Learning about Challengers *

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Abstract

We examine a political agency problem in repeated elections where an incumbent runs against a challenger from the opposing party, whose policy preferences are unknown by voters. We first ask: do voters benefit from attracting a pool of challengers with more moderate ideologies? When voters and politicians are patient, moderating the ideology distribution of centrist and moderate politicians (those close to the median voter) reduces voter welfare by reducing an extreme incumbent's incentives to compromise. We then ask: do voters benefit from informative signals about a challenger's true ideology? We prove that giving voters informative, but sufficiently noisy, signals always harm voters, because they make it harder for incumbents to secure re-election.

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1 Introduction

This paper examines how changes in the ideology distribution of individuals running for office or in the information voters have about candidates affect equilibrium outcomes and voter welfare. These are important determinants of equilibrium behavior for two reasons. First, in the predominately two-party system in the United States, an incumbent officeholder typically runs for re-election against an untried challenger drawn from the opposing party. Voters know far more about an incumbent because they see her policy choices in office. In contrast, a challenger is a risky option whose true preferences are unknown. As a result, an incumbent can implement policies away from the median voter's preferred policy and still win re-election. Second, in this two-party system, the likely policy choices that a challenger might select if elected serve as the chief device with which voters can discipline office-holders to control this political agency problem. The fear of losing to the opposing party's candidate who may implement policies far from an incumbent's ideal policy provides a key inducement to incumbents to moderate policy choices when politicians cannot commit to policies.

We begin by addressing a basic question: when do voters benefit from attracting a better pool of challenging candidates? Concretely, when do voters gain if challengers are more likely to hold views closer to those of the median voter? We then ask: do voters benefit from receiving an informative, but noisy, signal about a challenger's ideology prior to an election? That is, do voters benefit from learning about a challenger's views of the world, so that they can more precisely predict her likely policy choices if elected?

These changes in the ideology distribution of candidates or the information available to voters may reflect changes in the political environment—e.g., changes in the degree of ideological polarization between competing interest groups, the behavior of media outlets covering politics, or the institutions governing primary elections, campaign financing and spending—studied by the political economy literature. Our analysis complements this literature and provides insights into the possible equilibrium implications of such changes.

Our core model builds on the infinite horizon, repeated elections models of Duggan (2000) and Bernhardt et al. (2009). It features a pool of politicians with ideologies symmetrically distributed around the median voter, divided into a left-wing $[-1, 0]$ and a right-wing $[0, 1]$ party. We investigate the welfare of voters who incur quadratic disutility from policies that deviate from their preferred policies. Equilibrium outcomes are characterized by two ideology cutoffs, w and c, where $0 < w < c < 1$. When in office, "centrist politicians" with ideology $i \in [-w, w]$ implement their preferred policies and are re-elected. "Moderate politicians" with ideology $i \in (w, c)$ choose to compromise and adopt policy w in order to win re-election, while politicians in $(-c, -w)$ compromise to $-w$. "Extreme politicians" with ideology $i \in [-1, -c] \cup [c, 1]$ implement their preferred policies, but lose re-election.

In an ideal world, with no other agency problems, social welfare would be maximized by a pool of politicians whose interests were *perfectly* aligned with the median voter's, and hence would *want* to adopt the median's preferred policies. But, in practice, selecting challengers is a complex, noisy process, resulting in significant variation in the realized preferences of challengers, and substantial voter uncertainty about a challenger's preferred policies. In this context, the welfare effects of attracting more politicians with ideologies close to the median and fewer politicians with ideologies far from the median are less clear. The direct benefit from attracting a better population of challengers is obvious—when replacing an incumbent, most voters want to elect as moderate a challenger as possible. However, improved selection of challengers also adversely feeds back to affect incentives of office holders to moderate policy choices to win re-election. In particular, an office holder does not mind losing by as much if she is likely to be replaced by a moderate rather than an extremist from the opposing party: better challengers weaken the threat of replacement that voters use to discipline incumbents. This gives rise to a negative indirect effect of moderating challengers—cutoff c falls, as more incumbents choose to implement extreme policies.

Do voters benefit from attracting a more moderate pool of challengers? We show that if voters and politicians are sufficiently *impatient*, then given any two ideology distributions of challengers, voters always prefer the more moderate distribution.[1] Intuitively, when incumbent politicians do not care much about the future, they do not care much about compromising to be reelected. The indirect negative welfare effects for voters of changes in cutoff c are then small relative to the direct benefits of ideology moderation.

