correlation between character and extremism, and a *negative* correlation between character and the probability of winning the election.

We then investigate how the distribution of candidate valences affects expected policy outcomes and voter welfare. We first show that a first-order stochastic dominance improvement in the distribution of valences raises expected payoffs of *all* voters from an untried candidate. We then show that in the stationary distribution of officeholders, a first-order stochastic dominance improvement also increases the expected per-period payoff of *all* voters—voters with extreme ideologies still benefit even though higher valence incumbents locate more extremely than lower valence ones. This result precisely reflects that an untried challenger becomes more attractive after a first-order stochastic dominance improvement, so that to win reelection, an incumbent of *any* given valence level must compromise by more, implementing a more moderate policy, closer to the median voter.

We next explore the effects of second-order stochastic dominance shifts in the distribution of valences when ideologies are uniformly distributed and voters hold quadratic loss functions, and hence trade off in the same way between valence and expected policies. We prove that greater valence heterogeneity leads to more extreme expected policy outcomes. Nonetheless, all voters gain from such heterogeneity. Intuitively, all voters share the positive option value that the median voter places on electing a challenger who *might* have a high valence.

However, when loss functions are not quadratic, the decisive median voter trades off valence and expected policies differently from voters with more extreme ideologies—reflecting that candidates can locate further away from a more extreme voter. Numerically, we find that when voters are less risk averse, citizens with more extreme ideologies are more willing to forsake moderate platforms for higher valence, and so they gain even more than the median voter from the option of an untried challenger. Most obviously, with Euclidean preferences, the voter with the most extreme ideology is risk neutral over ideological gambles from two untried candidates. In sharp contrast, when voters are more risk averse, those with more extreme ideologies especially fear the policy gamble associated with untried candidates, and are less willing than the median voter to forsake low valence candidates with moderate platforms for the possibility of drawing a high valence candidate. As a result, a majority of voters (those with extreme ideologies) may prefer an economy of "average" politicians whose unique valence corresponds to the average valence in the economy with heterogeneity in valence.

Finally, we extend the model to allow interest groups to search to identify candidates with higher valence, and explore how an interest group's ideology affects their search efforts and equilibrium expected valence and policy. We find that interest groups with more extreme ideologies spend less effort on search, decreasing the expected utility of *all* voters. In essence, this result reflects that extreme interest groups are hurt less by low valence candidates who also have extreme ideologies, and who locate extremely as a result. This reduced search causes the median voter to set slacker re-election standards, thereby increasing expected extremism in the policies of re-elected officials; but it also induces more incumbents to compromise, reducing extremism. We find simple conditions under which the first effect dominates, so that less search effort by more extreme interest groups endogenously gives rise to policies that, on average, are more extreme.

Thus, we provide a complete theoretical explanation for the intuitive conjectures that politics driven by more extreme interest groups reduce the welfare of all voters and can give rise to more extreme policy choices. Indeed, alternative theoretical explanations may be hard to find. Consider, for instance, a standard model of elections in which the two candidates choose policy platforms before the election, and improve their chances of winning as contributions from their support groups grow. As long as support groups have single-peaked utilities, they become more willing to contribute to their candidate when the candidate supported by the opposing interest group chooses more extreme platforms. Provided utilities are concave, this incremental willingness to contribute exceeds the incremental willingness induced by the interest group's candidate moving toward the group's bliss point. Thus, each candidate's loss from moving away from the median platform exceeds the gain, with the result that platforms converge to the median in equilibrium.

The paper is organized as follows. We next relate our paper to the literature. Section 3 presents the model. Section 4 characterizes the equilibrium. Section 5 details how valence affects policy choices. Section 6 derives how changes in the valence distribution affect welfare. Section 7 endogenizes valence via search by interest groups. An appendix contains all proofs.

2 Related Literature

Theory. Since Stokes (1963), a vast literature has examined the role of valence in politics, primarily in single-election frameworks. The term valence is typically used to represent non-policy advantages of a politician, i.e., attributes that the electorate values independently of ideology and policy choices. Some valence characteristics can be observed by voters prior to an election (looks, charisma, rhetorical skills, etc.), while others can only be learned by voters after seeing the politician perform in office. Our paper focuses on the second group of valence characteristics. We have in mind attributes such as honesty (i.e., a politician is not corrupt), dedication, efficient use of public resources, competence in providing service for constituents and in managing non-policy issues, such as cutting red tape for local businesses and constituents, and attracting external resources to the district (both government-funded projects, and new businesses that provide local jobs).

In one class of models of valence, candidate valences are known before the election and campaign policies are binding. Ansolabehere and Snyder (2000) consider a setting with purely-office motivated candidates where the median voter's identity is public information. They show that if the valence advantage is not too large, then in the pure-strategy Nash equilibrium, the valence-advantaged candidate chooses a moderate policy and always wins the election. Aragones and Palfrey (2002) show that if, instead, the median voter position is unknown, then the valence-advantaged candidate adopts a mixed strategy with a distribution of policies closer to the expected median voter, and is more likely to win the election. Groseclose (2001) allows each candidate to have a known policy preference, symmetric around the median voter, and finds an analogous result: the valence-advantaged candidate chooses a pure-strategy policy that is closer to the expected median voter and is more likely to win.

More recent papers maintain the single-election framework, but find opposite results when a candidate's type is private information. In Kartik and McAfee (2007), by definition, candidates with "character" cannot compromise—their platform/policy is always their ideology—and such "character" is also assumed to raise the utility of all voters. Candidates without character are purely office motivated, and can costlessly locate moderately. As a result, Kartik and McAfee generate a positive correlation between character and extremism, and find that candidates without character are more likely to win. Callander and Wilkie (2007) consider a more general model in which campaign platforms are not binding and some candidates face a convex, but not infinite, cost of making campaign promises further from their preferred policy, and generate similar results. In their model, voters do not directly value character, but rather derive endogenously a preference for candidates with high lying costs due to the implications for subsequent policy choices. Callander and Wilkie observe that one can interpret this as a valence advantage. Callander (2008) investigates a model where candidates have private information about their motivation. Policy-motivated candidates have higher costs of compromising. In equilibrium, office-motivated candidates locate closer to the median voter and are more likely to win.

In sum, there is no consensus about the theoretical correlation between valence and extremism in single-election models. When valence is known by the electorate, there is a *negative* correlation between valence and extremism. Higher valence candidates exploit this advantage by moving closer to the median voter to increase the probability of winning. When valence is unknown, the assumed *exogenous* correlation between valence and the cost of compromising generates a *positive* correlation between valence and extremism, and a consequent lower probability that high valence candidates win the election.

Our model integrates valence into a version of the repeated election framework introduced by Duggan (2000). In Duggan (2000), voters observe an incumbent's policy choice in office and can forecast likely future actions, but have less information about challengers; and this gives rise to cutoff rules that characterize how the median voter selects between candidates, and the platforms that incumbents with different ideologies adopt. Other papers have extended Duggan's repeated election framework: Banks and Duggan (2008) consider a multi-dimensional policy space, Bernhardt, Dubey and Hughson (2004) introduce term limits, and Bernhardt et al. (2009) consider untried candidates drawn from multiple parties rather than at large. By integrating valence, we show how the endogenous cost of compromising and the re-election standard varies with valence levels, and derive the consequences for voter welfare.

Meirowitz (2007) examines valence in a very different repeated election two party model, in which each period one party draws an independent and identically distributed net valence advantage. Policy preferences and valence advantage are known before election. When in office, a party has private information about the feasible set of policies. Meirowitz finds that a party with a net valence advantage can select policies closer to its ideal point.

The seminal model of the role of interest groups in elections is Aldrich (1983), who studies a static model. More recently, Austen-Smith (1987) proposes a model that links interest groups contributions with campaign advertising; in contrast to our model, policy is fixed in his analysis (see also Baron (1994)). Grossman and Helpman (1996) study a model of interest group influence on policy in which: (i) interest groups can credibly commit to transfers contingent on the policies chosen by candidates, and (ii) there exist naive voters whose vote depends only on the campaign expenditures that follow from interest group contributions. In contrast, in our model, voters are rational and forward looking, and interest groups cannot sign contracts with candidates. Grossman and Helpman (1999) study a model of interest group endorsements, where some partisan voters who share the view of an interest group use its endorsement as a cue for voting choices. Prat (2002a) studies a model in which a single interest group is privately informed about candidate valences, and in equilibrium, the interest group contributes to high-valence candidates in exchange for favorable policies; Prat (2002b) extends this analysis to multiple opposing interest groups. Unlike this paper, the analysis is set in a common agency framework in which lobbies can make contributions contingent on policy choices.[1] Snyder and Ting (2008) study a different repeated model of elections with interest groups and find that re-election rates may be higher as interest groups become more extreme. Integrating over the different valence levels, we find that re-election rates may rise or fall with the extremism of interest groups; however, *conditional* on valence type, re-election rates in our model always increase with extremism of interest groups.

Empirics. In a recent article, Stone and Simas (2010) observe that the literature exploring the relationship between valence and policy uncovers inconsistent empirical results. In part, this reflects that it is a challenge to measure valence. The most often-used proxy for valence is a dummy identifying whether a candidate held an elected office prior to the election (see Jacobson, 1989): by construction, incumbents have high valence, whereas challengers may or may not. Ansolabehere, Snyder and Stewart (2001) use this proxy in a large study on 1996 House elections, finding that after controlling for voters' ideologies, incumbents are more moderate than open seat candidates, who, in turn, are more moderate than challengers. Groseclose (2001) and others cite Fiorina's (1973) evidence *against* the marginality hypothesis[2] as consistent with the idea that valence is negatively correlated to extremism. However, more recent researchers (Griffin (2006) and Ansolabehere, Snyder and Stewart (2001)) use different measures of the degree of electoral competition and find evidence in *favor* of the marginality hypothesis.

Our explicitly dynamic theoretical analysis highlights why these proxies for valence (incumbency and degree of electoral competition) may bias the estimation. Incumbents are endogenously different from challengers in many ways: voters know more about the characteristics of incumbents; incumbents who run for re-election are an endogenously-selected group (they were previously elected by voters *and* they chose to run for re-election); and incumbents implement policies *before* knowing the attributes of future challengers. On average, re-elected incumbents should have higher valence and more moderate policies than challengers. Therefore, using incumbency as a proxy for valence can bias the estimation in non-trivial ways, as it is correlated with valence *and* the endogenous policy choices of incumbents. Similar problems arise when one uses the degree of electoral competition and the marginality hypothesis to measure the impact of valence on extremism (e.g., incumbent's

[1] Further afield, Coate (2004) shows that contribution limits and matching public financing can be Pareto improving, even if campaigns financed by interest groups are informative, whereas Ashworth (2006) studies a model where interest groups are not ideological and demand favors from endorsed elected officials.

[2] According to the marginality hypothesis, electorally-weak incumbents tend to moderate more than electorally-strong incumbents.

vote share is endogenous, and Presidential vote share should be a proxy for voter's ideology, not for the valence of local politicians).

Our analysis may provide a road map for future empirical analyses of the intricate relation between valence, extremism and re-election. We propose three hypotheses for validation: (i) a positive correlation between policy extremism and valence for re-elected incumbents, (ii) a negative correlation between policy extremism and valence for first-timers, and (iii) a positive overall correlation between policy extremism and valence in Congress. Our predictions relate an incumbent's own valence[3] to the incumbent's choices when in office.[4] Hence, we contrast incumbents of different valence levels, and not incumbents and challengers as in most of the literature.

3 The Model

There is an interval $[-a, +a]$ of citizen candidates, each indexed by her private ideology x, distributed across society according to the c.d.f. F, with an associated single-peaked density f that is differentiable and symmetric about the median voter's ideology, $x = 0$. Ideologies are private information to candidates. Each citizen candidate is also characterized by a valence $v \in V = [V_L, V_H]$, where $0 \leq V_L \leq V_H$; all qualitative results extend if the valence set has a finite number of elements. Valence is uncorrelated with candidate ideology, and is distributed in the population according to the continuously differentiable c.d.f. G with support V. Valence is initially private information of a candidate before she holds office, but her performance in office reveals her valence to the electorate.

At any date t, an office holder with ideology x and valence v selects a policy $p(x, v) \equiv y$. The time-t utility of a citizen x depends on the implemented policy y, according to $u_x(y, v) = L_x(y) + v$, where $L_x(y) \equiv l(|x - y|)$ is a symmetric, single-peaked loss function that is $\mathcal{C}^2$, with $l' < 0$ and $l'' \leq 0$. We normalize $l(0) = 0$ without loss of generality. Note that u satisfies the single-crossing property: $\partial u_x/\partial y$ is increasing in x. Period utilities are discounted by factor $\delta \in (0, 1)$. In addition to the period utility $u_x(y, v)$, an office-holder receives an ego rent of $\rho \geq 0$ each period in office. Each period, after adopting her policy, with probability $q \in [0, 1)$

[3] Politicians' valences should be measured with proxies that are exogenous to voters' election decisions, such as the judgments of independent expert informants, as in Stone and Simas (2010). However, we focus on incumbent's own valence, and not on the valence difference between incumbent and challenger used by Stone and Simas.

[4] Policy extremism could be estimated using roll-call votes (see Poole and Rosenthal (1997)) and controlling for district median voter ideology via surveys or vote shares in presidential elections.

an incumbent receives an exogenous shock and cannot run for re-election. One can interpret this re-election shock as an unanticipated retirement for health or family issues.[5] An incumbent who did not receive this shock then decides whether to run for re-election or not.

Citizens are divided into two parties, a left-wing party L, and a right-wing party R. Party L consists of all citizen-candidates with ideologies $x < 0$, and party R has all possible candidates with ideologies $x > 0$. At date 0, an office holder is randomly determined. In any subsequent date-t majority rule election, an incumbent who runs for re-election faces a challenger from the opposing party. The valence of an untested challenger is not known by voters, but its distribution $G(v)$ is common knowledge. If the incumbent receives a re-election shock or decides not to run for re-election, then both parties compete with untried candidates.

We assume that citizens adopt the weakly dominant strategy of voting for the candidate whom they believe will provide them strictly higher discounted lifetime utility if elected—citizens vote sincerely. We assume that a voter who is indifferent between an incumbent and an untried challenger selects the incumbent. We will identify conditions under which the median voter is decisive in equilibrium. Focusing on symmetric equilibria, we assume that in elections between two untried candidates, the indifferent median voter randomizes, selecting each candidate with equal probability.[6]

In summary, the sequence of events at any period t is:

1. An office holder with valence v and ideology x implements her policy choice $y = p(x, v)$.
2. The incumbent realizes a re-election shock
   (a) With probability q, the incumbent cannot run for re-election;
   (b) With probability $1-q$, the incumbent is able to run for re-election and optimally chooses whether to run for re-election or not.
3. Opposing party draws an untried candidate.
4. Given the information about candidates (party affiliation for challengers; party affiliation, valence and past policy choices for incumbents), citizens vote for their preferred candidate.
5. The winning politician assumes office.

[5] All of our analysis holds for $q = 0$. We allow for $q > 0$ to capture the empirical fact that a small percentage of senior incumbents do not run for re-election for reasons that are outside of our model.

[6] In our working paper draft, we show that if a monotonicity condition on re-election cutoffs $c(v)$ holds, then all qualitative findings hold when the outcome of an election between two untried candidates is determined by the actions of the departing incumbent; i.e., an untried candidate from the exiting incumbent's party wins if and only if the incumbent would have won, had she run for re-election. Numerically, we establish that this monotonicity condition holds in two valence settings for power loss functions $L_x(y) = -|x-y|^z$, with $z \in [1, 4]$, and uniform or truncated normal distributions for ideologies.

4 Equilibrium

We focus on symmetric, stationary and stage-undominated perfect Bayesian equilibrium (PBE). We view symmetry and stage undomination as natural equilibrium requirements. Stationarity permits a tractable representation of equilibrium that highlights the features of the trade off between valence and ideology, and the equilibrium behavior of incumbents of different valence levels. A stationary policy strategy p for an office holder prescribes that at any time t, she selects a policy that depends only on her ideology x and valence v. The policy strategy is symmetric if $p(x,v) = -p(-x,v)$.

Under the three sufficient conditions of Theorem 1 that we state momentarily, there is a unique symmetric, stage-undominated, stationary perfect Bayesian equilibrium. This equilibrium is completely summarized by threshold functions $w, c : V \to (0, a)$, where for each $v \in V$, $0 < w(v) < c(v) < a$ for party R, and $-a < -c(v) < -w(v) < 0$ for party L. Incumbents from party R with valence v and centrist ideology $x \in [0, w(v)]$ and extremist incumbents $x \in (c(v), a]$ adopt their preferred policy $y = x$ when in office. Moderate politicians $x \in (w(v), c(v)]$ do not adopt their preferred policy, as they would then lose office. Instead, they compromise and adopt the most extreme policy that still allows them to win re-election, i.e., they locate at $w(v)$. In the next election, centrist and moderate incumbents are re-elected, while extremists choose not to run for re-election — they would lose for sure, and would prefer that a new face represent their party. The characterization is symmetric for party L. Figure (1) depicts the thresholds for an office-holder with valence v.

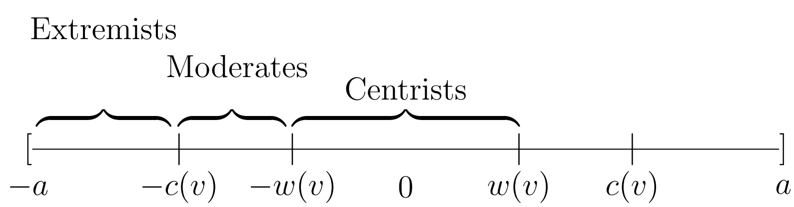

Figure 1: Thresholds for office-holders with valence v

Before we present the theorem, we describe the roles that each of these sufficient conditions serves. The first sufficient condition says that voters are not too risk averse. If this sufficient condition is violated and voters are too risk averse, the compromise set might not be connected: some incumbents with less extreme ideologies and some with very extreme ideologies might compromise, while a group of incumbents with intermediate ideologies choose not to compromise (see the discussion following Lemma A.5 in the appendix for details). Analytically, our sufficient condition holds for Euclidean and quadratic loss functions. Numerically, we solved the model for two valences, uniform and truncated normal distributions for ideologies, and loss function $L_x(y) = -|x-y|^z$. We find that the results are robust to higher levels of risk aversion, e.g., with $z = 3$ or 4.

To guarantee that equilibrium threshold functions are interior, $0 < w(v) < c(v) < a$, we also require that ego rents are not so high that a high valence incumbent with the most extreme ideology a would compromise to win re-election, and that valences are not so dispersed that low valence candidates cannot win re-election, even if they adopt the median voter's preferred policy, $y = 0$. These are natural requirements to avoid an uninteresting equilibrium in which low valence politicians always lose re-election and high valence politicians always win.

To prove equilibrium existence, we establish a fixed point of a function that maps the set of feasible median voter's expected payoff from an untried candidate back to itself. The key to establishing existence and uniqueness of equilibrium (see Lemma A.11) is to provide conditions under which the cutoff functions $w(v)$ and $c(v)$ display appropriate monotonicity properties. Lemma A.9 establishes that the functions $w(\cdot)$ are always monotone in that increases in the expected payoff from an untried candidate cause the median voter to set a tighter standard for each v. It further provides sufficient conditions under which the compromise functions $c(\cdot)$ display a common monotonicity property, so that a change in $w(\cdot)$ that strictly increases some $c(v)$ does not cause some other $c(v')$ to decrease. This common monotonicity property always holds when utility is quadratic, and, more generally, it holds for other utility functions whenever valence heterogeneity $V_H - V_L$, is sufficiently small.

Theorem 1 *Consider the class of symmetric, stationary, stage-undominated perfect Bayesian equilibrium (PBE). There exist bounds $M'' < 0$, $0 < M'''$, $0 < \overline{\rho}$ and $0 < \overline{v}$ such that if*

C1. *voters are not too risk averse, $M'' \leq l'' \leq 0$ and $|l'''| \leq M'''$;*

C2. *the ego rent is not too high, $\rho \leq \overline{\rho}$;*

C3. *valence heterogeneity is not too large, $V_H - V_L \leq \overline{v}$,*

then a unique equilibrium exists. The median voter is decisive, and every equilibrium is completely summarized by threshold functions $w, c : V \to (0, a)$, where for each $v \in V$, $0 < w(v) < c(v) < a$ for party R, and symmetric thresholds $-w(v)$ and $-c(v)$ for party L.

Numerically we solve the model for Euclidean, quadratic, cubic and quartic preferences, with two valence types and ideologies distributed as uniform and truncated normal, and verify that conditions **C1** to **C3** are not too restrictive. For the remainder of the paper we focus on equilibria with the properties described in Theorem 1. To simplify presentation we write $w_v \equiv w(v)$ and $c_v \equiv c(v)$.

4.1 Equilibrium Characterization

The proof of Theorem 1 in the Appendix characterizes the equilibrium behavior of voters and incumbents. Next we briefly describe key features of the equilibrium.

Let $U_x(y, v|w, c)$ denote the equilibrium continuation utility that a voter with ideology x expects to derive from a date-t office-holder with valence v who adopts platform y, if the incumbent is able to be re-elected each time she runs for office. Define $U_x^j(w, c)$ to be the equilibrium continuation utility that x expects to derive from an untried representative from party $j = L, R$, and let $\overline{U}_x(w, c) \equiv (U_x^R(w, c) + U_x^L(w, c))/2$ represent the payoff x expects from an untried challenger drawn from at large. Integrating over the possibility of an election shock, the continuation payoff that x expects from an incumbent who is able to win re-election is

$$\begin{aligned} U_x(y, v|w, c) &= u_x(y, v)(1-\delta) + \delta\left[q\frac{U_x^L(w, c) + U_x^R(w, c)}{2} + (1-q)U_x(y, v|w, c)\right] \\ &\equiv k\, u_x(y, v) + k\frac{\delta q}{(1-\delta)}\overline{U}_x(w, c), \end{aligned} \tag{1}$$

where $k \equiv \frac{(1-\delta)}{[1-\delta+\delta q]}$. An incumbent who would not win re-election implements as policy her own extreme ideology and steps down from office. Hence, voter x derives an expected continuation payoff from an extremist politician of $(1-\delta)u_x(y, v) + \delta\overline{U}_x(w, c)$.

For any citizen with ideology x, the PBE continuation expected value from electing a challenger from party L is:

$$\begin{aligned} U_x^L(w, c) &= \int_V \left\{ 2\int_{-w_v}^{0}\left[ku_x(y, v) + k\frac{\delta q}{1-\delta}\overline{U}_x(w, c)\right]dF(y)\right. \\ &+ 2\int_{-c_v}^{-w_v}\left[ku_x(-w_v, v) + k\frac{\delta q}{1-\delta}\overline{U}_x(w, c)\right]dF(y) \\ &+ \left. 2\int_{-a}^{-c_v}\left[(1-\delta)u_x(y, v) + \delta\overline{U}_x(w, c)\right]dF(y)\right\}dG(v). \end{aligned} \tag{2}$$

To understand this expression, recognize that for each challenger valence $v \in V$, the challenger's ideology y will turn out to be either (a) centrist, $y \in [-w_v, 0]$; (b) moderate, $y \in [-c_v, -w_v)$; or (c) extremist, $y \in [-a, -c_v)$. A centrist candidate adopts her own ideology as policy and is re-elected every time she runs for office, which provides an expected continuation payoff of $U_x(y, v|w, c) = ku_x(y, v) + k\frac{\delta q}{1-\delta}\overline{U}_x(w, c)$ to a voter with ideology x. A moderate candidate compromises to $-w_v$ and also wins re-election so that $U_x(-w_v, v|w,c) = ku_x(-w_v, v) + k\frac{\delta q}{1-\delta}\overline{U}_x(w,c)$. Finally, an extremist candidate adopts her own ideology, steps down from office, and is replaced by a new candidate. Hence, voter x derives an expected continuation payoff from an extremist politician of $(1-\delta)u_x(y,v)+\delta\overline{U}_x(w,c)$. We analogously define the payoff U_x^R that voter x expects to derive from a challenger from party R.

If the date-t incumbent from party L with valence v adopts platform y, then a voter with ideology x votes for the incumbent if and only if $U_x(y,v|w,c) \geq U_x^R(w,c)$. Similarly, voter x selects an incumbent from party R if and only if $U_x(y,v|w,c) \geq U_x^L(w,c)$. The median voter is decisive whenever an incumbent is re-elected if and only if the median voter prefers the incumbent to the challenger. That is, an incumbent from party L with valence v who adopts policy y is re-elected if and only if $U_0(y,v|w,c) \geq U_0^R(w,c)$, and an incumbent from party R is re-elected if and only if $U_0(y,v|w,c) \geq U_0^L(w,c)$.

The equilibrium functions $\{w, c\}$ obey the following recursive equations. First, for any $v \in V$,

$$U_0\left(w_v, v|w,c\right) = U_0^L\left(w,c\right) = U_0^R\left(w,c\right) = \overline{U}_0\left(w,c\right). \tag{3}$$

This recursive condition describes the voting rule for the decisive median voter. In particular, an incumbent with valence v who implements policy w_v leaves the median voter indifferent between the incumbent and a random challenger from the opposite party. In light of symmetry, the median voter is indifferent between random challengers from either party.

From equation (1) for the median voter, re-electing an incumbent with valence v who adopts policy w_v results in an expected discounted lifetime payoff of

$$U_0(w_v, v|w,c) = k(v + L_0(w_v)) + k\frac{\delta q}{1-\delta}\overline{U}_0(w,c). \tag{4}$$

From equilibrium condition (3) we have $U_0(w_v, v|w,c) = \overline{U}_0(w,c)$, so simplifying (4) yields

$$\begin{aligned} & U_0(w_v, v|w,c) = v + L_0(w_v), \\ \Rightarrow \quad & v + L_0(w_v) = U_0^L\left(w,c\right) = U_0^R\left(w,c\right) = \overline{U}_0\left(w,c\right), \forall v \in V. \end{aligned} \tag{5}$$

The second recursive equation describes the compromise decision for the marginal incumbent with valence v and ideology c_v. For any $v \in V$,

$$U_{c_v}\left(w_v, v|w,c\right) + \rho k = \left(v+\rho\right)\left(1-\delta\right) + \delta\overline{U}_{c_v}\left(w,c\right). \tag{6}$$

An incumbent from party R with valence v and ideology c_v is indifferent between (i) compromising to policy w_v to win if she runs for re-election, and (ii) adopting her own ideology

c_v as a policy and stepping down from office, since she would lose re-election to a challenger from the opposing party. An analogous recursive equation describes a party L incumbent with valence v and ideology $-c_v$.

Conditions (5) and (6) together with the continuation payoff function $\overline{U}_x(w, c)$ define equilibrium cutoff functions (w, c).

5 Policy Choices

In this section, we explore how valence affects policy choices, re-election, and expected extremism.

Proposition 1 *Take any equilibrium (w, c). For any $v_H, v_L \in V$,*

1. *Higher valence office-holders can take more extreme policies and win re-election,*

$$v_H > v_L \Rightarrow w_H > w_L;$$

2. *The probability of re-election strictly increases in valence,*

$$v_H > v_L \Rightarrow c_H > c_L;$$

3. *The compromise set strictly increases in valence,*

$$v_H > v_L \Rightarrow c_H - w_H > c_L - w_L.$$

The first result reflects that the decisive median voter is prepared to trade off valence for policy—she values valence and hence is willing to tolerate more extreme policies from higher valence incumbents. The second result reflects that an office-holder with higher valence v_H and ideology c_L is more willing to compromise than a lower valence v_L politician with the same ideology. This is because (a) her higher valence generates a higher payoff when in office, and (b) it is less costly for her to compromise, as she can win with a more extreme policy, $w_H > w_L$.

The third result says that if $v_H > v_L$ then $c_H - c_L > w_H - w_L$. To understand this stronger result, consider a low valence incumbent with ideology c_L and a high valence incumbent with ideology $x_H = c_L + (w_H - w_L)$. In terms of the distance between incumbent's ideology and re-election standard w_v, both incumbents face the same cost of compromising to win re-election.

However, incumbent x_H faces a higher cost than incumbent c_L of *not* compromising and then being replaced by an untried challenger—incumbent x_H is further from most untried challengers than c_L, including any untried challenger from the opposing party. Lemma A.2 shows that, as a result, x_H faces a higher cost of being replaced. Moreover, the higher valence v_H generates more utility than v_L when incumbent x_H is in office. Together, the higher benefit from compromising plus the higher cost of not compromising makes the higher valence office-holder x_H more willing to compromise, which results in a larger compromise set.

When we consider the group of re-elected incumbents or losing office holders, these results imply that the expected policies of higher valence representatives are *more* extreme. That is, on average, re-elected high valence office holders adopt more extreme policies than re-elected low valence office-holders; and losing office holders with high valence adopt more extreme policies than losing office holders with low valence. The result for re-elected (senior) office-holders emerges because the median voter sets slacker re-election standards for higher valence candidates that allow them to adopt more extreme policies and be re-elected. Among losing candidates, the set of extremist incumbents $(c_v, a]$ is decreasing in valence because higher valence candidates are more willing to compromise, which implies that, on average, higher valence candidates who lose locate more extremely.

However, these results do not imply that expected extremism increases with valence in the population. This is because for any fixed valence level, on average, losing incumbents adopt more extreme policies than re-elected officials; and since the number of extreme politicians falls with valence, so does the ratio of losing–to–re-elected officials. The next proposition shows that for politicians in their first term in office, the "lemons effect" dominates when the ideology distribution does not decline too sharply on the intermediate portion of its support, $[w(V_L), c(V_H)]$.[7]

Proposition 2 (Valence & Extremism: First-Term) *If the density function of ideologies does not decrease too steeply, then the expected extremism of a first term representative strictly decreases with the politician's valence. That is, there exists a lower bound* $\underline{f} < 0$ *such that if* $f(c_v) - f(w_v) \geq \underline{f}$ *then* $\frac{\partial EPol(v)}{\partial v} < 0$.

Proposition 2 only addresses a subset of representatives—those in their first term in office. Our model is intrinsically dynamic so that we must also account for the re-election of good candidates and the replacement of bad ones—over time, the likelihood of having an

[7] In particular, the center or tails of the distribution could be steeply sloped. Numerically, the result holds for truncated normal distributions.

extremist in office falls because extremists are not re-elected. From a long-run perspective, the relevant distribution is the stationary distribution of office-holders, or equivalently the cross-sectional distribution of policies and valence in a large congress.[8]

While Proposition 2 established that the expected extremism of first-term representatives falls with valence, Proposition 3 shows that this relationship is reversed in the steady-state distribution of a large congress whenever the probability q that an incumbent does not run for re-election for exogenous reasons is below an upper bound $\underline{q}$, which is strictly bounded away from zero. Empirically, less then 10% of incumbents in the United States Congress do not run for re-election.[9]

Proposition 3 (Valence & Extremism: Large Congress) *Consider the long-run stationary distribution of office holders. There exists a turnover probability bound $\underline{q} > 0$ such that if $q \leq \underline{q}$, then the expected policies of higher valence representatives are* ***more*** *extreme.*

Proposition 3 shows that, in a large congress, higher valence office holders are more likely to implement more extreme policies, even though valence and ideology are ex ante uncorrelated in the population, and we do not impose exogenous costs of compromising. Indeed, this result emerges despite the fact that high valence candidates compromise more (Proposition 1.3). The result is driven by the median voter's willingness to re-elect high valence office holders with more extreme policies (Proposition 1.1).

Propositions 2 and 3 show how important it is to consider the implications of incentives in a dynamic framework, when investigating the correlation between valence and extremism. They show that the sign of the correlation *varies* across incumbents with different seniority.

6 Ex-Ante Welfare

We consider two notions of voter welfare: (a) the ex-ante expected discounted lifetime payoff from electing an untried challenger drawn from either party with equal probability to serve as

[8]As in Bernhardt et al. (2004), we ignore the issue of how aggregation of ideologies in Congress affects policy outcomes. We simply assume that, at each election, voters behave as if only the ideology of their representative determines policy outcomes.

[9]An extensive numerical investigation in the quadratic preference, uniform ideology, two valence framework of Section 7 reveals that the upper bound $\underline{q}$ significantly exceeds 10% as long as the equilibrium cutoffs are not too close to the boundaries; that is, as long as w_L and c_H are not too close to zero and a, respectively. Therefore, we believe Proposition 3 characterizes the empirically-relevant scenario. If the conditions of Propositions 2 and 3 are violated, then our model implies that for first-term representatives the correlation between valence and extremism is more negative (or less positive) then the correlation in a large congress.

a first-term representative, and (b) the expected period payoff integrating over valences and policy choices using the long-run stationary distribution of office-holders. These notions arise from the dynamic nature of our model, and correspond to the frameworks used to analyze the correlation between valence and extremism in Propositions 2 and 3. We focus on how exogenous changes in the distribution of valences affect equilibrium strategies and voter welfare.

As a preliminary, we observe that a location shift of the valence distribution that raises the valence of each candidate type by a constant amount α has no strategic impact: adding α to utility function $u_x(y, v)$ results in a simple monotonic transformation $v + \alpha + L_x(y)$, which represents the same underlying preferences. That is, facing the better valence distribution $G'(v + \alpha) = G(v), \forall v \in V$, a voter with ideology x votes for an incumbent with valence $v + \alpha$ who locates at y if and only if the voter would vote for the incumbent v who locates at y when facing distribution $G(v)$. In essence, from a strategic standpoint the mean of the valence distribution is a strategically irrelevant lump sum transfer to all agents; what matters is the distribution of valences around the mean. It follows that one can normalize the lowest valence to zero, $v_L \equiv 0$.

More intriguing questions are: how are voters' payoffs and politicians' expected policy choices affected by more complicated shifts in valence distribution? In particular, how is voter welfare affected by first and second order stochastic improvements in the valence distribution, and when do such stochastic changes lead to greater expected extremism in policy choices? To address these questions, we first focus our analysis on the case where the loss function is quadratic, ideologies are uniformly distributed, and there are no ego rents from holding office. With quadratic preferences, all voters share the same ordering over changes in the mean and variance of the valence distribution. We then discuss how the qualitative results are affected when voters have non-quadratic preferences over ideology so that they trade off differently between valence and expected policy outcomes.

Our previous results revealed that incumbents with higher valences compromise to more extreme policies, and in the stationary distribution of office holders, they adopt more extreme policies. In fact, we show below that some FOSD improvements in the valence distribution also increase the expected extremism of candidates. As a result, one might conjecture that some (extreme) voters might be hurt by an increase in the probability of high valence candidates. Moreover, one might conjecture that increasing valence variance decreases the expected voter welfare, as voters are risk averse. The next results show that these conjectures are false.