In sharp contrast, if voters and politicians are *patient*, moderating the ideology distribution of centrist and moderate politicians, while keeping constant the ideology distribution

[1]For politicians with right-wing ideologies $i \geq 0$, ideology distribution F'_R is more moderate than F_R if F_R first order stochastically dominates F'_R. The opposite holds for the symmetric left-wing politicians.

of more extreme politicians, hurts voters. To understand why, observe that (i) although centrist and moderate politicians have ideologies $i \in (-c, c)$, they only implement policies in the smaller set $[-w, w]$; and (ii) when players are patient, the compromise cutoff w is close to the median voter, but c is far away. As a result, moderation in the *ideology* distribution of centrist and moderate politicians only provides a small direct benefit to the median voter, since these politicians already implement *policies* close to the median. In contrast, ideology c is far from the median voter and the policy set $[-w, w]$, so moderation of centrist and moderate politicians has a large direct positive impact on a right-wing incumbent's expected payoff from being replaced by a left-wing challenger. This causes enough extreme incumbents to cease compromising that the direct benefit is swamped, reducing voter welfare.[2]

What happens when ideology moderation shifts the distribution of extreme politicians? A naïve conjecture would be that when the proportion of extreme politicians is reduced, voter welfare would always rise. This conjecture is false. We consider a class of linear ideology distributions, for which moderation implies a rotation that shifts extreme ideologies closer to the median. We show that such moderating shifts reduce voter welfare as long as politicians and voters are sufficiently patient.

Our findings indicate that one must be cautious when evaluating the welfare impacts of changes to the processes that select challengers. Many institutional or strategic changes may lead to a pool of more moderate challengers—a shift from a closed to open primary system that draws more independents; increased party filtering of challengers to improve electability; or increased concerns of primary voters for more moderate candidates, with better chances in the general election. Alternatively, reductions in the opportunity cost of running for office, e.g., increased compensation, may differentially appeal to "good" (moderate) challengers. Our findings highlight that differentially drawing particularly attractive, moderate challengers robustly and paradoxically reduces voter welfare whenever voters and officeholders are patient. Indeed, a robust empirical feature of electoral competition is that "weak" incumbents who adopt extreme policies are more likely to draw "good" challengers. Paradoxically, this endogenous response raises the incumbent's incentives to adopt extreme policies, harming voters.

[2]When voters are impatient, holding constant any discount factor, we prove that any moderation in the ideology distribution of politicians sufficiently close to the median voter always reduces voter welfare.

Having characterized how a more *moderate distribution* of challenger ideologies affects outcomes, we next characterize how more *information* about challengers affects outcomes. Specifically, we derive the impact of a political campaign process prior to an election that reveals information to voters about a challenger's attributes. We want to understand whether and when voter learning via the electoral process improves political outcomes. We suppose that voters receive a binary (good or bad) signal about a challenger's true preferences, for example, reflecting an endorsement by an informed interest group. A good signal indicates that a challenger's ideology is more likely to be closer to the median, while a bad signal means that it is more likely to be farther away.

Now, incumbents with sufficiently extreme ideologies must compromise by more to defeat a challenger following a good signal about the challenger than following a bad one. We identify sufficient conditions such that all incumbents who compromise do so by enough to win re-election against *all* challengers. In particular, this is so whenever the signal about the challenger is sufficiently noisy. Intuitively, when signals are noisy, re-election standards vary little with the signal realization—by compromising a little more, an incumbent gains a discrete increase in the probability of re-election.

We then derive a very negative result for the impact of learning about challengers: more accurate signals about challengers *always* harm voters provided that incumbents who compromise do so by enough to defeat all challengers. To understand why, suppose the political process conveys no information to voters about challengers other than party affiliation. Then voters would be better off if they could commit to *relaxing* the policy standard for re-election, i.e., if they could commit to re-electing incumbents who compromise by slightly less than what is required for re-election in equilibrium. At the equilibrium standard, the median voter is just indifferent between re-electing the incumbent and trying the risky challenger. The median voter does not internalize that *if* he set a slacker standard, then more extreme incumbents would choose to compromise to win re-election, rather than locate extremely. The welfare gain from greater compromise is first-order, and the cost from slightly inefficient replacement is second-order. Now consider slightly informative signals about a challenger. A good signal about a challenger induces voters to set a *stricter* re-election standard, and all incumbents who choose to compromise do so to that stricter standard. Because the compromise costs of re-election are raised, more extreme incumbents choose to adopt their preferred extreme policies and lose, and this hurts voters. Moreover, increasing the signal's accuracy and making it harder to secure reelection further decreases voter welfare, as long as incumbents who compromise do so by enough to defeat all challengers.

Informative signals about the challenger harm voters by more when they are more patient or when politicians are likely to have extreme ideologies. In both scenarios, more incumbent types compromise. This means that an incumbent politician who is just indifferent between compromising and not has a more extreme ideology. Hence, changes that make the re-election cutoff stricter reduce an incumbent's payoff from compromising by more, making her more likely to stop compromising and, instead, to adopt as policy her own extreme ideology.

There are many factors that affect the amount of noise in the information about challenging candidates that reaches voters: the media's coverage of campaigns, the laws governing campaign expenditures (e.g., the Citizen's United ruling), increased exposure to social media, and so on. As we show, one must be cautious when evaluating the welfare impacts of such changes: voters may benefit from very informative signals about challengers, but sufficiently noisy signals are always worse than no information.

The paper's outline is as follows. We next review the literature. Section 2 presents our base model. Section 3 analyzes how the distribution of politician ideologies affects outcomes and welfare. Section 4 considers campaigns that provide informative signals about a challenger. Section 5 concludes. All proofs are in the Appendix.

1.1 Related Literature

Our paper relates to a literature on the equilibrium consequences of noisy signals about candidates. In the single-election models of Carrillo and Castanheira (2008) and Boleslavsky and Cotton (2014), voters trade off valence and policy. Noisy information about candidates' valence affects their choices of policy platforms and may induce increased polarization. The authors show how variations in the signal's informativeness can increase or decrease voter welfare. Eguia and Nicolò (2011) explore how voter information about candidate platforms affects the subsequent provision of inefficient local public goods (pork) by the elected government. They show that if the electorate is well informed about the platforms to which candidates commit, electoral competition leads candidates to provide excessive pork. Other papers examine signaling games where politicians choose policies and actions that convey information about their private types—e.g., Kartik and McAfee (2007), Callander and Wilkie (2007), and Van Weelden and Morelli (2012).

In our dynamic model, noisy signals about future challengers negatively affects the behavior of current incumbents. Incumbents anticipate that voters will be more tempted to elect a future candidate if they observe a "good" signal (i.e., the challenger is more likely to be moderate), which makes current reelection standards harder. Moreover, losing to a "good" challenger is less costly for the incumbent, since she is expected to be more moderate. These two effects combined reduce an incumbent's incentive to compromise by too much, reducing voter welfare. Dewan and Hortala-Vallve (2014) also study a dynamic model in which information about a challenger makes it harder for an incumbent to guarantee reelection, possibly reducing voter welfare. Interestingly, the mechanism driving their negative welfare result is the opposite of ours: in their model, voters lose because some incumbents inefficiently choose a risky policy to increase their reelection chances, while in our model voters lose because some incumbents give up on reelection and choose an extreme policy.

Our paper also relates to a recent literature on how some degree of ideological extremism may benefit voters. In Van Weelden (2013, 2014), voters care about one policy dimension and the expropriation of resources. He assumes that voters can *perfectly* select a challenger's ideology. He shows that it is optimal to select slightly-ideologically irresponsible challengers in order to raise the cost of replacement to an office holder and thereby induce them to reduce their theft of resources. In Bernhardt et. al (2009), voters want policies to adapt to a random state of the world. When candidates propose platforms that have a certain degree of polarization, the possibility of choosing between the candidates allows voters to select the policy that better adapts to the realized state. In our framework, there is no need to adapt policies to unknown states, nor do voters trade off different policy dimensions. However, we provide broad conditions under which a more extreme distribution of politicians' ideologies benefits patient voters. In our model, under these conditions, the threat of being replaced in the future by a more extreme challenger indirectly benefits voters by increasing an incumbent's incentives to compromise to such an extent that it dominates the always negative direct effect on voters of selecting politician ideologies from a more extreme distribution.

2 Basic Model

We build on the infinite horizon, repeated elections models of Duggan (2000) and Bernhardt et al. (2009). When politicians care about policies but cannot commit to future choices, an incumbent's agency problem is limited by the threat of voters to elect a challenger from the opposing party.[3]

There is a continuum of infinitely-lived voters, each indexed by his private ideology $x \in [-1, +1]$, and distributed according to a probability density function that is continuous and symmetric about the median voter's ideology, $x = 0$. There is a continuum of infinitely-lived politicians, each indexed by her private ideology $i \in [-1, +1]$. Politicians are divided into a left-wing party L and a right-wing party R. Party R consists of politicians with ideologies $i \in [0, 1]$, distributed according to the probability density function f_R, with associated cumulative distribution function F_R. Party L consists of politicians with ideologies $i \in [-1, 0]$, distributed according to the probability density f_L, with associated c.d.f. F_L, where $f_L(i) = f_R(-i)$.

At any date $t = 0, 1, 2, \ldots$, an office holder with ideology i selects a policy $p(i) \equiv y$, providing voter x a date-t payoff of $u(x, y) = -(x - y)^2$, and delivering a date-t payoff of $u(i, y) = -(i - y)^2$ to politician i. Period utilities are discounted by factor $\delta \in [0, 1)$. At date 0, an office holder is randomly drawn from one of the parties. At any subsequent date-t the incumbent runs for re-election against an untried challenger drawn from the opposing party, with the incumbent winning if and only if he wins at least half of the votes.[4] The

[3]Duggan (2000) studies a model without parties where the ideology of each challenger is drawn from "at-large" (i.e., from the same distribution). Bernhardt et al. (2009) introduce parties so that the distribution of a challenger' ideology depends on her party affiliation. In this paper we focus on the case where each incumbent runs for reelection against an untried challenger from the opposing party. Bernhardt et al. (2009) show that the resulting "party competition effect" strictly benefits all voters. In an on-line Appendix, we study the at-large case, without parties. While all results regarding the impact of noisy signals about challengers extend to at-large settings, the result that a more moderate distribution of challenger ideologies can reduce voter welfare does *not* hold with at-large selection of challengers when voters and politicians have quadratic preferences.

[4]To maintain the "party competition effect", as in Bernhardt et al. (2009) we assume that either incumbents always run for re-election, or that the indifferent median voter votes for the untried candidate of the party whose incumbent is not running for re-election if and only if the median voter would vote to re-elect the incumbent.

challenger's ideology is not known by voters, but its distribution is common knowledge.