Proposition 4 *Consider a quadratic loss function, uniform ideology distribution, and $\rho = 0$. Let $EPol(\cdot)$ represent the absolute value of expected policy outcomes. The following results hold for valence distributions $G(v)$ and $G'(v)$:*

1. *If $G'(v)$ first order stochastically dominates $G(v)$, then*

$$\overline{U}'_x(w', c') > \overline{U}_x(w, c), \forall x \in [-a, a]; \tag{7}$$

2. *If $G(v)$ second order stochastically dominates $G'(v + \alpha)$ for some $\alpha \geq 0$ then*

$$\begin{aligned} \overline{U}'_x(w', c') &> \overline{U}_x(w, c) + \alpha, \forall x \in [-a, a], & (8) \\ EPol(G') &> EPol(G). & (9) \end{aligned}$$

Valence is valued and, for untried candidates, is negatively correlated with extremism. Hence, an improvement in the valence distribution raises the payoff that the median voter expects to derive from an untried candidate. The untried candidate becomes more attractive, inducing the decisive median voter to set tighter re-election standards for *all* valence levels: re-election cutoffs w_v move closer to the median voter. However, there is an indirect offsetting effect—the decline in w_v is accompanied by a decline in c_v, making this proposition far from trivial to establish. In particular, a politician with valence v and ideology c_v has (a) a higher cost of compromising, since w_v is now closer to the median voter, and (b) a lower cost of being replaced by a challenger, who now has a higher expected valence and faces tighter re-election standards. As a result, more politicians choose to locate extremely and lose, and this hurts all voters. However, we prove that the direct positive effect dominates—if not, the median voter would be worse off and hence set looser re-election standards, which would increase the incentives of extremist incumbents to compromise, raising median voter welfare, a contradiction.[10]

It is even more challenging to establish this welfare result for the stationary distribution of office holders. Recall from Proposition 3 that when incumbents are likely to run

[10] In a working paper draft we prove that even with non-quadratic utilities and non-uniform ideology distributions, the median voter always gains from a first-order stochastic improvement in valences as long as the cutoff function $c(\cdot)$ exhibits a stronger monotonicity property in $w(\cdot)$ than that required to ensure existence (see Lemma A.9). Specifically, changes in the economy that decrease(increase) the median voter's expected payoff from an untried candidate do not raise(reduce) the expected payoffs of officeholders with moderate ideologies by too much. By contradiction, suppose the median voter is hurt by an FOSD improvement. Then $w(v)$s shift out and the $c(v)$s do not shift in (by the monotonicity property). Integrating over the possible ideologies of the challenger, the direct benefit of an FOSD improvement plus any indirect benefits associated with outward shifts in $c(v)$ exceed the negative welfare effect of an outward shift in $w(v)$s, a contradiction. Notice that the monotonicity condition holds for quadratic utility because shifts in the valence distribution have the same effect on each voter's expected payoff.

for re-election (q is small), valence is positively correlated with extremism in the stationary distribution of office holders, and the increased measure of more extreme high valence incumbents in a large congress could hurt the voters. However, after a first order stochastic improvement in valence distribution, all re-elected officials who compromise locate closer to the median voter. When incumbents are likely to run for re-election, enough representatives in the large congress are returning centrist/compromising incumbents that the valence improvement and tighter re-election standards benefits all voters.

We now investigate why all risk averse voters prefer the "riskier" distribution G' over the second order stochastically dominant distribution G. With more heterogeneity in valence, untried candidates are more likely to have extreme valence values. Compared to candidates with valence close to the mean, higher valence candidates compromise to more extreme policies, while low valence candidates are more likely to adopt extreme policies. Why then do voters still prefer the "gamble"? The answer is that those losses are more than compensated by the gains from the "competition" between good and bad candidates: (a) lower valence candidates must take more moderate positions to win re-election, (b) high valence candidates are more willing to compromise, and most importantly, (c) there is a positive option value associated with an untried challenger who *could* have a high valence—the decisive median voter has the option of voting extremist, lower valence types out of office, in the hope of drawing a centrist/moderate high valence candidate. Since low valence incumbents are more likely to be ousted from office, in the long-run, heterogeneity raises the expected valence in the cross-section of office holders. The value of this future expected benefit exceeds the immediate costs associated with the reduced willingness of low valence candidates to compromise, so that all voters prefer to have heterogeneity in valences.

The next result describes the consequences of Proposition 4 in a two-type valence setting, establishing how the probability p of drawing a high valence candidate affects expected voter payoffs and expected policy changes.

Corollary 1 *Consider a quadratic loss function, uniform ideology distribution, and $\rho = 0$. In a two-valence economy where an untried candidate has valence v_H with probability $p \in [0, 1]$ and has valence $v_L < v_H$ with probability $(1 - p)$,*

1. *The expected payoff $\overline{U}_x(w, c|p)$ of each voter x strictly increases in p, at rate greater than $(v_H - v_L)$ for any $p < 1/2$, and at rate less than $(v_H - v_L)$ for any $p > 1/2$;*
2. *Expected policy $EPol(p)$ is a single-peaked function of p, symmetric about $p = 1/2$.*

For $p < 1/2$, a marginal increase in p results in both a FOSD improvement and an increase in the variance of valence distribution. Combining these two effects results in an increase in utility greater than $v_H - v_L$. For $p > 1/2$, the FOSD benefit of a marginal increase in p is mitigated by the decrease in variance, so that utility increases less than $v_H - v_L$. Higher variance induces higher expected extremism, therefore the single-peaked/symmetric result on $EPol(p)$ follows from the single-peaked/symmetric change in variance around $p = 1/2$.

6.1 Non-Quadratic Preferences and Voter Welfare

When loss functions are quadratic, our welfare characterization holds for all voters, since the expected payoff of each voter can be expressed as a function of the median voter's expected payoff. Consequently, all voters share the median voter's preferences over changes in the mean and variance of the valence distribution.

However, what happens when voter loss functions are not quadratic? How does the extent of voter risk aversion interact with ideology to determine voter preferences over different valence distributions? Who benefits and who loses?

To address these questions, we investigate outcomes numerically when ideologies are drawn from uniform or truncated normal distribution, loss functions take the form $L_x(y) = -|x - y|^z$ for $z \in [1, 4]$, and there are two valences. We find that *all* voters benefit from a stochastic improvement in the valence distribution. What drives this finding is that higher valence candidates are more willing to compromise (Proposition 1.3). Moreover, the higher expected valence of challengers induces the median voter to set more demanding re-election standards. Hence, incumbents of *all* valence levels must compromise to more moderate policies to win re-election, and this increases the ex ante welfare of all voters.

We also find that the median voter is always better off when we increase variance by moving from a one-valence economy to a two-valence economy, where the expected valence in the two economies is the same. However, while the median voter always gains from increased dispersion in valences, voters with different ideologies trade off differently between valence and expected policy. The decisive median voter is more willing to accept a more extreme position from a high valence incumbent from party R than *any* voter in party L: voters in party L are further from the incumbent, and due to the concavity of the loss function, are less willing to trade off extremism for valence.

We now retrieve the intuition that even though the median voter gains from heterogeneity in candidate qualities, because voters trade-off valence for policy differently, voters with more extreme ideologies may be hurt. When we increase valence heterogeneity, it increases the long-run expected valence, which benefits all voters by the same amount. However, the relative impact of changes in equilibrium policies depends on the extent of voter risk aversion. To make this point, we consider loss functions $L_x(y) = -|x - y|^z$ with $z \geq 1$.

Euclidean loss function, $z = 1$. One can prove that when voters have Euclidean loss function, changes that induce more extreme expected equilibrium policies hurt the median voter (and voters close to her) by more than extreme voters close to a. This is because extreme voters are "almost" risk neutral with respect to changes in the (symmetric) policy, and hence almost indifferent to mean zero shifts in policy. The introduction of heterogeneity increases the expected equilibrium valence, which benefits all voters by the same amount; and since greater valence heterogeneity also yields more extreme policies, it follows that voters with sufficiently extreme ideologies gain *more* than the median voter (and voters close to the median).

Quadratic loss function, $z = 2$. When voters have quadratic loss functions, $\overline{U}_x(w, c) = \overline{U}_0(w, c) - x^2$. Therefore, valence heterogeneity raises every voter's expected ex ante payoff from an untried challenger by the *same* amount as the median voter.

Cubic loss function, $z = 3$. When voters are highly risk averse, with cubic loss functions, we establish numerically that a shift from one-valence to a two-valence environment hurts all voters with sufficiently extreme ideologies: there exists an $\overline{x} > 0$ such that a voter with ideology x is hurt if and only if $|x| > \overline{x}$. For example, when ideologies are uniformly distributed on $[-10, 10]$, $\delta = .3$, $v_L = 0$, $v_H = 1$, $p = 1/2$, $\rho = q = 0$, we find that $\overline{x} = 3$; i.e., even though the median voter gains from valence heterogeneity, 70% of voters would prefer the economy of "average" politicians to the one with heterogeneity in valences.

7 Valence Search

We conclude by extending the model to endogenize the probability an untried candidate has high valence. To do this, we introduce two symmetric Interest Groups (IG) with ideologies $-i$ and $+i$. IG $-i$ supports party L while IG i supports party R. The interest groups have the same utility function as voters $\{-i, +i\}$. There are two possible valence levels, $v_H > v_L \geq 0$. In each election, an interest group can undertake a costly search to try to identify an untried challenger from its supported party who has high valence. To identify with probability $p \in [0, 1]$ an untried candidate with high valence, the IG incurs a cost $\alpha c(p)$, where $\alpha > 0$ and $c(p)$ is $\mathcal{C}^2$, $c' > 0$ for $p > 0$, $c'' \geq 0$, with boundary conditions $c(0) = c'(0) = 0$ and

$c(1) > \frac{v_H - v_L + a^2}{\alpha}$ that guarantee interior solutions. Incumbents keep their valences for their entire political career, so that if an incumbent runs for re-election, her supporting IG does not search. While voters and the opposing IG do not see the realized search effort, in equilibrium they correctly forecast the probability p^* that an untried candidate has high valence. We focus on a setting where ideologies are uniformly distributed, the loss function is quadratic, $l(|x|) = -|x|^2$, there are no ego rents ($\rho = 0$), and the IGs employ symmetric strategies.

In equilibrium, the opposing IG never searches when an incumbent with valence v adopts a centrist policy $|y| \leq w_v$: the challenger is sure to lose. The opposing IG is only willing to search if the incumbent chose an extreme policy $|y| > w_v$ and will not be re-elected. In this case, voters and IGs must form consistent beliefs about the equilibrium re-election cutoff w_v that leaves the median voter indifferent between re-electing the incumbent and electing an untried candidate who has high valence with probability $\tilde{p}$. But the cutoff w_v depends on equilibrium beliefs about $\tilde{p}$ — $\tilde{p}$ can take any value $\tilde{p} \in [0, p^*]$, where p^* is the optimal valence search level of IGs when IGs expect that the incumbent will not be re-elected—it follows that there is a continuum of equilibria indexed by $\tilde{p}$. We focus on the equilibrium where equilibrium search $\tilde{p} = p^*$ is the highest—this equilibrium yields the highest expected utility for all voters. Thus, w_v leaves the median voter indifferent between re-electing an incumbent with valence v who adopts policy w_v and electing an untried candidate from the opposing party who has high valence v_H with probability p^*.

Our previous analysis can be used to characterize the equilibrium—all equilibrium equations remain the same—but now we must use the endogenous equilibrium probability p^*. When an incumbent steps down and an untried candidate will be elected, the search effort of an interest group supporting party R is pinned down by the first-order condition

$$\alpha c'(p^*) \quad = \quad \frac{1}{2}\Big[U_i^R(v_H|w,c) - U_i^R(v_L|w,c)\Big]. \tag{10}$$

Equation (10) states that the marginal search cost equals its marginal expected benefit, which is the expected payoff difference from drawing a high valence challenger rather than a low valence one[11]. For an IG whose ideology is close to the median voter's, there are three benefits from increasing the probability of a high valence candidate: (a) valence itself, (b) untried, high valence candidates are more likely to adopt policies closer to the median voter (Proposition 2), and (c) reduced turnover (Proposition 1.2). An IG with a more extreme ideology receives the same direct benefit from valence, but the other two factors move in

[11]The marginal benefit is multiplied by $1/2$ because the challenger from party R wins the election with probability $1/2$. See the detailed discussion about equilibrium search in the proof of Proposition 5 in the Appendix.

opposite directions. An extreme right-wing IG prefers its supported candidate to adopt more extreme, right-wing policies—a moderate high valence candidate is less beneficial. However, turnover hurts more extreme interest groups, so they value the reduced turnover of high valence candidates. The next proposition shows that the preference for extreme policies dominates. Moreover, less search[12] implies smaller p^*, and by Proposition 4, this implies that untried candidates yield lower payoffs to *all* voters.

Proposition 5 (Valence Search) *More extreme interest groups (i larger) search strictly less for valence, thereby hurting all voters.*

Next we explore how the extremism of IGs affects equilibrium policies. As the ideologies of IGs grow more extreme they reduce the search for high valence candidates. As a result, the decisive median becomes worse off and sets slacker re-election standards. Therefore,

Corollary 2 *Conditional on valence type, extremism of re-elected officials is positively correlated with extremism of interest groups.*

How does Corollary 2 extend unconditionally, when we integrate over all possible ideology and valence types? From Corollary 1, the (absolute value of the) expected policy of an untried candidate is a single-peaked function of the equilibrium probability p^*, symmetric about $p^* = 1/2$. Therefore, there is more extremism if and only if the equilibrium probability p^* of identifying a high valence candidate is sufficiently high. That is,

Corollary 3 *Extremism of untried candidates is* ***positively*** *correlated with extremism of interest groups if and only if the marginal cost of valence search is sufficiently low: there exists an* $\bar{\alpha} > 0$ *such that more extreme interest groups give rise to more polarized platforms if and only if the search cost parameter* α *is less than* $\bar{\alpha}$*.*

Finally, the last step in the proof of Proposition 5 implies that a higher search cost parameter α reduces valence search and p^* in equilibrium, hurting all voters. Consequently, for given IGs with ideologies $\{-i, i\}$, a small increase in search cost parameter α would give rise to more polarized platforms if and only if α is sufficiently low, so that $p^* > 1/2$.

[12] Our result only states that more extreme IGs expend less searching for candidates with high valence, but we do not make any claims about total expenditures. We do not model advertisements or campaign expenditures—areas where empirical evidence suggests that more extreme IGs spend more money.

8 Conclusion

This paper develops a dynamic citizen-candidate model of repeated elections, in which candidates are distinguished by both their ideology and valence. From an incumbent's performance in office, voters can infer her valence and forecast her future policy choices. An incumbent is opposed by an untried challenger, about whom voters only know her party affiliation. Voters base re-election choices on this information. We show how reputation/re-election concerns drive policy choices, and serve to endogenize the costs of locating extremely. We prove that higher valence incumbents are more likely to compromise and win re-election, even though they compromise to more extreme policies. However, this does not imply that valence is negatively correlated with extremism: we find a *negative* correlation for first-term representatives, and a *positive* correlation for re-elected officials. This novel result may help explain the conflicting empirical findings regarding the correlation between valence and extremism.

We then determine how the distribution of candidate valences affects equilibrium policy choices and voter welfare. We show that even though voters may trade off differently between valence and expected policy, all voters benefit from a first-order stochastic improvement in the distribution of valences because it raises the expected payoff from an untried challenger, thereby forcing incumbents of any given valence to compromise by more in order to win re-election. In sharp contrast, while the median voter always benefits from greater dispersion in valences due to the embedded option to elect an untried challenger, voters with more extreme ideologies only benefit when they are not too risk averse. Lastly, we expand our model, endogenizing the determination of the valence of challenging candidates by supposing that interest groups or activist groups may undertake costly searches to identify candidates with better skills. We derive a complete theoretical explanation for the intuitive conjectures that activists with more extreme ideologies lower voter welfare, and can give rise to policies that, on average, are more extreme.

A maintained assumption of our model was that a politician's valence did not vary with her tenure. However, one might believe that valence may rise with tenure say due to greater pork provision by more senior incumbents, as in Bernhardt et al. (2004), or because, due to learning-by-doing, politicians become better at providing for their constituents. When valence increases with tenure, it follows routinely that voters set slacker re-election standards for more senior incumbents. As a result, following a given re-elected politician over time, a researcher will uncover a positive correlation between extremism and tenure (*seniority effect*), as more senior incumbents need not moderate by as much to win re-election. However, if one compares the cohorts of first-term versus senior representatives, there is an opposing *group selection effect* because extremist first-term representatives are ousted from office. Disentangling and measuring these two effects, and their consequences for the relationship between extremism and valence, is an important, albeit complicating, task for empirical researchers.

9 Appendix

Proof: [**Theorem 1**] To simplify presentation and to be consistent with our stationary equilibrium concept, we focus on stationary out-of-equilibrium beliefs—whatever policy a representative implements today, voters believe that she will continue to implement the same policy in the future. More generally, there is a broad set of out-of-equilibrium beliefs that support our equilibrium path. In essence, all we need are beliefs that a candidate with valence v who locates more extremely than equilibrium re-election cutoff w_v at some date t will never locate more moderately than w_v in the future.

When an incumbent chooses not to run for re-election, both parties run with untried candidates, and the previous policy choices of the exiting incumbent do not affect the new election's outcome. Moreover, from concavity of the loss function, an incumbent optimally chooses to run for re-election if and only if she expects to win. Therefore, in equilibrium, we can divide incumbents into three groups. Define $W_v^L \subseteq [-a, 0]$ as the party L win set for candidates with valence v. In equilibrium, an incumbent with ideology $x \in W_v^L$ and valence v implements her own ideology as policy; if not affected by the re-election shock, she runs for re-election and wins. Define $C_v^L \subseteq [-a, 0]$ as the party L compromise set for candidates with valence v. In equilibrium, an incumbent with ideology $x \in C_v^L$ and valence v does not adopt her own ideology as policy—she compromises to policy $p(x, v) = \arg\min_{w \in W_v^L} l(|x - w|)$, i.e., to the least costly policy that allows her to win re-election. Define the compromise function $c^L(x, v) = \arg\min_{w \in W_v^L} l(|x - w|)$. From symmetry, for $x < 0$, $c^L(x, v) = -c^R(-x, v)$. Define $E_v^L \subseteq [-a, 0]$ as the party L extremist set for candidates with valence v. In equilibrium, an incumbent with ideology $x \in E_v^L$ and valence v implements as policy her own ideology and does not run for re-election. Analogously define the symmetric sets W_v^R, C_v^R and E_v^R for party R. Notice that W_v^L, C_v^L, and E_v^L partition $[-a, 0]$. Define the complete win set as $W = \left\{(x, v) \in [-a, a] \times V | x \in W_v^L \cup W_v^R\right\}$, and define C and E analogously.

Let $U_x(y, v|W, C)$ denote the equilibrium continuation utility that a voter with ideology x expects to derive from a date-t office-holder with valence v who adopts platform y, if the incumbent is re-elected every time she runs for office — that is, if the incumbent belongs to the win set or compromise set. Define $U_x^j(W,C)$ to be the equilibrium continuation utility that x expects to derive from selecting an untried representative from party $j \in \{L,R\}$, and let $\overline{U}_x(W,C) \equiv [U_x^R(W,C) + U_x^L(W,C)]/2$ represent the payoff x expects from an untried challenger drawn from at large. Integrating over the possibility of a re-election shock, the continuation payoff that x expects from an incumbent is

$$\begin{aligned} U_x(y,v|W,C) &= u_x(y,v)(1-\delta) + \delta\left[q\frac{U_x^L(W,C)+U_x^R(W,C)}{2} + (1-q)\,U_x(y,v|W,C)\right] \\ &\equiv k\ u_x(y,v) + k\frac{\delta q}{(1-\delta)}\overline{U}_x(W,C), \end{aligned} \tag{11}$$

where $k \equiv \frac{(1-\delta)}{[1-\delta+\delta q]}$. Notice that $k \in (0,1]$. An office-holder with valence v who adopts extremist platform y and does not run for re-election yields to voter x an equilibrium continuation utility $(1-\delta)u_x(y,v) + \delta\overline{U}_x(W,C)$.

For any voter x, integrating over the three possible sets, the expected payoff from electing an untried candidate from party L is

$$\begin{aligned} U_x^L(W,C) &= \int_V \left\{2\int_{W_v^L}\left[k\ u_x(y,v) + k\frac{\delta q}{1-\delta}\overline{U}_x(W,C)\right]dF(y)\right. \\ &+ 2\int_{C_v^L}\left[k\ u_x\left(c^L(y,v),v\right) + k\frac{\delta q}{1-\delta}\overline{U}_x(W,C)\right]dF(y) \\ &+ \left. 2\int_{E_v^L}\left[(1-\delta)u_x(y,v) + \delta\overline{U}_x(W,C)\right]dF(y)\right\}dG(v). \end{aligned}$$

Define $\beta(v) \equiv \delta(1-q)2\int_{E_v^L} dF(y)$, which is $\delta(1-q)$ times the probability that a candidate from party L belongs to the extremist set *given* that the candidate has valence v. Define $\beta \equiv \delta(1-q)\int_V 2\int_{E_v^L}\ dF(y)\ dG(v)$, which is $\delta(1-q)$ times the (unconditional) probability that a random candidate from party L belongs to the extremist set. Notice that $\beta \in [0,1)$.

Add and subtract $k\frac{\delta q}{1-\delta}\overline{U}_x(W,C)\int_V 2\int_{E_v^L} dF(y)dG(v)$ to $U_x^L(W,C)$. Since $\delta - k\frac{\delta q}{1-\delta} = k\delta(1-q)$, we can rewrite

$$\begin{aligned} U_x^L(W,C) &= k\frac{\delta q}{1-\delta}\overline{U}_x(W,C) + k\ \beta\overline{U}_x(W,C) + \int_V\left\{2\int_{W_v^L} k\ u_x(y,v)dF(y)\right. \\ &+ 2\int_{C_v^L} k\ u_x\left(c^L(y,v),v\right)dF(y) + 2\int_{E_v^L}(1-\delta)u_x(y,v)\,dF(y)\left.\right\}dG(v). \end{aligned} \tag{12}$$

Analogously,

$$\begin{aligned} U_x^R(W,C) &= k\frac{\delta q}{1-\delta}\overline{U}_x(W,C) + k\ \beta\overline{U}_x(W,C) + \int_V\left\{2\int_{W_v^R} k\ u_x(y,v)dF(y)\right. \\ &+ 2\int_{C_v^R} k\ u_x\left(c^R(y,v),v\right)dF(y) + 2\int_{E_v^R}(1-\delta)u_x(y,v)\,dF(y)\left.\right\}dG(v). \end{aligned}$$

Exploiting symmetry, for any voter x, the expected payoff from electing an untried candidate drawn from at large is

$$\begin{aligned}
\overline{U}_x(W,C) &= \frac{U_x^L(W,C)+U_x^R(W,C)}{2} = k\frac{\delta q}{1-\delta}\overline{U}_x(W,C)+k\,\beta\overline{U}_x(W,C)\\
&+ \int_V\left\{2\int_{W_v^L} k\frac{\left[u_x(y,v)+u_x(-y,v)\right]}{2}dF(y)\right.\\
&+ 2\int_{C_v^L} k\frac{\left[u_x\left(c^L(y,v),v\right)+u_x(c^R(-y,v),v)\right]}{2}dF(y)\\
&+ \left.2\int_{E_v^L}(1-\delta)\frac{\left[u_x(y,v)+u_x(-y,v)\right]}{2}dF(y)\right\}dG(v).
\end{aligned}$$

Since $1-k\frac{\delta q}{1-\delta}-k\beta=k(1-\beta)$, we have

$$\begin{aligned}
\overline{U}_x(W,C) &= \frac{1}{1-\beta}\int_V\left\{2\int_{W_v^L}\frac{\left[u_x(y,v)+u_x(-y,v)\right]}{2}dF(y)\right.\\
&+ 2\int_{C_v^L}\frac{\left[u_x\left(c^L(y,v),v\right)+u_x(c^R(-y,v),v)\right]}{2}dF(y)\\
&+ \left.2\int_{E_v^L}(1-\delta(1-q))\frac{\left[u_x(y,v)+u_x(-y,v)\right]}{2}dF(y)\right\}dG(v). \qquad (13)
\end{aligned}$$

Substitute (13) into the term $k\,\beta\overline{U}_x(W,C)$ in (12). After some algebra, one can solve for

$$\begin{aligned}
U_x^L(W,C) &= k\frac{\delta q}{1-\delta}\overline{U}_x(W,C)\\
&+ \frac{k}{1-\beta}\int_V\left\{2\int_{W_v^L}\frac{\left[(2-\beta)u_x(y,v)+\beta u_x(-y,v)\right]}{2}dF(y)\right.\\
&+ 2\int_{C_v^L}\frac{\left[(2-\beta)u_x\left(c^L(y,v),v\right)+\beta u_x\left(c^R(-y,v),v\right)\right]}{2}dF(y)\\
&+ \left.2\int_{E_v^L}(1-\delta(1-q))\frac{\left[(2-\beta)u_x(y,v)+\beta u_x(-y,v)\right]}{2}dF(y)\right\}dG(v). \quad (14)
\end{aligned}$$

For each pair valence v and ideology $y\le 0$, $U_x^L(W,C)$ is a weighted average between the period payoff derived from an incumbent with *negative* ideology y and its symmetric positive counterpart $-y$, where more weight is given to the negative ideology. $U_x^R(W,C)$ is defined symmetrically, where most weight is given to positive ideologies. Equal weight is given to both parties in (13), when a candidate is drawn at large.

In equilibrium, the expected per-period valence is

$$E^*(v)\equiv\int_V\frac{v[1-\beta(v)]}{1-\beta}dG(v).$$

Notice that $V_L \leq E^*(v) \leq V_H$. Using this definition, rewrite equation (14) as

$$\begin{aligned} U_x^L(W,C) &= kE^*(v) + k\frac{\delta q}{1-\delta}\overline{U}_x(W,C) \\ &+ \frac{k}{1-\beta}\int_V \left\{ 2\int_{W_v^L} \frac{[(2-\beta)l(|x-y|) + \beta l(|x+y|)]}{2} dF(y) \right. \\ &+ 2\int_{C_v^L} \frac{[(2-\beta)l(|x-c^L(y,v)|) + \beta l(|x+c^L(y,v)|)]}{2} dF(y) \\ &+ \left. [1-\delta(1-q)]2\int_{E_v^L} \frac{[(2-\beta)l(|x-y|) + \beta l(|x+y|)]}{2} dF(y) \right\} dG(v). \quad (15) \end{aligned}$$

A voter with ideology x votes for an incumbent from party R with valence v who adopts policy y if and only if this incumbent yields a higher expected payoff than an untried candidate from party L. That is, voter x votes for the incumbent from R when $U_x(y,v|W,C) \geq U_x^L(W,C)$. Define $\mathbf{S}_x^R$ as the retrospective $R-$set of voter with ideology x: the set of {implemented policy, valence} pairs of an incumbent from party R that x would re-elect over a random challenger from the opposite party (party L), and define $\mathbf{S}_x^L$ analogously:

$$\begin{aligned} \mathbf{S}_x^R &= \left\{(y,v)|U_x(y,v|W,C) - U_x^L(W,C) \geq 0\right\}, \\ \mathbf{S}_x^L &= \left\{(y,v)|U_x(y,v|W,C) - U_x^R(W,C) \geq 0\right\}. \end{aligned}$$

The next lemma proves that if the heterogeneity in valences is not too large then a majority of voters prefer to re-elect even a low valence incumbent from party $j \in \{R, L\}$ who adopts policy $y = 0$ over an untried candidate from the opposing party — in particular, all voters from the incumbent's party j vote for re-election.

Lemma A. 1 *There exists an upper bound $\overline{v}$, $0 < \overline{v}$, such that if $V_H - V_L \leq \overline{v}$, then for any valence $v \in V$ a majority of voters prefers to re-elect an incumbent who adopts policy $y = 0$ over an untried candidate from the opposing party. In particular, all voters from the incumbent's party vote fore re-election when $y = 0$: for all $v \in V$, we have $(0,v) \in \mathbf{S}_x^R$, $\forall x \in [0,a]$, and $(0,v) \in \mathbf{S}_x^L$, $\forall x \in [-a,0]$.*

Proof: We first prove that a majority of voters prefers to re-elect an incumbent v from party R who adopts policy $y = 0$. Take any valence $\tilde{v} \in V$. We must show that $U_x(0,\tilde{v}|W,C) -$

$U_x^L(W,C) > 0$ for a majority of voters $x \in [-a,a]$. Using (11) and (15),

$$\begin{aligned}
U_x(0,\tilde{v}|W,C) \quad - \quad U_x^L(W,C) \;&=\; k\tilde{v} + k\frac{\delta q}{1-\delta}\overline{U}_x(W,C) - kE^*(v) - k\frac{\delta q}{1-\delta}\overline{U}_x(W,C) \\
&+\; kl(|x|) - \frac{2k}{1-\beta}\int_V \left\{ \int_{W_v^L} \frac{[(2-\beta)l(|x-y|) + \beta l(|x+y|)]}{2} dF(y) \right. \\
&+\; \int_{C_v^L} \frac{[(2-\beta)l(|x-c^L(y,v)|) + \beta l(|x+c^L(y,v)|)]}{2} dF(y) \\
&+\; \left. [1-\delta(1-q)]\int_{E_v^L} \frac{[(2-\beta)l(|x-y|) + \beta l(|x+y|)]}{2} dF(y) \right\} dG(v).
\end{aligned}$$

Rewriting

$$\begin{aligned}
U_x(0,\tilde{v}|W,C) \quad - \quad U_x^L(W,C) \;&=\; k[\tilde{v} - E^*(v)] \qquad (16)\\
+\; \frac{2k}{1-\beta}\int_V &\left\{ \int_{W_v^L} \left[l(|x|) - \frac{[(2-\beta)l(|x-y|) + \beta l(|x+y|)]}{2} \right] dF(y) \right. \\
+\; \int_{C_v^L} &\left[l(|x|) - \frac{[(2-\beta)l(|x-c^L(y,v)|) + \beta l(|x+c^L(y,v)|)]}{2} \right] dF(y) \\
+\; [1-\delta(1-q)] &\left. \int_{E_v^L} \left[l(|x|) - \frac{[(2-\beta)l(|x-y|) + \beta l(|x+y|)]}{2} \right] dF(y) \right\} dG(v).
\end{aligned}$$

Concavity of the loss function implies that the term inside the integrals is strictly positive for all voters x sufficiently close to the median voter $x = 0$. Moreover, for any voter $x > 0$ such that the term inside the integral is strictly negative, symmetry implies that there exists a voter $x' = -x$ such that the term is strictly positive. Hence, the policy-related payoff term in (16) is strictly positive for a majority of voters. If the valence related payoff term is non-negative, $\tilde{v} - E^*(v) \geq 0$, we are done—notice that this condition always holds if there is a unique valence, $V_H = V_L$. If $\tilde{v} - E^*(v) < 0$, then it suffices to show that

$$\begin{aligned}
E^*(v) - \tilde{v} \;<\; \frac{1}{1-\beta}\int_V &\left\{ 2\int_{W_v^L} \left[l(|x|) - \frac{[(2-\beta)l(|x-y|) + \beta l(|x+y|)]}{2} \right] dF(y) \right. \qquad (17)\\
+\; 2\int_{C_v^L} &\left[l(|x|) - \frac{[(2-\beta)l(|x-c^L(y,v)|) + \beta l(|x+c^L(y,v)|)]}{2} \right] dF(y) \\
+\; [1-\delta(1-q)]2 &\left. \int_{E_v^L} \left[l(|x|) - \frac{[(2-\beta)l(|x-y|) + \beta l(|x+y|)]}{2} \right] dF(y) \right\} dG(v)
\end{aligned}$$

for a majority of voters. The RHS is strictly positive for a majority of voters. Since $E^*(v) - \tilde{v} \leq V_H - V_L$, there exists an upper bound $\overline{v} > 0$ such that equation (17) holds for a majority of voters under the gross sufficient condition $V_H - V_L \leq \overline{v}$, establishing that for any $v \in V$, $0 \in W_v^R$. An analogous argument holds for an incumbent from party L: for any $v \in V$, $0 \in W_v^L$.

This result implies that an incumbent with ideology $y \leq 0$ will not adopt a policy $p(y, v) > 0$, because she can win by locating at zero. Therefore, $y \leq 0$ implies $c^L(y, v) \leq 0$ and $c^R(-y, v) \geq 0$. Since $2 - \beta > \beta$, more weight is given to the negative policy in (16). Hence, concavity of the loss function implies that the policy-related payoff term in (16) is strictly positive for every party R voter $x \in [0, a]$. Following the argument above, we can show that if the valence set is not too large then $(0, v) \in \mathbf{S}_x^R$, $\forall x \in [0, a]$. An analogous argument holds for party L, concluding the proof. ■

Lemma A. 2 *The more moderate is a citizen's ideology, the higher is her expected utility from a challenger, whether selected from the opposing party or from a random party.*

In particular, for any pair $x', x \in [0, a]$ with $x' > x$,

$$U_x^L(W, C) > U_{x'}^L(W, C), \tag{18}$$

$$\overline{U}_x(W, C) > \overline{U}_{x'}(W, C), \tag{19}$$

$$U_x^L(W, C) - U_{x'}^L(W, C) > \overline{U}_x(W, C) - \overline{U}_{x'}(W, C). \tag{20}$$

Proof: Consider $x', x \in [0, a]$ with $x' > x$. From equation (13), using concavity of the loss function it follows that $\overline{U}_x(W, C) > \overline{U}_{x'}(W, C)$. In particular, moderate citizen x loses less than extreme citizen x' for every candidate draw from the opposing party, as the moderate is closer. While x' loses less for realizations of the same party that exceed $\frac{x'+x}{2}$, because $l'' \leq 0$, for every gain (smaller loss) that x' gets from an extreme office-holder from the same party, x gains at least as much from the symmetric extreme office-holder from the other party.

This result and the same argument on equation (15) imply that $U_x^L(W, C) > U_{x'}^L(W, C)$.

To show that $U_x^L(W, C) - U_{x'}^L(W, C) > \overline{U}_x(W, C) - \overline{U}_{x'}(W, C)$, it suffices to show that $U_x^L(W, C) - U_{x'}^L(W, C) > U_x^R(W, C) - U_{x'}^R(W, C)$ for $x' > x \geq 0$. Again, this follows from the concavity of $l(\cdot)$ and the fact that for any policy $y > (x' + x)/2$ voter x' loses less than x. ■

The next lemmas characterize the win and compromise sets, and prove that the median voter is decisive.

Lemma A. 3 *For each $v \in V$, the win set is connected, $W_v \equiv W_v^R \cup W_v^L = [-w_v, +w_v]$.*

Proof: Fix valence $v \in V$. From Lemma A.1, $0 \in W_v$. Suppose that $y > 0 \in W_v$, which implies that the incumbent is from party R. We now show that all citizens who vote for y also vote for any $y' \in [0, y]$. For each citizen $x \leq y'$ who votes for y, $U_x(y, v|W, C) \geq U_x^L(W, C)$ and since $U_x(y', v|W, C) \geq U_x(y, v|W, C)$, she also votes for y'. Every voter $x \geq y'$ also votes for y' since $U_x(y', v|W, C) \geq U_x(0, v|W, C) \geq U_x^L(W, C)$ where the last inequality follows from Lemma A.1. Therefore, y' receives at least as many votes as y and $y' \in W_v$. The same argument applies to any $y < 0 \in W_v$. ■

Lemma A. 4 *The retrospective set of the median voter is contained in the win set:*

1. *If $(y, v) \in \mathbf{S}_0^R$ then $y \in W_v^R$;*

2. *If $(y, v) \in \mathbf{S}_0^L$ then $y \in W_v^L$.*

Proof: Let $(y, v) \in \mathbf{S}_0^R \Rightarrow U_0(y, v|W, C) \geq U_0^L(W, C)$ and $y \geq 0$. Every voter $x \geq y$ votes for y since $U_x(y, v|W, C) \geq U_x(0, v|W, C) \geq U_x^L(W, C)$ where the last inequality comes from Lemma A.1. Every voter $x \in [0, y]$ also votes for y since $U_x(y, v|W, C) \geq U_0(y, v|W, C) \geq U_0^L(W, C) \geq U_x^L(W, C)$ where the last inequality comes from Lemma A.2. Therefore, x wins at least half of the votes and belongs to the win set. The same argument applies for $y \leq 0$. ■

Fix a $v \in V$. From Lemma A.3, an incumbent with valence v and ideology $x \in [0, w_v]$ adopts her own policy and is re-elected, and an incumbent with ideology $x > w_v$ who chooses to compromise adopt policy w_v because $w_v = \arg\min_{y \in W_v^R}(|x - y|)$. Similarly, an incumbent $x < -w_v$ who compromises adopts policy $-w_v$. For an incumbent with valence v and ideology $x > w_v$, the value of compromising to win if she runs for re-election is $U_x(w_v, v|W, C) + k\rho$, while the value of adopting her own ideology is $(1-\delta)(v+\rho)+\delta\overline{U}_x(W, C)$. For an incumbent with valence v and ideology $x > w_v$, define $\Psi(x, v|W, C)$ to be the net value of compromising:

$$\Psi(x, v|W, C) \equiv \delta(1-q)k(v+\rho) + k\,l(x - w_v) - \delta(1-q)k\overline{U}_x(W, C). \tag{21}$$

The incumbent compromises to w_v if and only if $\Psi(x, v|W, C) \geq 0$. For incumbent $x = w_v$, $\Psi(x, v|W, C) > 0$. Therefore, the necessary condition for the compromise set C_v^R to be connected is that $\Psi(x, v|W, C)$ crosses zero at most once for $x \in [w_v, a]$. A sufficient condition is that $\Psi(x, v|W, C)$ is concave in the range $x \in [w_v, a]$.