In order to guarantee the existence of an equilibrium, we assume that

(A.1) Distribution F_R has support $[0, 1]$ and an absolutely continuous density f_R, with $f_R(i) \geq \underline{f}$ for all $i \in [0, 1]$, for some $\underline{f} > 0$.

2.1 Equilibrium and Voter Welfare

Equilibrium Concept: We focus on the class of symmetric, stationary perfect Bayesian equilibria described by Duggan (2000) for at-large selection, and extended to party selection by Bernhardt et al. (2009). In this equilibrium class, voters use simple strategies:[5] voters act as though they are "pivotal" in the current election, and voter x votes to re-elect the incumbent if and only if her most recent policy choice satisfied a voter-specific utility standard. In equilibrium, this utility standard corresponds to the expected discounted payoff of voter x if the challenger is elected. Therefore, voters' equilibrium behavior is consistent with both retrospective and prospective voting. Stationarity implies that this re-election standard is time-invariant and history independent. Politicians also use simple strategies: along the equilibrium path, in each period in office, politician i implements the same policy $p(i)$.[6] When choosing $p(i)$, an incumbent politician cares about the current and future implications of her policy, in particular, the re-election consequences of her actions, and the discounted expected payoff from being replaced by a challenger. Symmetry implies $p(i) = -p(-i)$ for all $i \in [0, 1]$. Consequently, in equilibrium each voter can correctly predict an incumbent's future behavior simply by observing her most recent policy choice, and will vote for the challenger if and only if the expected payoff from electing the challenger exceeds that from re-electing the incumbent.

Equilibrium existence and uniqueness follow directly from Bernhardt et al. (2009). Along the equilibrium path, outcomes are characterized by a re-election standard $w \in (0, 1)$ and by a compromise threshold $c \in (w, 1)$. The median voter is decisive: an incumbent is re-elected if and only if she implements a policy y that is sufficiently close to the median voter's preferred policy, $y \in [-w, w]$. Incumbents with ideology $i \in [-w, w]$ are called "centrists". Centrist

[5]See Banks and Duggan (2008) for a detailed discussion.

[6]One also needs to specify reasonable beliefs and behavior following out-of-equilibrium-path policy choices. See Duggan (2000, Theorem 1) for details.

i implements her preferred policy $p(i) = i$ and is re-elected. Incumbents $i \in [1, -c] \cup [c, 1]$ are called "extremists". Extremists implement their preferred policies, but are ousted from office. Incumbents with ideology $i \in (w, c)$ are called "moderates." They do not adopt their preferred policies, as they would then lose office. Instead, they compromise and adopt the most extreme policy that still allows them to win re-election, $p(i) = w$. Similarly, incumbents $i \in (-c, -w)$ compromise to $p(i) = -w$. Figure 1 depicts such thresholds.

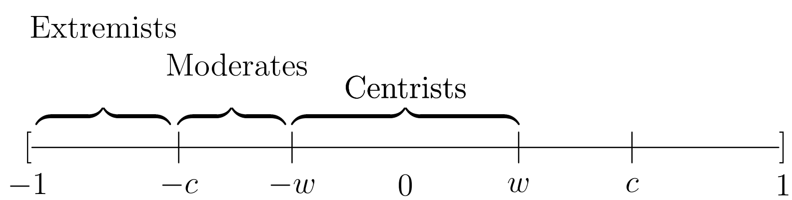


Figure 1: Equilibrium Thresholds

Voter Welfare: To measure voter welfare, we use the median voter's expected discounted payoff from electing an untried challenger. In a symmetric equilibrium with quadratic preferences, the payoff derived by a voter from electing an untried challenger from either party with equal probability equals that of the median voter minus a constant that reflects the distance of the voter's ideology to the median. That is, ex ante (at time $t = 0$), all voters share the median voter's preference ordering over different symmetric equilibria.[7] Thus, focusing on the payoff of voters and disregarding the payoffs of incumbent politicians, our welfare concept is Pareto efficiency.

3 The Value of Moderation

In this section we address the question: what is the value of selecting challengers from a pool of politicians with more moderate ideologies? We want to identify the conditions under which voters benefit from selecting from a pool of politicians whose world views tend to better reflect those of the median voter. In practice, the distribution of challenger ideologies depends on the opportunity cost of running for office, which may differentially impact

[7]Loosely speaking, with quadratic utility and a symmetric equilibrium, all voters would benefit ex ante from reductions in the variance of the implemented policies. See Banks and Duggan (2006) for a more general welfare result on majority preferences over lotteries when voters have quadratic utility.

potential candidates with more extreme or more moderate ideologies. Similarly, changes in the institutions governing the process of primary selection of candidates, or changes in the ideological composition of primary voters may affect the ideological distribution of selected challengers. Substantial uncertainty remains after the selection of a challenger, reflecting both the small pool of interested candidates in most primary elections, and the limited information that primary voters have about these untried politicians.

3.1 Analysis

We refer to politicians with ideologies further from the median voter as "more extreme", and those with ideologies closer to the median as "more moderate." Consistent with this idea, we use the following definition to compare different distributions of politicians' ideologies:

Definition: If ideology distribution F_R first order stochastically dominates[8] ideology distribution F'_R, $F_R \succ_{FOSD} F'_R$, then we say that distribution F'_R is "more moderate" than distribution[9] F_R. Equivalently, we say that F_R is "more extreme" than F'_R.