Lemma A. 5 *There exists a bound $M''' > 0$ such that if $|l'''| \leq M'''$ then $\Psi(x, v|W, C)$ is concave. Hence, for each valence $v \in V$, the compromise set consists of two symmetric, connected intervals around the win set, i.e., $C_v^L = [-c_v, -w_v]$ and $C_v^R = [w_v, c_v]$.*

Proof: Fix a $\tilde{v} \in V$. For $x > w_v$, after some algebra we can rewrite $\Psi(x, \tilde{v}|W, C)$ as

$$\begin{aligned}
& \Psi(x, \tilde{v}|W, C) \\
= \ & \delta(1-q)k[\tilde{v} + \rho - E^*(v)] \\
+ \ & \frac{k}{1-\beta} \int_V \Bigg\{ 2 \int_{-w_v}^{0} \left[l(x - w_v) - \delta(1-q) \frac{[(2-\beta)l(x-y) + \beta l(x+y)]}{2} \right] dF(y) \\
+ \ & 2 \int_{C_v^L} \left[l(x - w_v) - \delta(1-q) \frac{[(2-\beta)l(x+w_v) + \beta l(x-w_v)]}{2} \right] dF(y) \\
+ \ & [1 - \delta(1-q)] 2 \int_{E_v^L} \left[l(x - w_v) - \delta(1-q) \frac{[(2-\beta)l(x-y) + \beta l(|x+y|)]}{2} \right] dF(y) \Bigg\} dG(v) .
\end{aligned}$$

The second derivative with respect to x is

$$\begin{aligned}
& \frac{\partial^2}{\partial x^2} \Psi(x, \tilde{v}|W, C) \\
= \ & \frac{k}{1-\beta} \int_V \Bigg\{ 2 \int_{-w_v}^{0} \left[l''(x - w_v) - \delta(1-q) \frac{[(2-\beta)l''(x-y) + \beta l(x''+y)]}{2} \right] dF(y) \\
+ \ & 2 \int_{C_v^L} \left[l''(x - w_v) - \delta(1-q) \frac{[(2-\beta)l''(x+w_v) + \beta l''(x-w_v)]}{2} \right] dF(y) \\
+ \ & [1 - \delta(1-q)] 2 \int_{E_v^L} \left[l''(x - w_v) - \delta(1-q) \frac{(2-\beta)[l''(x-y) + \beta l''(|x+y|)]}{2} \right] dF(y) \Bigg\} dG(v) .
\end{aligned}$$

If $l''' = 0$, then l'' is a constant $l'' \leq 0$ and $\frac{\partial^2}{\partial x^2}\Psi(x, \tilde{v}|W, C) = kl''(1-\delta(1-q)) \leq 0$. Therefore, there exists a bound $0 < M'''$ such that if $|l'''| \leq M'''$ then $\Psi(x, v|W, C)$ is concave.

In particular, these conditions are satisfied by both Euclidean and quadratic loss functions. The condition requires that the risk aversion of citizens cannot grow too quickly (the second derivative cannot fall too fast), else compromise sets may not be connected—some representatives may prefer to lose the election rather than compromise, while representatives with more extreme ideologies may become so risk averse that they prefer to compromise. For example, suppose voter's loss function $l(|x - y|)$ is piecewise linear in policy distance $|x - y|$, dropping off precipitously when someone locates further than $\hat{y}$ from a voter's bliss point, $|x - y| > \hat{y}$. Then because some untried challengers can choose policies further than $\hat{y}$ from extremists, the increasing disutility can induce officeholders with extreme ideologies to compromise, but not those with more moderate ideologies. ■

Lemma A. 6 *If $U_x(0,v|W,C) - U_x^R(W,C)$ does not increase in x for any $x > 0$, then the win set is contained in the retrospective set of the median voter,*

1. *If $y \in W_v^R$, then $(y,v) \in \mathbf{S}_0^R$;*
2. *If $y \in W_v^L$, then $(y,v) \in \mathbf{S}_0^L$.*

Proof: First notice that if $U_x(0,v|W,C) - U_x^R(W,C)$ does not increase in x for any $x > 0$, then $U_x(y,v|W,C) - U_x^R(W,C)$ also does not increase in x for any $x > 0$ and $y < 0$, since $U_x(y,v|W,C)$ decreases at least as fast as $U_x(0,v|W,C)$ from concavity. We will show that if $y \notin \mathbf{S}_0^L$, then $y \notin W_v^L$. Let $y \notin \mathbf{S}_0^L \Rightarrow 0 > U_0(y,v|W,C) - U_0^R(W,C)$ and $y < 0$. For every voter $x > 0$, the assumption implies that $U_0(y,v|W,C) - U_0^R(W,C) \geq U_x(y,v|W,C) - U_x^R(W,C)$, which implies $U_x^R(W,C) > U_x(y,v|W,C)$. All voters with ideology $x \in [0,a]$ vote for the challenger and the incumbent will not be re-elected. Therefore, $y \notin W_v^L$. Analogously, we can show that any $y \notin \mathbf{S}_0^R$ and $y > 0$ does not belong to the win set. ■

Lemma A. 7 *There exists a lower bound $M'' < 0$ such that if $M'' \leq l'' \leq 0$ then $U_x(0,v|W,C) - U_x^R(W,C)$ does not increase in x for any $x > 0$.*

Proof: Fix a $\tilde{v} \in V$. For $x > 0$, after some algebra, one can solve for

$$\begin{aligned}
& U_x(0,\tilde{v}|W,C) - U_x^R(W,C) \\
= \; & k[\tilde{v} - E^*(v)] + \frac{k}{1-\beta}\int_V \left\{ 2\int_0^{w_v} \left[l(x) - \frac{[(2-\beta)l(|x-y|) + \beta l(x+y)]}{2} \right] dF(y) \right. \\
+ \; & 2\int_{w_v}^{c_v} \left[l(x) - \frac{[(2-\beta)l(|x-w_v|) + \beta l(x+w_v)]}{2} \right] dF(y) \\
+ \; & \left. [1-\delta(1-q)]2\int_{c_v}^{a} \left[l(x) - \frac{[(2-\beta)l(|x-y|) + \beta l(x+y)]}{2} \right] dF(y) \right\} dG(v).
\end{aligned}$$

The first derivative with respect to x is

$$\begin{aligned}
& \frac{\partial}{\partial x}[U_x(0,\tilde{v}|W,C) - U_x^R(W,C)] \\
= \; & \frac{k}{1-\beta}\int_V \left\{ 2\int_0^{w_v} \left[\frac{\partial}{\partial x}l(x) - \frac{[(2-\beta)\frac{\partial}{\partial x}l(|x-y|) + \beta\frac{\partial}{\partial x}l(x+y)]}{2} \right] dF(y) \right. \\
+ \; & 2\int_{w_v}^{c_v} \left[\frac{\partial}{\partial x}l(x) - \frac{[(2-\beta)\frac{\partial}{\partial x}l(|x-w_v|) + \beta\frac{\partial}{\partial x}l(x+w_v)]}{2} \right] dF(y) \\
+ \; & \left. [1-\delta(1-q)]2\int_{c_v}^{a} \left[\frac{\partial}{\partial x}l(x) - \frac{[(2-\beta)\frac{\partial}{\partial x}l(|x-y|) + \beta\frac{\partial}{\partial x}l(x+y)]}{2} \right] dF(y) \right\} dG(v).
\end{aligned}$$

If $l'' = 0$, this first derivative is indeed negative, because $l'(x) < 0$ and the absolute value of $\frac{\partial}{\partial x} l(|x - y|)$ is constant in x, y. Therefore, the term inside each integral is zero if $x \geq y$ and *strictly negative* if $x < y$. Therefore, there is a lower bound $M'' < 0$ such that if $M'' \leq l'' \leq 0$ then $U_x(0, \tilde{v}|W, C) - U_x^R(W, C)$ decreases in x.

The condition $\frac{\partial}{\partial x}\left[U_x(0, \tilde{v}|W, C) - U_x^R(W, C)\right] \leq 0$ is satisfied by Euclidean and quadratic loss functions. ■

Therefore, combining Lemmas A.4–A.7, the median voter is decisive and her retrospective set is defined as follows. From symmetry, $U_0^R(W, C) = U_0^L(W, C) = \overline{U}_0(W, C)$. An incumbent with valence $v \in V$ belongs to the retrospective set of the median voter if and only if she implements policy y such that

$$ku_0(y, v) + k\frac{\delta q}{1 - \delta}\overline{U}_0(W, C) - \overline{U}_0(W, C) \geq 0 \quad \Leftrightarrow \quad ku_0(y, v) - k\overline{U}_0(W, C) \geq 0$$
$$\Leftrightarrow \quad v + L_0(|y|) \geq \overline{U}_0(W, C). \qquad (22)$$

Define the threshold function $w : V \to [0, a]$ as the most extreme policy $w(v)$ taken by an incumbent with valence v from party R such that the median voter would vote to re-elect the incumbent. That is, $w(v) = |l^{-1}(U_0(W, C) - v)|$ where $l^{-1}(\cdot)$ denotes the inverse function of $l(\cdot)$. The retrospective set of the median voter is $\mathbf{S}_0 = \{(y, v)|v \in V, \ y \in [-w(v), w(v)]\}$. The following Lemma guarantees that solutions are interior.

Lemma A. 8 *There exists $\overline{v} > 0$ and $\overline{\rho} > 0$ such that if $V_H - V_L \leq \overline{v}$ and $\rho \leq \overline{\rho}$, then every equilibrium (w, c) is interior, $0 < w_v < c_v < a$, for each $v \in V$.*

Proof: From Lemma A.1, $w_v > 0$ holds since $U_x(0, v|W, C) > U_x^L(W, C)$ for a strict majority of voters when $\overline{v}$ is sufficiently small. $c_v > w_v$ follows from the result that the net value of compromising for w_v is $\Psi(w_v, v|W, C) > 0$. Bounding office benefits, $\rho \leq \overline{\rho}$, appropriately ensures that $a > c_v$. ■

Using (22), the decisive median voter defines re-election cutoffs $\overline{U}_0(W, C) = v + L_0(w_v), \forall v \in V$. Moreover, from (21), each compromise cutoff $c_v \in (w_v, a)$ solves $\Psi(c_v, v|W, C) = 0$. Hence, under the conditions of the theorem, *every* equilibrium is fully characterized by functions $w, c : V \to (0, a)$ that satisfy the following equations for all $v \in V$:

$$U_0(w_v, v|W, C) = U_0^R(W, C) = U_0^L(W, C) = \overline{U}_0(W, C) = v + L_0(w_v), \tag{23}$$

$$k[v + L_{c_v}(w_v)] + k\tfrac{\delta q}{1-\delta}\overline{U}_{c_v}(W, C) + \rho k = (v + \rho)(1 - \delta) + \delta\overline{U}_{c_v}(W, C). \tag{24}$$

As an intermediate step to proving existence and uniqueness of equilibrium, we now prove Proposition 1 from page 15.

Proof: [**Proposition 1**] Let $v_H, v_L \in V$ and $v_H > v_L$. From equation (23), $v_H + L_0(w_H) = v_L + L_0(w_L)$, so that $L_0(w_L) - L_0(w_H) = v_H - v_L > 0$, i.e., $L_0(w_L) > L_0(w_H)$, and $l' < 0$ implies $w_H > w_L$.

From our equilibrium characterization, $c_H > w_H$. Thus, trivially if $c_L \le w_H$ then $c_H > c_L$. It remains to show that $c_H > c_L$ holds when $c_L > w_H$. Assume $c_L > w_H$. In equilibrium, an office-holder with valence v_L and ideology c_L is indifferent between compromising to policy w_L and adopting her own ideology. From the indifference equation (24)

$$k[v_L + L_{c_L}(w_L)] + k\frac{\delta q}{1-\delta}\overline{U}_{c_L}(w, c) + \rho k \quad = \quad (v_L + \rho)(1 - \delta) + \delta\overline{U}_{c_L}(w, c) \,. \tag{25}$$

The LHS of (25) is the expected payoff of compromising and the RHS is the expected payoff of adopting her own ideology. It suffices to show that an office-holder with ideology $x = c_L$ and valence v_H strictly prefers compromising to adopting her own ideology, i.e.,

$$k[v_H + L_{c_L}(w_H)] + k\frac{\delta q}{1-\delta}\overline{U}_{c_L}(w, c) + \rho k > (v_H + \rho)(1 - \delta) + \delta\overline{U}_{c_L}(w, c) \,. \tag{26}$$

Subtracting equation (25) from (26), we must show that

$$\begin{aligned} k[v_H - v_L + L_{c_L}(w_H) - L_{c_L}(w_L)] \quad &> \quad (v_H - v_L)(1 - \delta), \\ \Leftrightarrow (v_H - v_L)(k - 1 + \delta) + k[L_{c_L}(w_H) - L_{c_L}(w_L)] &> 0. \end{aligned} \tag{27}$$

The first term is strictly positive since $k > 1 - \delta$. Furthermore, $c_L > w_H > w_L$ implies that $(c_L - w_H) < (c_L - w_L)$. Therefore, $L_{c_L}(w_H) > L_{c_L}(w_L)$ and the second term is also strictly positive. Thus, the inequalities in (26) and (27) hold, establishing $c_H > c_L$.

To show that $c_H - w_H > c_L - w_L$, subtract the indifference equation (25) for a low valence candidate from the indifference condition for a high valence office holder with ideology c_H,

$$k[v_H + L_{c_H}(w_H)] + k\frac{\delta q}{1-\delta}\overline{U}_{c_H}(w, c) + \rho k \quad = \quad (v_H + \rho)(1 - \delta) + \delta\overline{U}_{c_H}(w, c) \,, \tag{28}$$

to obtain

$$\begin{aligned} & k[v_H - v_L + L_{c_H}(w_H) - L_{c_L}(w_L)] + k\frac{\delta q}{1-\delta}[\overline{U}_{c_H}(w, c) - \overline{U}_{c_L}(w, c)] \\ = \quad & (v_H - v_L)(1 - \delta) + \delta[\overline{U}_{c_H}(w, c) - \overline{U}_{c_L}(w, c)]. \end{aligned} \tag{29}$$

Rewrite this as

$$k[L_{c_H}(w_H) - L_{c_L}(w_L)] = (1-\delta-k)(v_H - v_L) + k\delta(1-q)[\overline{U}_{c_H}(w,c) - \overline{U}_{c_L}(w,c)]. \quad (30)$$

$k > 0$ implies that $c_H - w_H > c_L - w_L$ if and only if the LHS of equation (30) is strictly negative. Hence, we must show that the RHS is strictly negative. The term $(1-\delta-k)(v_H - v_L)$ is strictly negative, and $k\delta(1-q) > 0$. So it remains to show that $\overline{U}_{c_H}(w,c) - \overline{U}_{c_L}(w,c) < 0$; but this follows from Lemma A.2 and the result $c_H > c_L$. ■

Lemma A. 9 *Fix the parameters of the model, and take any equilibrium thresholds (w, c) and (w', c'). Then*

1. *The change in the threshold function w is strictly monotone. That is, for every pair of valences $v, \tilde{v} \in V$,*

$$w'_v > w_v \Rightarrow w'_{\tilde{v}} > w_{\tilde{v}}; \quad (31)$$

2. *There exists a $\bar{v} > 0$ such that if $V_H - V_L \leq \bar{v}$, then the change in the threshold function c is weakly monotone. That is, for every pair of valences $v, \tilde{v} \in V$,*

$$c'_v > c_v \Rightarrow c'_{\tilde{v}} \geq c_{\tilde{v}}. \quad (32)$$

Proof: Fix the parameters of the model and let (w, c) and (w', c') be equilibrium thresholds. From (23), $\overline{U}_0(w,c) = v + L_0(w_v)$. Hence, $v + L_0(w_v) = \tilde{v} + L_0(w_{\tilde{v}})$ and $v + L_0(w'_v) = \tilde{v} + L_0(w'_{\tilde{v}})$ for every $v, \tilde{v} \in V$. Therefore,

$$L_0(w'_v) - L_0(w_v) \quad = \quad L_0(w'_{\tilde{v}}) - L_0(w_{\tilde{v}}),$$

for every $v, \tilde{v} \in V$. Since $l' < 0$, if for any $v \in V$ we have an increase from w_v to $w'_v > w_v$ then for all other valences $\tilde{v} \in V$ we must have $w'_{\tilde{v}} > w_{\tilde{v}}$.