In equilibrium, politicians with ideologies further from the median voter implement more extreme policies. This raises the basic question: do voters benefit from drawing challenging candidates from a more moderate distribution of ideologies? If so, when do they benefit?

A more moderate ideology distribution has a positive direct effect and a negative indirect effect. Holding constant the equilibrium strategy of incumbents,[10] the direct impact of having a more moderate distribution of politicians' ideologies is that the median voter now expects a higher payoff from electing a challenger, who is expected to implement policies closer to the median. From an incumbent's perspective, however, more ideologically-moderate challengers mean that losing re-election is less costly, since a challenger is less likely to implement an extreme policy if elected. This makes incumbents *less* willing to compromise (reduces

[8]Recall that $F_R \succ_{FOSD} F'_R$ if and only if $F_R(i) \leq F'_R(i)$ for all i, and the inequality is strict for some i.

[9]Throughout this Section we focus our discussion on the comparison between economies featuring right-wing ideology distributions F_R and F'_R. It is implicit that we always consider symmetric parties, that is, we always consider symmetric left-wing ideology distributions F_L and F'_L.

[10]This amounts to holding fixed the cutoffs w and c that describe how a politician's ideology maps into her policy choice, $p(i)$.

compromise cutoff c), and this indirect effect on incumbents' strategies reduces voter payoffs. So the question is: which effect dominates?

Lemma B.1 in the on-line Appendix shows that given any pair of ideology distributions F_R and F'_R, with $F_R \succ_{FOSD} F'_R$, if voters are sufficiently impatient, then they all prefer the more moderate distribution of ideologies. When voters are sufficiently impatient, changes in the ideology distribution have a small indirect impact on equilibrium cutoffs w and c, since voters and politicians are not very concerned about the future. Consequently, the direct positive effect of drawing challengers from a more moderate distribution of ideologies dominates.

But, what if voters and politicians are patient? Our first contribution is to define a comprehensive sufficient condition for patient voters to prefer a more extreme distribution of politicians's ideology.

Definition: Given $k \in (0,1)$, ideology distribution F_R *k-dominates* F'_R, $F_R \succ_k F'_R$, if $F_R \succ_{FOSD} F'_R$ and $f_R(i) = f'_R(i)$ for all $i \in [k,1]$.

To understand the definition, consider an economy A with ideology distribution F_R and an economy B with distribution F'_R, such that $F_R \succ_k F'_R$. Recall that we symmetrically define left-wing candidates, and that the overall distribution of politicians' ideologies has support $[-1,1]$. Cutoff k divides politicians into two groups: a group with ideologies $(-k,k)$ in the middle of the distribution, and a group with ideologies $[-1,-k] \cup [k,1]$ that comprises the tails of the distribution. Both economies feature the same distribution of politicians in the tails, but economy B has a more moderate ideology distribution (in the FOSD sense) in the middle of its support. That is, as we move from the "more extreme" economy A to the "more moderate" economy B, the ideology distribution of politicians $(-k,k)$ around the median voter moves closer to the median, while the ideology distribution of tail politicians remains the same.[11]

Proposition 1 proves that, if F_R k-dominates F'_R, then sufficiently patient voters prefer the more *extreme* ideology distribution F_R.

[11]Thus, k-dominance implies first order stochastic dominance, but not conversely. The only additional constraint imposed by k-dominance is on the set of tail ideologies $[-1,-k] \cup [k,1]$. For k close to one, this set is very small. Hence, k-dominance describes a large subset of FOSD cases.

Proposition 1 *Suppose $F_R \succ_k F'_R$ and both distributions satisfy (A.1). If voters and politicians are sufficiently patient, then all voters prefer the more extreme ideology distribution F_R. The preference is strict if the distribution of centrist ideologies is not the same.*[12]

To understand the intuition, first consider the role of patience in the proposition. Fix any $k \in (0, 1)$ and any pair of ideology distributions such that $F_R \succ_k F'_R$. Let $\{w, c\}$ and $\{w', c'\}$ be the respective equilibrium cutoffs, where it is implicit that cutoffs depend on the discount factor δ. Recall that politicians with ideology $i \in [0, w]$ are centrists, $i \in [w, c]$ are moderates, and $i \in [c, 1]$ are extremists. Lemma B.3 in the on-line Appendix shows that, when voters are patient, $\min\{c, c'\}$ is close to one and $\max\{w, w'\}$ is close to zero. This has two important consequences. First, if agents are sufficiently patient, then $k < \min\{c, c'\}$. Consequently, the ideology distribution of extreme politicians $i \in [\min\{c, c'\}, 1]$ is the same in both distributions. In other words, as we move from F_R to F'_R, we only shift the ideology distribution of centrist and moderate politicians. Second, if agents are sufficiently patient, the ideologies of politicians c and c' are far from the median voter.

The intuition behind the result is then the following. Consider changing the economy from the more extreme distribution F_R to the more moderate distribution F'_R. Consider the median voter's decision of whether to reelect a right-wing incumbent, or to elect an untried left-wing challenger. Moderation of centrist and moderate politicians provides a small direct benefit to the median voter, since these politicians are already implementing policies close to the median voter. Now consider the decision of a right-wing incumbent with an extreme ideology around c. When politicians are patient, the cutoff ideology c is far from the median voter. As a result, the moderation of centrist and moderate politicians has a large direct positive impact on the expected payoff of this extreme right-wing incumbent if she is replaced by a left-wing challenger. This causes a large decrease in c: enough extreme incumbents cease compromising that the net result is that voter welfare is harmed. Paradoxically, improvements in the distribution of political ideologies reduce welfare by more precisely when politicians tend to be extreme. This reflects that decreases in c are more harmful when more politicians have extreme ideologies close to c, i.e., when $f_R(c)$ is high.

[12]That is, if $f_R(i) \neq f'_R(i)$ for some $i < w$. This always holds if $f_R(0) \neq f'_R(0)$, since a politician with the median ideology is always a centrist, $0 < w$ for all $\delta \in (0, 1)$.

Given $k \in (0,1)$ and distributions $F_R \succ_k F_R'$, Proposition 1 requires voters to be sufficiently patient. But what does sufficiently patient mean? Loosely speaking, when k is closer to one, agents must be more patient in order to ensure that we only change the ideology distribution of centrist and moderate politicians, $k < \min\{c, c'\}$. When k is closer to zero, the result in Proposition 1 extends when agents are less patient. We now establish that for any fixed discount $\delta \in (0,1)$ and F_R', voters prefer the more extreme ideology distribution whenever k is sufficiently small: fixing $\delta \in (0,1)$ and F_R', there exists a strictly positive cutoff k^* such that if $F_R \succ_k F_R'$ and $k \leq k^*$, then all voters prefer the more extreme ideology distribution F_R.

Proposition 2 *Fix any discount $\delta \in (0,1)$ and any ideology distribution F_R' that satisfies (A.1). Let $\{w', c'\}$ be the equilibrium cutoffs given $\{\delta, F_R'\}$. Define $\underline{f} = \min_{i\in[0,1]} f_R'(i)$, $\overline{f} = \max_{i\in[0,1]} f_R'(i)$, and*

$$k^* \quad = \quad \min\left\{w', \frac{\delta \underline{f}\,[c'^2 - w'^2]}{[4 + 2\delta\overline{f}\,]}, \sqrt{[c'^2 - w'^2]\,[c' - w']\,\underline{f}}\right\}. \tag{1}$$

Then, if distribution F_R satisfies (A.1) and $F_R \succ_k F_R'$ for some $k \in (0, k^*]$, all voters strictly prefer the more extreme ideology distribution F_R.

This result has two important implications. First, *every* moderation of ideologies that only involves politicians whose ideologies are close enough to the median voter harms all voters. Second, since given any fixed discount factor we have $k^* > 0$, *there always exist* more extreme ideology distributions of politicians that benefit all voters.

Propositions 1 and 2 consider ideology shifts that do not change the ideology distribution of politicians $i \in [c', 1]$. But, what happens when ideology moderation shifts the distribution of these extreme politicians? Bernhardt et al. (2009, Proposition 2) present the following example. They start from uniformly distributed ideologies F_R, and consider a more moderate distribution F_R' that takes the form of eliminating all probability mass on the most extreme ideologies and redistributing it uniformly across the remaining more moderate ideologies. In that case, moderation *always* benefits voters: such an extreme form of moderation has such a large direct positive payoff impact on the median voter that it always dominates the negative impact of a lower compromise cutoff c. That is, eliminating the worst possible office holders is always welfare enhancing.

However, not every ideology moderation involving extreme politicians is beneficial. To better understand when moderating shifts in the distribution of challengers benefit voters, we next numerically solve for equilibrium voter welfare when the density of right-wing ideologies f_R is linear. This lets us capture the degree of moderation with a single parameter. Similar to Proposition 1, we find that moderation hurts voters when agents are sufficiently patient.

3.2 Linear densities

Suppose that party R consists of politicians with ideologies $i \in [0, 1]$, distributed according to the probability density function

$$f_R(i) \quad = \quad \alpha + 2(1 - \alpha)i, \tag{2}$$

where $\alpha \in (0, 2)$. The associated cumulative distribution function is $F_R(i) = \alpha i + (1 - \alpha)i^2$. Party L consists of politicians with ideologies $i \in [-1, 0]$, distributed according to

$$f_L(i) \quad = \quad \alpha - 2(1 - \alpha)i. \tag{3}$$

The associated cumulative distribution function is $F_L(i) = 1 - F_R(|i|)$, for $i \in [-1, 0]$.