Moreover, for $V_H - V_L$ sufficiently small, (i) thresholds w_v are arbitrarily close to each other, and (ii) thresholds c_v are arbitrarily close to each other. Result (i) follows directly from the median voter's indifference condition. Result (ii) follows from result (i), and the fact that an incumbent's expected utility from being replaced by an untried candidate is a continuous function of her own ideology.

Since equation (31) holds, without loss of generality, let $w'_v \geq w_v$ for all $v \in V$. First suppose that $c'_v < c_v$ for some $v \in V$. In equilibrium (w, c), the incumbent with valence v and ideology c_v is indifferent between compromising or not,

$$k[v + L_{c_v}(w_v)] + k\frac{\delta q}{1-\delta}\overline{U}_{c_v}(w, c) + \rho k \quad = \quad (v+\rho)(1-\delta) + \delta\overline{U}_{c_v}(w, c)\,. \tag{33}$$

In equilibrium (w', c'), incumbent c'_v is indifferent between compromising or not, which implies that incumbent $c_v > c'_v$ strictly prefers to not compromise,

$$k[v + L_{c_v}(w'_v)] + k\frac{\delta q}{1-\delta}\overline{U}_{c_v}(w', c') + \rho k \quad < \quad (v+\rho)(1-\delta) + \delta\overline{U}_{c_v}(w', c')\,. \tag{34}$$

Subtract equation (33) from (34). After some algebra, we have

$$L_{c_v}(w'_v) - L_{c_v}(w_v) < \delta(1-q)[\overline{U}_{c_v}(w', c') - \overline{U}_{c_v}(w, c)]. \tag{35}$$

From continuity of the loss function, for any valence $\tilde{v}$ sufficiently close to v and any ideology $c_{\tilde{v}}$ sufficiently close to c_v we have

$$L_{c_{\tilde{v}}}(w'_{\tilde{v}}) - L_{c_{\tilde{v}}}(w_{\tilde{v}}) \leq \delta(1-q)[\overline{U}_{c_{\tilde{v}}}(w', c') - \overline{U}_{c_{\tilde{v}}}(w, c)].$$

This implies that the incumbent with valence $\tilde{v}$ and ideology $c_{\tilde{v}}$ also (weakly) prefers not to compromise in equilibrium (w', c'). Therefore, $c'_{\tilde{v}} \leq c_{\tilde{v}}$ for every $\tilde{v} \in V$ if $V_H - V_L$ is sufficiently small. An analogous argument holds for the case $c'_v > c_v$.

When the loss function is quadratic, equation (32) holds for any $V_H - V_L$. ■

Lemma A. 10 *Take any ideology x, $|x| < a$, and valence $v \in V$. There exists bounds $\overline{v} > 0$ and $\tilde{M}'' < 0$ such that if $V_H - V_L \leq \overline{v}$ and $\tilde{M}'' \leq l'' \leq 0$ then equilibrium (w, c) imply*

$$v + \frac{L_x(w_v) + L_x(-w_v)}{2} \geq \big[1 - \delta(1-q)\big]\left[v + \frac{L_x(c_v) + L_x(-c_v)}{2}\right] + \delta(1-q)\overline{U}_x(w, c). \tag{36}$$

Proof: Define

$$\Gamma(x) \equiv v + \frac{L_x(w_v) + L_x(-w_v)}{2} - \big[1 - \delta(1-q)\big]\left[v + \frac{L_x(c_v) + L_x(-c_v)}{2}\right] - \delta(1-q)\overline{U}_x(w, c). \tag{37}$$

Take any ideology x, $|x| < a$. From symmetry, we can focus on $x \geq 0$. For the median voter, $v + L_0(w_v) = v + L_0(-w_v) = \overline{U}_0(w, c)$, therefore $\Gamma(0) > 0$.

Consider $l'' = 0$ (Euclidean loss function). It is easy to show that for any $x \in (0, a)$, $\frac{\partial \overline{U}_x(w,c)}{\partial x} \in (-1, 0)$. Therefore, $\Gamma(x)$ increases in $x \in [0, w_v]$ and decreases in $x \in [w_v, a)$: $\frac{\partial \Gamma(x)}{\partial x} = -\delta(1-q)\frac{\partial \overline{U}_x(w,c)}{\partial x} > 0$ for $x \in [0, w_v]$; $\frac{\partial \Gamma(x)}{\partial x} = -1-\delta(1-q)\frac{\partial \overline{U}_x(w,c)}{\partial x} < 0$ for $x \in [w_v, c_v]$; and $\frac{\partial \Gamma(x)}{\partial x} = -1 + (1-\delta(1-q)) - \delta(1-q)\frac{\partial \overline{U}_x(w,c)}{\partial x} < 0$ for $x \in [c_v, a)$.

Consequently, it is sufficient to show that if $\Gamma(\cdot)$ crosses zero at some $\Gamma(x') \in [0, a)$, then $x' \geq x$. At $x = a$, $\overline{U}_a(w,c) = E^*(v) - a \Rightarrow \Gamma(a) = v - a - (1-\delta(1-q))(v-a) - \delta(1-q)[E^*(v) - a] = \delta(1-q)[v - E^*(v)]$. If $v - E^*(v) \geq 0$ then we are done. Otherwise, for any given $x < a$, we require the upper bound on valence to be sufficiently small so that at the x' such that $\Gamma(x') = 0$ we have $x' \geq x$.

This implies that there is a lower bound $\tilde{M}'' < 0$ such that if $\tilde{M}'' \leq l'' \leq 0$ then equation (36) holds. In particular, equation (36) holds for a quadratic loss function: if the loss function is quadratic, then $\frac{\partial \overline{U}_x(w,c)}{\partial x} = -2x \Rightarrow \frac{\partial \Gamma(x)}{\partial x} = 0$, so that $\Gamma(x) > 0$ for all x.

Notice that result 2 always holds with quadratic utility, because changes in continuation values affect all voters in the same way (see equation 44). ■

Lemma A. 11 *If conditions C1 to C3 of Theorem 1 hold, then the system*

$$U_0(w_v, v|w,c) = U_0^R(w,c) = U_0^L(w,c) = \overline{U}_0(w,c) = v + L_0(w_v) \tag{38}$$

$$U_{c_v}(w_v, v|w,c) + k\rho = (1-\delta)(v+\rho) + \delta\overline{U}_{c_v}(w,c) \tag{39}$$

$\forall v \in V$ has a unique solution (w,c).

Proof: Existence follows from a fixed point argument on the expected discounted utility of the median voter from electing an untried candidate. Provided that $V_H - V_L$ is sufficiently small, the median voter's expected utility from an untried challenger is contained in the interval $D_0 \equiv [V_H + L_0(a), V_L]$. For every $u_0 \in D_0$, equation (38) defines a unique vector of re-election cutoffs w. This mapping $w(u_0)$ is continuous on u_0 and defines a compact, convex set of cutoffs $[w_L(u_0), w_H(u_0)]$. Given the re-election cutoff vector $w(u_0)$ and any arbitrary compromising cutoff vector $c \in D_c \equiv [W_L(u_0), a] \times \ldots \times [W_H(u_0), a]$, it is straightforward to compute expected utilities $\overline{U}_x$ and U_x^L for each citizen-candidate. For each valence $v \in V$, one can find the most extreme compromising ideology $c'_v \in [W_v(u_0), a]$ such that

$$U_{c'_v}(w_v, v|w,c) + k\rho \geq (1-\delta)(v+\rho) + \delta\overline{U}_{c'_v}(w,c). \tag{40}$$

Let c' be the vector of all c'_v; notice that, for any fixed $w(u_0)$, there exists a unique c' for each c. We need to show that, for any given $u_0 \in D_0$, the mapping implied by condition (40) has a fixed point $c' = c$. To see this, notice that this compromising condition defines a mapping from the compact convex set of feasible compromising thresholds, $D_c \equiv [W_L(u_0), a] \times \ldots \times [W_H(u_0), a]$, to itself. Moreover, from the perspective of a current incumbent, both the expected utility from an untried candidate and the cost of compromising is continuous in the incumbent's ideology/valence. Hence, the compromising condition defines a continuous mapping from D_c into a compact/connected subset of D_c. Therefore, a fixed point $c(u_0)$ exists. Moreover, one can follow the argument in the second part of Lemma A.9 to define the sufficient conditions under which this fixed point is unique and continuous on u_0. Together, $w(u_0)$ and $c(u_0)$ define a unique expected utility u'_0 for the median voter. u'_0 is continuous on $w(u_0)$ and $c(u_0)$. For $V_H - V_L$ sufficiently small, then one can show that u'_0 always belongs to D_0. Therefore, we have a continuous function from D_0 into itself, and there exists a fixed point in the discounted expected utility of the median voter from electing an untried candidate.

To prove uniqueness, by contradiction, suppose (w, c) and (w', c') are both equilibria, $(w, c) \neq (w', c')$. Exploiting Lemma A.9, without loss of generality, let $w'_v \geq w_v$. Furthermore, for $V_H - V_L$ sufficiently small, the threshold function c is weakly monotone. Hence, it suffices to consider the following two cases.

Case 1) Suppose $c'_v \geq c_v$ for every $v \in V$. $w'_v \geq w_v$ implies that the median voter is (weakly) worse off, $\overline{U}_0(w', c') \leq \overline{U}_0(w, c)$. We show that if incumbents do not become more extreme by reducing the thresholds c_v, then the more extreme positions w'_v do not decrease the expected utility of the median voter sufficiently, violating her equilibrium condition $\overline{U}_0(w', c') = v + L_0(w'_v)$. By definition,

$$\begin{aligned}
\overline{U}_0(w', c') \quad - \quad \overline{U}_0(w, c) \;=\; & \int_V \Bigg\{ 2\int_0^{w_v} \left[k[0] + k\frac{\delta q}{1-\delta}[\overline{U}_0(w', c') - \overline{U}_0(w, c)] \right] dF(y) \\
+ \quad & 2\int_{w_v}^{w'_v} \left[k[l(y) - l(w_v)] + k\frac{\delta q}{1-\delta}[\overline{U}_0(w', c') - \overline{U}_0(w, c)] \right] dF(y) \\
+ \quad & 2\int_{w'_v}^{c_v} \left[k[l(w'_v) - l(w_v)] + k\frac{\delta q}{1-\delta}[\overline{U}_0(w', c') - \overline{U}_0(w, c)] \right] dF(y) \\
+ \quad & 2\int_{c_v}^{c'_v} \left[k[v + l(w'_v)] + k\frac{\delta q}{1-\delta}\overline{U}_0(w', c') - (1-\delta)[v + l(y)] - \delta\overline{U}_0(w, c) \right] dF(y) \\
+ \quad & 2\int_{c'_v}^{a} \left[(1-\delta)[0] + \delta\left[\overline{U}_0(w', c') - \overline{U}_0(w, c)\right] \right] dF(y) \Bigg\} dG(v).
\end{aligned}$$

We now replace terms in the RHS by strictly smaller terms to show that the RHS is strictly positive, a contradiction to $\overline{U}_0(w', c') - \overline{U}_0(w, c) \leq 0$. For each $v \in V$, exploit concavity and replace the expression inside the first two integrals with the smaller number $\left[k[l(w'_v) - l(w_v)] + k\frac{\delta q}{1-\delta}[\overline{U}_0(w', c') - \overline{U}_0(w, c)]\right]$, strictly smaller if $w'_v > w_v$. From equilibrium, $l(w'_v) - l(w_v) = \overline{U}_0(w', c') - \overline{U}_0(w, c)$ and since $k(1 + \frac{\delta q}{1-\delta}) = 1$, the term inside each of the first three integrals simplifies to $\overline{U}_0(w', c') - \overline{U}_0(w, c)$. In the fourth integral, replace the term $-(1-\delta)[v + l(y)]$ with the strictly smaller number $-(1-\delta)[v + l(w_v)]$. Exploiting the equilibrium condition, replace $v + l(w'_v)$ with $\overline{U}_0(w', c')$ and replace $v + l(w_v)$ with $\overline{U}_0(w, c)$. Again, the expression simplifies to $\overline{U}_0(w', c') - \overline{U}_0(w, c)$ and we have

$$\overline{U}_0(w', c') - \overline{U}_0(w, c) \;>\; [\overline{U}_0(w', c') - \overline{U}_0(w, c)] \int_V \left\{ 2\int_0^{c'_v} dF(y) + 2\delta \int_{c'_v}^a dF(y) \right\} dG(v).$$

Since $\int_V \left\{ 2\int_0^{c'_v} dF(y) + 2\delta \int_{c'_v}^a dF(y) \right\} dG(v) < 1$, it must be the case that $\overline{U}_0(w', c') - \overline{U}_0(w, c) > 0$, a contradiction to $w'_v \geq w_v$.

Case 2) Suppose $c'_{\tilde{v}} < c_{\tilde{v}}$ for at least one $\tilde{v} \in V$. From Lemma A.9, monotonicity implies $c'_v \leq c_v$ for all $v \in V$. We show that if incumbents implement more extreme policies and re-election cutoffs are slacker, then more incumbent types should compromise to avoid losing re-election, a contradiction.

Fix valence $\tilde{v}$. Under equilibrium (w, c), incumbent $c_{\tilde{v}}$ is indifferent between compromising and not. Under equilibrium (w', c'), incumbent $c_{\tilde{v}}$ strictly prefers not to compromise, since $c'_{\tilde{v}} < c_{\tilde{v}}$. From inequality (35),

$$L_{c_{\tilde{v}}}(w'_{\tilde{v}}) - L_{c_{\tilde{v}}}(w_{\tilde{v}}) \;<\; \delta(1-q)[\overline{U}_{c_{\tilde{v}}}(w', c') - \overline{U}_{c_{\tilde{v}}}(w, c)].$$

Since $c_{\tilde{v}} > c'_{\tilde{v}} > w'_{\tilde{v}} \geq w_{\tilde{v}}$, $L_{c_{\tilde{v}}}(w'_{\tilde{v}}) - L_{c_{\tilde{v}}}(w_{\tilde{v}}) \geq 0$. Next we show that $\overline{U}_{c_{\tilde{v}}}(w', c') - \overline{U}_{c_{\tilde{v}}}(w, c) \leq 0$, a contradiction.

$$\begin{aligned}
& \overline{U}_{c_{\tilde{v}}}(w', c') - \overline{U}_{c_{\tilde{v}}}(w, c) \\
= \;& 2\int_V \left\{ \int_0^{w_v} \left[k[0] + k\frac{\delta q}{1-\delta}[\overline{U}_{c_{\tilde{v}}}(w', c') - \overline{U}_{c_{\tilde{v}}}(w, c)] \right] dF(y) \right. \\
+ \;& \int_{w_v}^{w'_v} \left[k[L_{c_{\tilde{v}}}(y) + L_{c_{\tilde{v}}}(-y) - L_{c_{\tilde{v}}}(w_v) - L_{c_{\tilde{v}}}(-w_v)] + k\frac{\delta q}{1-\delta}[\overline{U}_{c_{\tilde{v}}}(w', c') - \overline{U}_{c_{\tilde{v}}}(w, c)] \right] dF(y) \\
+ \;& \int_{w'_v}^{c'_v} \left[k[L_{c_{\tilde{v}}}(w'_v) + L_{c_{\tilde{v}}}(-w'_v) - L_{c_{\tilde{v}}}(w_v) - L_{c_{\tilde{v}}}(-w_v)] + k\frac{\delta q}{1-\delta}[\overline{U}_{c_{\tilde{v}}}(w', c') - \overline{U}_{c_{\tilde{v}}}(w, c)] \right] dF(y) \\
+ \;& \int_{c'_v}^{c_v} \left[(1-\delta)[2v + L_{c_{\tilde{v}}}(y) + L_{c_{\tilde{v}}}(-y)] + 2\delta \overline{U}_{c_{\tilde{v}}}(w', c') \right. \\
- \;& \left. k[2v + L_{c_{\tilde{v}}}(w_v) + L_{c_{\tilde{v}}}(-w_v)] - k\frac{\delta q}{1-\delta}\overline{U}_{c_{\tilde{v}}}(w, c) \right] dF(y) \\
+ \;& \left. \int_{c_v}^a \left[(1-\delta)[0] + 2\delta \left[\overline{U}_{c_{\tilde{v}}}(w', c') - \overline{U}_{c_{\tilde{v}}}(w, c) \right] \right] dF(y) \right\} dG(v).
\end{aligned}$$

Shifting comparable continuation payoffs to the LHS yields

$$\left[\overline{U}_{c_{\tilde v}}(w',c') - \overline{U}_{c_{\tilde v}}(w,c)\right]\left[1 - 2\int_V \left\{\int_0^{c_v} k\frac{\delta q}{1-\delta} dF(y) - \int_{c_v}^a \delta dF(y)\right\} dG(v)\right]$$

$$\begin{aligned}
= \quad & 2\int_V \left\{\int_{w_v}^{w'_v} k\left[L_{c_{\tilde v}}(y) + L_{c_{\tilde v}}(-y) - L_{c_{\tilde v}}(w_v) - L_{c_{\tilde v}}(-w_v)\right] dF(y)\right. \\
+ \quad & \int_{w'_v}^{c'_v} k\left[L_{c_{\tilde v}}(w'_v) + L_{c_{\tilde v}}(-w'_v) - L_{c_{\tilde v}}(w_v) - L_{c_{\tilde v}}(-w_v)\right] dF(y) \\
+ \quad & \int_{c'_v}^{c_v} \big[(1-\delta)[2v + L_{c_{\tilde v}}(y) + L_{c_{\tilde v}}(-y)] + 2\delta\overline{U}_{c_{\tilde v}}(w',c') \\
- \quad & \left. k[2v + L_{c_{\tilde v}}(w_v) + L_{c_{\tilde v}}(-w_v)] - 2k\frac{\delta q}{1-\delta}\overline{U}_{c_{\tilde v}}(w',c')\big] dF(y)\right\} dG(v).
\end{aligned}$$