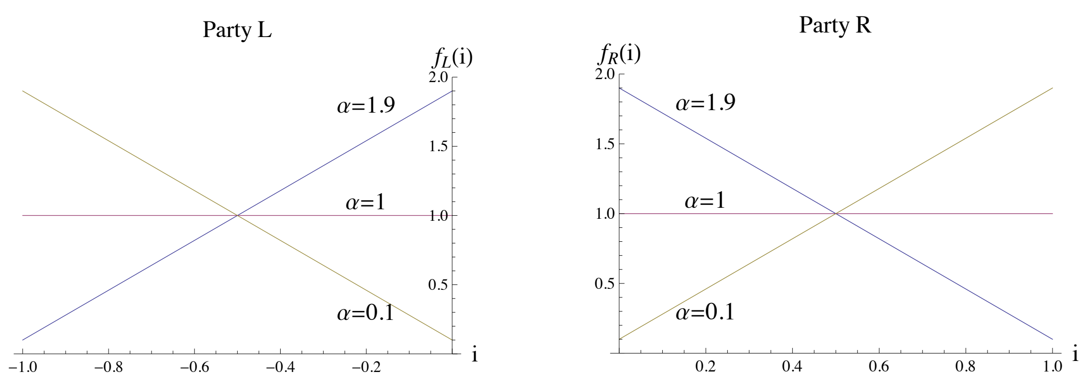


Figure 2: Probability density function of ideologies, for different values of α.

Parameter α captures the degree of ideological moderation. A higher α means that challengers are more likely to have ideologies closer to the median voter's. The distributions are uniform when $\alpha = 1$. The expected ideology of a party R candidate is $\int_0^1 i[\alpha+2(1-\alpha)i]\, di = \frac{2}{3} - \frac{\alpha}{6}$, and $\alpha = f_R(0) = f_L(0)$. Figure 2 illustrates these distributions for different values of α.

Figure 3 reveals how voter welfare varies with the ideology parameter α and the discount factor δ. The line in Figure 3 represents parameters such that the marginal value of increasing the moderation parameter α is zero. Above this line, the marginal value of increasing α is negative, and below this line, the value is positive. When voters are sufficiently impatient (low δ), a more moderate distribution of ideologies (marginal increase in α) benefits all voters, while the opposite is true when voters are sufficiently patient.

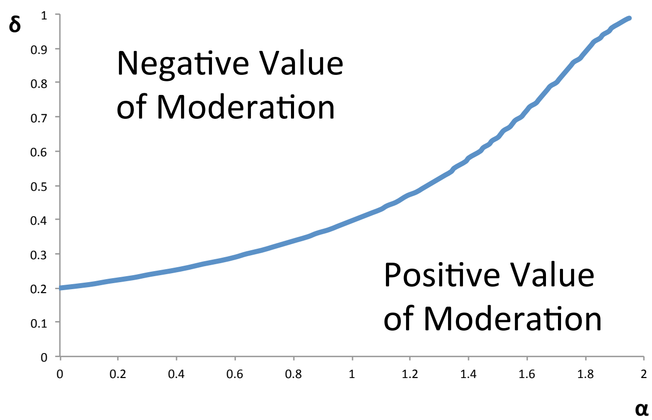


Figure 3: The marginal value of moderation (marginal value of increasing α)

When agents are even modestly patient, moderation has a non-monotonic impact on welfare. For any discount factor $\delta > 0.2$, voter welfare first falls in α, before rising. That is, when α is sufficiently low—so politicians are likely to be extreme—marginal increases in the moderation parameter α *increase* the expected extremism of policies implemented by incumbents, harming voters. Figure 3 shows that for plausible discount factors, $\delta > 0.5$, the marginal value of a more moderate pool of challengers is *negative unless* the distribution of political ideologies is already quite moderate (α much greater than one). This reflects that if enough politicians have moderate-to-extreme ideologies just below c, the extremism effect of the decrease in c dominates. Moreover, when many politicians have extreme ideologies and agents are patient, the compromising incentives generated by party competition are large—incumbents are very concerned about losing to a challenger from the opposing party. In this case, moderation of challengers sharply reduces the incentives to compromise generated by party competition.[13,14]

4 The "Value" of Information about Challengers

Bernhardt et al. (2009) introduces political parties to the repeated election framework of Duggan (2000). Party labels are informative because candidates from different parties have ideologies drawn from opposing sides of the ideological spectrum. When ousting an incumbent from office, this additional information allows voters to select a challenger from the opposing party. Incumbents dislike being replaced by someone with a more distant ideology, so they become more willing to moderate policy choices, raising voter welfare. Thus, in that model, the value of information (party labels) is *always* positive.

We next characterize how equilibrium outcomes are affected when, in addition to the information conveyed by party labels, voters receive a noisy, but informative, signal about a challenging candidate's ideology. When voters can partially distinguish between challengers, they are more willing to replace an incumbent when they receive a signal suggesting that the challenger is more likely to be a moderate, than when the signal suggests that she is an extremist. We want to understand whether voters benefit from such additional learning about challengers.

To do this we extend our basic model by introducing a noisy binary signal about a challenger's ideology. This signal's natural interpretation is as a binary signal "endorsement" or "no endorsement" by informed interest groups. Although we take this signal's origin as exogenous, we describe how one can endogenize the source and information content of the

[13] We numerically replicated Figure 3 using loss functions that take the form $u(x, y) = -|x - y|^z$, $z \in \{1, 2, 3\}$. In all cases, marginal increases in α reduce the median voter's payoff if δ is large and α is low. The region that represents a "negative value of moderation" for the median voter is smallest for Euclidean preferences, and largest for cubic preferences. This is because the threat provided by the ideology distribution of challengers has a bigger disciplining effect on incumbents when utility functions are more concave.