On the LHS,

$$0 < \left[1 - 2\int_V \left\{\int_0^{c_v} k\frac{\delta q}{1-\delta} dF(y) - \int_{c_v}^a \delta dF(y)\right\} dG(v)\right] < 1.$$

On the RHS, the first and second integrals are negative from the concavity of the loss function. To derive a contradiction, it is sufficient to show that

$$\begin{aligned}
& \int_V \left\{ \int_{c'_v}^{c_v} \big[(1-\delta)[2v + L_{c_{\tilde v}}(y) + L_{c_{\tilde v}}(-y)] + 2\delta\overline{U}_{c_{\tilde v}}(w',c') \right. \\
- \quad & \left. k[2v + L_{c_{\tilde v}}(w_v) + L_{c_{\tilde v}}(-w_v)] - 2k\frac{\delta q}{1-\delta}\overline{U}_{c_{\tilde v}}(w',c')\big] dF(y)\right\} dG(v) \le 0.
\end{aligned} \tag{41}$$

Since $c'_v < c_v$ for at least one valence, concavity of the loss function implies that the LHS of (41) is strictly less than

$$\begin{aligned}
& \int_V \left\{ \int_{c'_v}^{c_v} \big[(1-\delta)[2v + L_{c_{\tilde v}}(c'_v) + L_{c_{\tilde v}}(-c'_v)] + 2\delta\overline{U}_{c_{\tilde v}}(w',c') \right. \\
- \quad & \left. k[2v + L_{c_{\tilde v}}(w_v) + L_{c_{\tilde v}}(-w_v)] - 2k\frac{\delta q}{1-\delta}\overline{U}_{c_{\tilde v}}(w',c')\big] dF(y)\right\} dG(v).
\end{aligned}$$

Hence, it suffices to show

$$k[2v + L_{c_{\tilde v}}(w_v) + L_{c_{\tilde v}}(-w_v)] + 2k\frac{\delta q}{1-\delta}\overline{U}_{c_{\tilde v}}(w',c') \ge (1-\delta)[2v + L_{c_{\tilde v}}(c'_v) + L_{c_{\tilde v}}(-c'_v)] + 2\delta\overline{U}_{c_{\tilde v}}(w',c')$$

for every $v \in V$. Divide both sides by $2k = 2\frac{1-\delta}{1-\delta+\delta q} > 0$, and simplify the inequality to

$$v + \frac{L_{c_{\tilde v}}(w_v) + L_{c_{\tilde v}}(-w_v)}{2} \quad \ge \quad \left[1 - \delta(1-q)\right]\left[v + \frac{L_{c_{\tilde v}}(c'_v) + L_{c_{\tilde v}}(-c'_v)}{2}\right] + \delta(1-q)\overline{U}_{c_{\tilde v}}(w',c').$$

Since $w_v \le w'_v \Rightarrow L_{c_{\tilde v}}(w_v) + L_{c_{\tilde v}}(-w_v) \ge L_{c_{\tilde v}}(w'_v) + L_{c_{\tilde v}}(-w'_v)$, it suffices to show

$$v + \frac{L_{c_{\tilde v}}(w'_v) + L_{c_{\tilde v}}(-w'_v)}{2} \quad \ge \quad \left[1 - \delta(1-q)\right]\left[v + \frac{L_{c_{\tilde v}}(c'_v) + L_{c_{\tilde v}}(-c'_v)}{2}\right] + \delta(1-q)\overline{U}_{c_{\tilde v}}(w',c').$$

Let $x = c_{\tilde{v}} < a$, the result then follows from equation (36). This concludes the proof of Theorem 1. ■

Proof: [**Proposition 2**] Define the expected policy (in absolute value) of an untried candidate with valence v,

$$EPol(v) = 2\Big\{\int_0^{w_v} yf(y)dy + \int_{w_v}^{c_v} w_v f(y)dy + \int_{c_v}^{a} yf(y)dy\Big\}. \tag{42}$$

Taking derivatives with respect to v,

$$\begin{aligned}\frac{\partial EPol(v)}{\partial v} &= 2\Big\{\frac{\partial w_v}{\partial v}w_v f(w_v) + \frac{\partial c_v}{\partial v}w_v f(c_v) - \frac{\partial w_v}{\partial v}w_v f(w_v) + \int_{w_v}^{c_v}\frac{\partial w_v}{\partial v}f(y)dy - \frac{\partial c_v}{\partial v}c_v f(c_v)\Big\}\\ &= 2\Big\{\frac{\partial w_v}{\partial v}\int_{w_v}^{c_v} f(y)dy - \frac{\partial c_v}{\partial v}f(c_v)\Big[c_v - w_v\Big]\Big\}.\end{aligned}$$

We need to show $\frac{\partial EPol(v)}{\partial v} < 0$, that is,

$$\frac{\partial c_v}{\partial v}f(c_v)\Big[c_v - w_v\Big] \;>\; \frac{\partial w_v}{\partial v}\int_{w_v}^{c_v} f(y)dy,$$

or equivalently

$$\frac{\partial c_v}{\partial v}\Big[f(c_v) - f(w_v)\Big]\Big[c_v - w_v\Big] + \frac{\partial c_v}{\partial v}f(w_v)\Big[c_v - w_v\Big] \;>\; \frac{\partial w_v}{\partial v}\int_{w_v}^{c_v} f(y)dy. \tag{43}$$

From Proposition 1, $\frac{\partial c_v}{\partial v} > \frac{\partial w_v}{\partial v} > 0$. Moreover, $f(\cdot) > 0$, $c_v - w_v > 0$. From symmetry and single-peakedness, $f(y)$ weakly decreases with $y > 0$. Combining these observations, $f(w_v)\Big[c_v - w_v\Big] \geq \int_{w_v}^{c_v} f(y)dy$ and

$$\frac{\partial c_v}{\partial v}f(w_v)\Big[c_v - w_v\Big] \;>\; \frac{\partial w_v}{\partial v}\int_{w_v}^{c_v} f(y)dy.$$

Inequality (43) then holds if $\Big[f(c_v) - f(w_v)\Big]$ is not too negative, i.e., if the density $f(y)$ does not decrease too fast with $y > 0$. When F is uniform, f is a constant and the result holds. Hence, there exists a lower bound $\underline{f} < 0$ such that if $\Big[f(c_v) - f(w_v)\Big] \geq \underline{f}$ then $\frac{\partial EPol(v)}{\partial v} < 0$. ■

Proof: [**Proposition 3**] From Proposition 1, $w_H > w_L$ and $c_H - w_H > c_L - w_L$ for any $v_H > v_L \in V$. This implies that the expected policy of an incumbent *strictly increases* with valence in the subset of re-elected officials. If $q = 0$, then in the stationary distribution all office-holders are re-elected and the result holds. A small increase in q marginally increases the fraction of untried office holders in the stationary distribution, that is, politicians in their first term in office. If q is sufficiently small, then the proportion of re-elected office holders in the stationary distribution is sufficiently large and the result holds. ■

Proof: **[Proposition 4]** We first solve for the median voter's equilibrium payoff. When the loss function is quadratic, one can write the expected payoff of voter x as the following function of the median voter's expected payoff,

$$\overline{U}_x(w,c) \quad = \quad \overline{U}_0(w,c) - x^2. \tag{44}$$

From equations (23), (24), and the definition of $\overline{U}_0(w,c)$, equilibrium (w,c) solves the following system of equations:

$$\begin{aligned} \overline{U}_0(w,c) &= v - w_v^2, \ \forall v \in V, & (45)\\ \delta(1-q)v - (c_v - w_v)^2 &= \delta(1-q)\overline{U}_x(w,c), \ \forall v \in V, & (46)\\ \overline{U}_0(w,c) &= \int_V \left\{ 2\int_0^{w_v} [v - y^2] dF(y) + 2\int_{w_v}^{c_v} [v - w_v^2] dF(y) \right. & (47)\\ &\left. +2\int_{c_v}^{a} [(1-\delta(1-q))(v-y^2) + \delta(1-q)\overline{U}_0(w,c)] dF(y) \right\} dG(v), \end{aligned}$$

provided that solutions are interior, $0 < w_v < c_v < a$ for all $v \in V$; recall that solutions are interior when $V_H - V_L$ is sufficiently small. Normalize $v_L = 0 \Rightarrow \overline{U}_0(w,c) < 0$.

Rewrite w_v and c_v as functions of $\overline{U}_0(w,c)$. From (45),

$$w_v \quad = \quad [v - \overline{U}_0(w,c)]^{\frac{1}{2}}, \ \forall v \in V. \tag{48}$$

In (46), substitute $\overline{U}_x(w,c) = \overline{U}_0(w,c) - c_v^2 = v - w_v^2 - c_v^2$ and rearrange terms to write

$$0 \quad = \quad c_v^2[1-\delta(1-q)] - 2c_v w_v + w_v^2[1-\delta(1-q)]. \tag{49}$$

Solve the quadratic equation for c_v and select the unique solution such that $c_v > w_v$,

$$c_v \quad = \quad \theta w_v, \ \forall v \in V, \tag{50}$$

where $\theta \equiv \frac{1+\sqrt{\delta(1-q)(2-\delta(1-q))}}{1-\delta(1-q)}$. Notice that for any $\delta \in (0,1)$ and $q \in [0,1)$, we have $\theta > 1$.

Substitute $w_v = [v - \overline{U}_0(w,c)]^{\frac{1}{2}}$ and $c_v = \theta[v - \overline{U}_0(w,c)]^{\frac{1}{2}}$ into (47),

$$\begin{aligned}\overline{U}_0(w,c) &= \int_V \Bigg\{ 2\int_0^{[v-\overline{U}_0(w,c)]^{\frac{1}{2}}} [v - y^2] dF(y) \\ &\quad + 2\int_{[v-\overline{U}_0(w,c)]^{\frac{1}{2}}}^{\theta[v-\overline{U}_0(w,c)]^{\frac{1}{2}}} [v - [v - \overline{U}_0(w,c)]] dF(y) \\ &\quad + 2\int_{\theta[v-\overline{U}_0(w,c)]^{\frac{1}{2}}}^{a} [(1-\delta(1-q))(v-y^2) + \delta(1-q)\overline{U}_0(w,c)] dF(y) \Bigg\} dG(v) \quad (51)\end{aligned}$$

Exploiting the uniform distribution, $F(y) = \frac{y-a}{2a}$,

$$a^3\gamma = 3a\gamma\left[\int_V v dG(v) - \overline{U}_0(w,c)\right] + \int_V \left[v - \overline{U}_0(w,c)\right]^{\frac{3}{2}} dG(v), \quad (52)$$

where $\gamma = \frac{(1-\delta(1-q))}{2+(1-\delta(1-q))(\theta^3-3\theta)}$. Notice that $\theta > 1 \Rightarrow (\theta^3 - 3\theta) > -2$; therefore, $\gamma > 0$ and independent of valence distribution.

Analogously, the following must hold in equilibrium for valence distribution G',

$$a^3\gamma = 3a\gamma\left[\int_V v dG'(v) - \overline{U}'_0(w',c')\right] + \int_V \left[v - \overline{U}'_0(w',c')\right]^{\frac{3}{2}} dG'(v). \quad (53)$$

This implies

$$\begin{aligned}& 3a\gamma\left[\int_V v dG(v) - \overline{U}_0(w,c)\right] + \int_V \left[v - \overline{U}_0(w,c)\right]^{\frac{3}{2}} dG(v) \quad (54) \\ =\ & 3a\gamma\left[\int_V v dG'(v) - \overline{U}'_0(w',c')\right] + \int_V \left[v - \overline{U}'_0(w',c')\right]^{\frac{3}{2}} dG'(v).\end{aligned}$$

To prove (7), by contradiction, suppose $\overline{U}'_0(w',c') \leq \overline{U}_0(w,c) \Rightarrow -\overline{U}'_0(w',c') \geq -\overline{U}_0(w,c)$ and G' first order stochastically dominates G. Since G' has a strictly higher mean and $3a\gamma > 0$,

$$3a\gamma\left[\int_V v dG'(v) - \overline{U}'_0(w',c')\right] > 3a\gamma\left[\int_V v dG(v) - \overline{U}_0(w,c)\right]. \quad (55)$$

Moreover, for each $v \in V$, $v - \overline{U}'_0(w',c') \geq v - \overline{U}_0(w,c)$. Since G' first order stochastically dominates G and $\left[v - \overline{U}'_0(w',c')\right]^{\frac{3}{2}}$ strictly increases with v,

$$\int_V \left[v - \overline{U}'_0(w',c')\right]^{\frac{3}{2}} dG'(v) > \int_V \left[v - \overline{U}_0(w,c)\right]^{\frac{3}{2}} dG(v). \quad (56)$$

Together (55) and (56) contradict (54).

Turning to the second-order stochastic dominance argument, first recall the neutrality result of a constant valence transfer: a simple location shift by α raises utility by α, leaving policy unaffected. We can without loss of generality focus on a distribution $G(v)$ that second order stochastically dominates $G'(v)$. Accordingly, to prove (8), by contradiction, suppose that G second order stochastically dominates G', but $\overline{U}_0'(w', c') \leq \overline{U}_0(w, c) \Rightarrow -\overline{U}_0'(w', c') \geq -\overline{U}_0(w, c)$. Since G' and G have the same mean and $3a\gamma > 0$,

$$3a\gamma \left[\int_V v dG'(v) - \overline{U}_0'(w', c') \right] \geq 3a\gamma \left[\int_V v dG(v) - \overline{U}_0(w, c) \right]. \tag{57}$$

Moreover, for each $v \in V$, $v - \overline{U}_0(w', c') \geq v - \overline{U}_0(w, c)$. Since G' has higher variance and $\left[v - \overline{U}_0'(w', c') \right]^{\frac{3}{2}}$ is strictly convex,

$$\int_V \left[v - \overline{U}_0'(w', c') \right]^{\frac{3}{2}} dG'(v) > \int_V \left[v - \overline{U}_0(w, c) \right]^{\frac{3}{2}} dG(v). \tag{58}$$

Together (57) and (58) contradict (54).

We now prove $EPol(G') > EPol(G)$. By definition,

$$\begin{aligned} EPol(G) &= \int_V \left\{ 2 \int_0^{w_v} y dF(y) + 2 \int_{w_v}^{c_v} w_v dF(y) + 2 \int_{c_v}^{a} y dF(y) \right\} dG(v) \\ &= \int_V \left\{ 2 \int_0^{\sqrt{v - \overline{U}_0(w,c)}} y dF(y) + 2 \int_{\sqrt{v - \overline{U}_0(w,c)}}^{\theta \sqrt{v - \overline{U}_0(w,c)}} [\sqrt{v - \overline{U}_0(w, c)}] dF(y) \right. \\ &\quad \left. + 2 \int_{\theta \sqrt{v - \overline{U}_0(w,c)}}^{a} y dF(y) \right\} dG(v). \end{aligned} \tag{59}$$

Since F is uniform,

$$\begin{aligned} EPol(G) &= \int_V \left\{ \frac{v - \overline{U}_0(w, c)}{2a} + [v - \overline{U}_0(w, c)] \frac{(\theta - 1)}{a} + \frac{a^2}{2a} - \frac{\theta^2 (v - \overline{U}_0(w, c))}{2a} \right\} dG(v) \\ &= \frac{a}{2} - \left[\int_V v dG(v) - \overline{U}_0(w, c) \right] \frac{1}{2a} \left[1 + \theta^2 - 2\theta \right]. \end{aligned} \tag{60}$$

Similarly,

$$EPol(G') = \frac{a}{2} - \left[\int_V v dG'(v) - \overline{U}_0'(w', c') \right] \frac{1}{2a} \left[1 + \theta^2 - 2\theta \right]. \tag{61}$$

Notice that $1 + \theta^2 - 2\theta > 0$ and independent of G. Since $\int_V v dG(v) = \int_V v dG'(v)$, the result $EPol(G') > EPol(G)$ follows as we have established that if G second order stochastically dominates G' then $\overline{U}_0'(w', c') > \overline{U}_0(w, c)$. ■

Proof: [**Proposition 5**] We first consider the search effort choice at time t of an IG with ideology $i \geq 0$ supporting party R. IG i has equilibrium beliefs about future probabilities p_{t+1} of drawing a high valence candidate when a new candidate is elected[13], and must choose the optimal search effort at time t — that is, a probability p_t.

Given its equilibrium beliefs about p_{t+1}, the IG forms consistent beliefs about the equilibrium cutoff functions $\{w, c\}$. If at time $t-1$ the incumbent with valence v implemented policy $y \leq w_v$, the incumbent will be re-elected, so the IG will not search. If, instead, $y > w_v$, the incumbent optimally steps down (otherwise she would lose re-election) and both parties run untried candidates. In equilibrium voters correctly predict the symmetric search effort, so each untried candidate wins with equal probability.

In the following steps of the proof, we define the marginal benefit of valence search $\text{MB}_i^R(w,c)$ at period t given p_{t+1} and prove that: $\text{MB}_i^R(w,c)$ strictly decreases in i; $\text{MB}_i^R(w,c)$ strictly decreases in p_{t+1}; $\text{MB}_i^R(w,c) > 0$ for every $i \in [0,a]$ and p_{t+1}. These results together with the assumptions on the cost function $c(p_t)$ imply that for each ideology i there exists a unique solution $p^* = p_t = p_{t+1}$, and p^* strictly decreases with i.

Step 1: We first show that $\text{MB}_i^R(w,c)$ strictly decreases in i. For an IG with ideology $i \geq 0$, the expected payoff from a party R candidate with valence $v \in \{v_H, v_L\}$ is

$$\begin{aligned} U_i^R(v|w,c) \;\equiv\; & \frac{1}{a}\Bigg\{ \int_0^{w_v} \Big[k[v-(i-y)^2] + k\frac{\delta q}{(1-\delta)}\overline{U}_i(w,c)\Big]dy \\ & + \int_{w_v}^{c_v} \Big[k[v-(i-w_v)^2] + k\frac{\delta q}{(1-\delta)}\overline{U}_i(w,c)\Big]dy \\ & + \int_{c_v}^{a} \Big[(1-\delta)(v-(i-y)^2) + \delta\overline{U}_i(w,c)\Big]dy \Bigg\}. \end{aligned} \tag{62}$$

Since $\overline{U}_i(w,c) = \overline{U}_0(w,c) - i^2$ and $k + k\frac{\delta q}{(1-\delta)} = 1$, rewrite

$$\begin{aligned} U_i^R(v|w,c) \;=\; & \frac{1}{a}\Bigg\{ \int_0^{w_v} \Big[k[v-i^2+2iy-y^2] + k\frac{\delta q}{(1-\delta)}[\overline{U}_0(w,c)-i^2]\Big]dy \\ & + \int_{w_v}^{c_v} \Big[k[v-i^2+2iw_v-w_v^2] + k\frac{\delta q}{(1-\delta)}[\overline{U}_0(w,c)-i^2]\Big]dy \\ & + \int_{c_v}^{a} \Big[(1-\delta)(v-i^2+2iy-y^2) + \delta[\overline{U}_0(w,c)-i^2]\Big]dy \Bigg\} \end{aligned}$$

[13]Since we focus on stationary equilibria, equilibrium beliefs must be such that $p_{t+1} = p_{t+2} = \ldots$.

$$\begin{aligned}
&= \frac{1}{a}\Big\{ -i^2 a + 2ki\Big[\int_0^{w_v} y dy + \int_{w_v}^{c_v} w_v dy + \int_{c_v}^{a} (1-\delta(1-q)) y dy\Big] \\
&\quad + \int_0^{w_v} \big[k[v-y^2] + k\frac{\delta q}{(1-\delta)}\overline{U}_0(w,c)\big] dy \\
&\quad + \int_{w_v}^{c_v} \big[k[v-w_v^2] + k\frac{\delta q}{(1-\delta)}\overline{U}_0(w,c)\big] dy \\
&\quad + \int_{c_v}^{a} [(1-\delta)(v-y^2) + \delta\overline{U}_0(w,c)] dy\Big\}.
\end{aligned} \tag{63}$$

Notice that

$$\begin{aligned}
U_0^R(v|w,c) &= \frac{1}{a}\Big\{ \int_0^{w_v} \big[k[v-y^2] + k\frac{\delta q}{(1-\delta)}\overline{U}_0(w,c)\big] dy \\
&\quad + \int_{w_v}^{c_v} \big[k[v-w_v^2] + k\frac{\delta q}{(1-\delta)}\overline{U}_0(w,c)\big] dy \\
&\quad + \int_{c_v}^{a} [(1-\delta)(v-y^2) + \delta\overline{U}_0(w,c)] dy\Big\},
\end{aligned} \tag{64}$$

and

$$\begin{aligned}
&\int_0^{w_v} y dy + \int_{w_v}^{c_v} w_v dy + \int_{c_v}^{a} (1-\delta(1-q)) y dy \\
&= \frac{w_v^2}{2} + w_v(c_v - w_v) + \frac{(1-\delta(1-q))}{2}(a^2 - c_v^2) \\
&= \frac{(1-\delta(1-q))a^2 - \delta(1-q)w_v^2}{2},
\end{aligned} \tag{65}$$

where in the last equality we used equation (49) to substituted $w_v c_v = (1-\delta(1-q))(w_v^2 + c_v^2)/2$. Substitute (64) and (65) into (63),

$$U_i^R(v|w,c) = -i^2 + U_0^R(v|w,c) + ik(1-\delta(1-q))a - i\frac{k\delta(1-q)w_v^2}{a} \tag{66}$$

For an IG with ideology $i \geq 0$, the marginal benefit from valence search is

$$\begin{aligned}
\mathrm{MB}_i^R(w,c) &= U_i^R(v_H|w,c) - U_i^R(v_L|w,c) \\
&= U_0^R(v_H|w,c) - U_0^R(v_L|w,c) - i\frac{k\delta(1-q)(w_H^2 - w_L^2)}{a} \\
&= U_0^R(v_H|w,c) - U_0^R(v_L|w,c) - i\frac{k\delta(1-q)(v_H - v_L)}{a},
\end{aligned} \tag{67}$$

where the last equality follows from equilibrium condition $v_H - w_H^2 = v_L - w_L^2$. Since $v_H > v_L$ and $k\delta(1-q) \in (0,1)$,

$$\frac{\partial \mathrm{MB}_i^R(w,c)}{\partial i} = -\frac{k\delta(1-q)(v_H - v_L)}{a} < 0. \tag{68}$$

Step 2: We now prove that $\text{MB}_i^R(w,c)$ strictly decreases with p_{t+1}. Use $v - w_v^2 = \overline{U}_0(w,c)$ to rewrite (64),

$$\begin{aligned} U_0^R(v|w,c) &= \frac{1}{a}\bigg\{ \int_0^{w_v} \big[k[v-y^2] + k\frac{\delta q}{(1-\delta)}\overline{U}_0(w,c)\big]dy + \int_{w_v}^{c_v} \overline{U}_0(w,c)dy \\ &\quad + \int_{c_v}^{a} [(1-\delta)(v-y^2) + \delta\overline{U}_0(w,c)]dy \bigg\}. \end{aligned}$$

Subtract $U_0^R(v_L|w,c)$ from $U_0^R(v_H|w,c)$,

$$\begin{aligned} \text{MB}_0^R(w,c) &= \frac{1}{a}\bigg\{ \int_0^{w_L} k[v_H - v_L]dy + \int_{w_L}^{w_H} k[v_H - y^2 - v_L + w_L^2]dy + \int_{w_H}^{c_H} [0]dy \\ &\quad + \int_{c_L}^{c_H} (1-\delta)(\overline{U}_0(w,c) - v_L + y^2)dy + \int_{c_H}^{a} (1-\delta)(v_H - v_L)dy \bigg\}. \end{aligned}$$

Substitute $v_H - v_L = w_H^2 - w_L^2$ and $\overline{U}_0(w,c) = (v_L - w_L^2)$,

$$\begin{aligned} \text{MB}_0^R(w,c) &= \frac{k}{a}\bigg\{ \int_0^{w_L} [w_H^2 - w_L^2]dy + \int_{w_L}^{w_H} [w_H^2 - y^2]dy \\ &\quad + \int_{c_L}^{c_H} (1-\delta(1-q))(-w_L^2 + y^2)dy + \int_{c_H}^{a} (1-\delta(1-q))(w_H^2 - w_L^2)dy \bigg\}. \end{aligned}$$

Substitute $c_v = \theta w_v$ and take the integrals,

$$\begin{aligned} \text{MB}_0^R(w,c) &= \frac{k}{a}\bigg\{ [w_H^2 - w_L^2]w_L + w_H^2(w_H - w_L) - (\frac{w_H^3 - w_L^3}{3}) \\ &\quad + (1-\delta(1-q))\bigg[- w_L^2\theta(w_H - w_L) + \theta^3(\frac{w_H^3 - w_L^3}{3})\bigg] \\ &\quad + (1-\delta(1-q))(w_H^2 - w_L^2)(a - \theta w_H) \bigg\}. \end{aligned} \tag{69}$$

Simplify,

$$\begin{aligned} \text{MB}_0^R(w,c) &= k(1-\delta(1-q))[w_H^2 - w_L^2] + k(w_H^3 - w_L^3)\frac{2 + (1-\delta(1-q))(\theta^3 - 3\theta)}{3a} \\ &= k(1-\delta(1-q))\bigg[v_H - v_L + \frac{w_H^3 - w_L^3}{3a\gamma}\bigg] > 0. \end{aligned} \tag{70}$$

Notice that $w_v > 0 \Rightarrow w_v^3 = (w_v^2)^{3/2}$, and $\overline{U}_0(w,c) = v - w_v^2 \Rightarrow w_v^3 = (v - \overline{U}_0(w,c))^{3/2}$. Substitute $k(1-\delta(1-q)) = (1-\delta)$,

$$\text{MB}_0^R(w,c) = (1-\delta)\bigg[v_H - v_L + \frac{(v_H - \overline{U}_0(w,c))^{3/2} - (v_L - \overline{U}_0(w,c))^{3/2}}{3a\gamma}\bigg]. \tag{71}$$

Rewrite (67) as $\text{MB}_i^R(w,c) = \text{MB}_0^R(w,c) - i\frac{k\delta(1-q)(v_H-v_L)}{a}$. Taking the derivative with respect to probability p_{t+1},

$$\begin{aligned}\frac{\partial \text{MB}_i^R(w,c)}{\partial p_{t+1}} &= \frac{\partial \text{MB}_0^R(w,c)}{\partial p_{t+1}} \\ &= -\frac{\partial \overline{U}_0(w,c)}{\partial p_{t+1}}\frac{3}{2}(1-\delta)\left[\frac{(v_H-\overline{U}_0(w,c))^{1/2}-(v_L-\overline{U}_0(w,c))^{1/2}}{3a\gamma}\right] < 0,\end{aligned} \tag{72}$$

where the inequality follows from Proposition 4.1, $\frac{\partial \overline{U}_0(w,c)}{\partial p_{t+1}} > 0$.

Step 3: We now prove that $\text{MB}_i(w,c) > 0$ for every $p_{t+1} \in [0,1]$ and $i \in [0,a]$.

From the previous steps, $\text{MB}_i(w,c)$ strictly decreases in both $i \in [0,a]$ and p_{t+1}. Therefore, $\text{MB}_i(w,c) > 0$ for every $i \in [0,a]$ and p_{t+1} if and only if $\text{MB}_a(w,c) > 0$ when $p_{t+1} = 1$. Combining (67) and (71), we need to show that for $p_{t+1} = 1$,

$$\begin{aligned}(1-\delta)\left[v_H - v_L + \tfrac{(v_H-\overline{U}_0(w,c))^{3/2}-(v_L-\overline{U}_0(w,c))^{3/2}}{3a\gamma}\right] - k\delta(1-q)(v_H-v_L) > 0, \\ \Rightarrow v_H - v_L + \tfrac{(v_H-\overline{U}_0(w,c))^{3/2}-(v_L-\overline{U}_0(w,c))^{3/2}}{3a\gamma} > \tfrac{\delta(1-q)}{1-\delta(1-q)}(v_H-v_L).\end{aligned} \tag{73}$$

Since $v_H - v_L > 0$, for any $\delta(1-q) \leq 1/2$ condition (73) holds trivially, concluding this step of the proof.

We now prove (73) also holds for $\delta(1-q) > 1/2$. We exploit the result on the irrelevance of a valence shift and the fact that we are only considering the case $p_{t+1} = 1$ to write $\overline{U}_0(w,c) = v_H + \overline{U}_0(\overline{w},\overline{c})$, where $\overline{U}_0(\overline{w},\overline{c})$ is the median voter's expected payoff in an economy with a single valence $\overline{v} = 0$: $\overline{U}_0(\overline{w},\overline{c})$ is independent of $v_H - v_L$. Rewrite (73),

$$v_H - v_L + \frac{(-\overline{U}_0(\overline{w},\overline{c}))^{3/2} - (-(v_H-v_L)-\overline{U}_0(\overline{w},\overline{c}))^{3/2}}{3a\gamma} > \frac{\delta(1-q)}{1-\delta(1-q)}(v_H-v_L). \tag{74}$$

When $p_{t+1} = 1$, $-w_H^2 = -\overline{w}^2 = \overline{U}_0(\overline{w},\overline{c})$. Therefore $v_H - \overline{w}^2 = v_L - w_L^2 \Rightarrow w_L^2 = -(v_H - v_L) - \overline{U}_0(\overline{w},\overline{c})$. Cutoff w_L has an interior solution $w_L > 0$ if and only if the valence set is not too large, $(v_H - v_L) < -\overline{U}_0(\overline{w},\overline{c})$. Hence we only consider valence sets such that $(v_H - v_L) \in (0, -\overline{U}_0(\overline{w},\overline{c}))$ — notice that $\overline{U}_0(\overline{w},\overline{c}) < 0$ for any $\delta(1-q) \in (0,1)$.

The LHS of (74) is strictly increasing and strictly concave in $(v_H - v_L) \in (0, -\overline{U}_0(\overline{w},\overline{c}))$, while the RHS is strictly increasing and linear. At the lower limit $(v_H - v_L) = 0$, we have LHS=RHS. Therefore, if at the upper limit $(v_H - v_L) = -\overline{U}_0(\overline{w},\overline{c})$ we have LHS$\geq$RHS, the strict concavity of the LHS implies that (74) holds for every $(v_H - v_L)$ inside the bounds.

Rewriting (74), we need to show that at the upper limit $(v_H - v_L) = -\overline{U}_0(\overline{w}, \overline{c})$ we have

$$-\overline{U}_0(\overline{w}, \overline{c}) + \frac{(-\overline{U}_0(\overline{w}, \overline{c}))^{3/2} - (0)^{3/2}}{3a\gamma} \geq -\frac{\delta(1-q)}{1-\delta(1-q)}\overline{U}_0(\overline{w}, \overline{c})$$
$$\Rightarrow -3a\gamma\overline{U}_0(\overline{w}, \overline{c}) + (-\overline{U}_0(\overline{w}, \overline{c}))^{3/2} \geq -3a\gamma\frac{\delta(1-q)}{1-\delta(1-q)}\overline{U}_0(\overline{w}, \overline{c}). \tag{75}$$

Since $\overline{v} = 0$ with probability one, equilibrium condition (52) becomes

$$a^3\gamma \;=\; -3a\gamma\overline{U}_0(\overline{w}, \overline{c}) + (-\overline{U}_0(\overline{w}, \overline{c}))^{3/2}, \tag{76}$$

so we substitute $-3a\gamma\overline{U}_0(\overline{w}, \overline{c}) = a^3\gamma - (-\overline{U}_0(\overline{w}, \overline{c}))^{3/2}$ into (75),

$$a^3\gamma \geq -3a\gamma\frac{\delta(1-q)}{1-\delta(1-q)}\overline{U}_0(\overline{w}, \overline{c}) \;\;\Rightarrow\;\; \frac{1-\delta(1-q)}{3\delta(1-q)} \geq \frac{-\overline{U}_0(\overline{w}, \overline{c})}{a^2}. \tag{77}$$

Notice that in the worst case scenario where incumbents always adopt their own ideologies, $\overline{U}_0(\overline{w}, \overline{c}) = -\frac{a^3}{3a}$. Since for any $\delta(1-q) \in (0,1)$ some incumbents compromise, $\frac{-\overline{U}_0(\overline{w}, \overline{c})}{a^2} < 1/3$. Therefore, at $\delta(1-q) = 1/2$ the LHS of (77) is strictly greater than the RHS. Since both sides are continuous functions of $\delta(1-q)$, the LHS will be less than the RHS for some $\delta(1-q) \in (1/2, 1)$ if and only if there exist some $\delta(1-q) \in (1/2, 1)$ such that LHS=RHS. If such $\delta(1-q)$ exists then substituting $-\bar{U}_0(\bar{w}, \bar{c}) = \frac{a^2(1-\delta(1-q))}{3\delta(1-q)}$ into equation (76), we must have

$$a^3\gamma \;=\; 3a\gamma\frac{a^2(1-\delta(1-q))}{3\delta(1-q)} + \left(\frac{a^2(1-\delta(1-q))}{3\delta(1-q)}\right)^{3/2},$$
$$\Leftrightarrow 0 \;=\; \gamma\frac{(1-\delta(1-q))}{\delta(1-q)} + \left(\frac{(1-\delta(1-q))}{3\delta(1-q)}\right)^{3/2} - \gamma. \tag{78}$$

Recall that γ is only a function of $\delta(1-q)$, therefore (78) is only a function of $\tilde{\delta} \equiv \delta(1-q)$ and continuous. At $\tilde{\delta} = 1/2$, the RHS of (78) is strictly positive, $\left(\frac{1}{3}\right)^{3/2} > 0$. In the limit, as $\tilde{\delta} \to 1$ the RHS goes to zero. We use Mathematica to verify that the RHS of (78) is strictly decreasing and strictly convex for $\tilde{\delta} \in (1/2, 1)$. Hence it follows that (78) holds as a strict inequality for $\tilde{\delta} \in (1/2, 1)$, completing this step of the proof.

Step 4: Finally, we show that a unique equilibrium $p^* \in (0,1)$ exists, and it strictly decreases in i. Fix ideology i. Given any $p_{t+1} \in [0,1]$, the IG optimally chooses a $p_t \in [0,1]$ that maximizes expected payoff, $\frac{p_t}{2}MB_i(w,c) - \alpha c(p_t)$. We multiply $\mathrm{MB}_i^R(w,c)$ by 1/2 since the untried candidate from party R is elected with probability 1/2.

Existence of a unique equilibrium $p^* \in (0,1)$ follows from a fixed point argument on p_t and p_{t+1}. Search cost is a continuous, increasing function of $p_t \in [0,1]$, while the search benefit is a continuous, strictly decreasing function of $p_{t+1} \in [0,1]$. When $p_{t+1} = 0$, marginal benefit $MB_i(w,c) > 0$ is greater than marginal cost of $p_t = 0$, since $c'(0) = 0$. When $p_{t+1} = 1$, marginal cost $c'(1)$ is greater than the maximum possible marginal benefit. Therefore, a unique equilibrium $p^* = p_t = p_{t+1}$ exists, and it is interior, $p^* \in (0,1)$.

In equilibrium, Interest Group $i \geq 0$ optimally chooses p^* so that marginal benefit equals marginal cost,

$$\alpha c'(p^*) = \frac{1}{2}\mathrm{MB}_i^R(w,c). \tag{79}$$

Since $\mathrm{MB}_i^R(w,c)$ strictly decreases in i for every p_{t+1}, p^* strictly decreases in i. ■

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