[14] In contrast, with at large selection of challengers, and quadratic utility, moderations in the distribution of ideologies affect voters in the same way, regardless of their ideologies. As a result, the change in the median voter's expected payoff from electing a challenger exactly equals the change in an incumbent's expected payoff from losing re-election to a challenger. Hence, moderation in the distribution of political ideologies is always welfare enhancing.

signal by studying public endorsements from informed interest groups.

As before, right- and left-wing politicians are drawn from probability distributions F_R and F_L. After the incumbent implements her policy but before the election, voters observe a noisy public signal about the challenger's ideology.

Consider a left-wing incumbent facing a right-wing challenger. Recall that voters' prior belief is that the challenger has an ideology $i \in [0,1]$ drawn from the probability density function f_R. After a challenger is selected, voters learn about the challenger, observing a public signal Π_β about her ideology, where the index $\beta \in [0,1]$ captures the accuracy of signal Π_β relative to a benchmark likelihood function π in a way that we describe momentarily. Signal Π_β has two possible realizations, $s \in \{s_G, s_B\}$. Realization s_G is a "good" signal about the challenger's moderacy, while s_B is a "bad" signal: s_G implies that the challenger is more likely to have an ideology closer to the median voter than does s_B. Formally, realizations s_G and s_B generate updated posterior beliefs $f_R^{G\beta}$ and $f_R^{B\beta}$, respectively, that both satisfy (A.1) and the monotone likelihood ratio property.

Our central focus is on how the degree of informativeness of the signal about the challenger affects equilibrium behavior and welfare. To this end, we want to distinguish the informativeness β of the signal, from the unconditional probability that the challenger generates good signal s_G. Accordingly, public signal Π_β places weight $\beta \in [0,1]$ on an informative benchmark likelihood function π and remaining weight $1-\beta$ on the prior probability of a good signal. The benchmark likelihood function $\pi : [0,1] \to (0,1)$ defines the probability $\pi(i)$ that a challenger with ideology i generates signal realization s_G. To capture that signal s_G is a good signal about the challenger's moderacy and guarantee that posterior beliefs satisfy (A.1) we assume:

(A.2) The benchmark likelihood function $\pi : [0,1] \to (\underline{\pi}, \overline{\pi})$ is absolutely continuous and weakly decreasing, strictly decreasing on some interval $[0,h)$, with $0 < \underline{\pi} < \overline{\pi} < 1$ and $h > 0$.

The likelihood function π_β of signal Π_β is then

$$\pi_\beta(i) = \beta\pi(i) + (1-\beta)\rho, \tag{4}$$

where $\rho = \int_0^1 \pi(i) f_R(i)di$ is the prior probability of a good signal. Thus, a higher β indicates a more informative signal. When $\beta = 1$, the likelihood function π_β is as informative of i as is the benchmark π, and when $\beta = 0$, the likelihood function π_β is completely uninformative.

Further, the unconditional probability of signal realization s_G does not vary with β:

$$Pr(s = s_G) = \int_0^1 \pi_\beta(i) f_R(i) di = \beta \int_0^1 \pi(i) f_R(i) di + (1-\beta)\rho = \beta\rho + (1-\beta)\rho = \rho.$$

This structure isolates the effects of a change in signal accuracy β from those of the prior probability ρ of signal realization s_G.[15]

When signal Π_β generates realization $s \in \{s_G, s_B\}$, Bayes' rule yields the following updated probability density functions,

$$f_R^{G\beta}(i) = \frac{\pi_\beta(i) f_R(i)}{\rho} = \beta \frac{\pi(i) f_R(i)}{\rho} + (1-\beta) f_R(i) = \beta f_R^{G1}(i) + (1-\beta) f_R(i),$$
$$f_R^{B\beta}(i) = \frac{[1-\pi_\beta(i)] f_R(i)}{1-\rho} = \beta \frac{[1-\pi(i)] f_R(i)}{1-\rho} + (1-\beta) f_R(i) = \beta f_R^{B1}(i) + (1-\beta) f_R(i),$$

Thus, posteriors are a weighted average of the maximum feasible information captured by $f_R^{G1} = \frac{\pi(i) f_R(i)}{\rho}$ and $f_R^{B1} = \frac{[1-\pi(i)] f_R(i)}{1-\rho}$, and the prior belief f_R, where the weight on the information in the signal is β. It is straightforward to compute the corresponding cumulative density functions $F_R^{G\beta}$ and $F_R^{B\beta}$. When the challenger is a left-wing politician, the symmetric signal is defined analogously.

Remark 1: Suppose F_R satisfies (A.1) and π satisfies (A.2). Then, for any signal Π_β with $\beta \in (0, 1]$,

- Densities $f_R^{G\beta}$ and $f_R^{B\beta}$ satisfy (A.1) for some lower-bounds $\underline{f}^{G\beta} > 0$ and $\underline{f}^{B\beta} > 0$;
- Densities $f_R^{G\beta}$ and $f_R^{B\beta}$ satisfy the monotone likelihood ratio property (MLRP): $\frac{f_R^{B\beta}(i)}{f_R^{G\beta}(i)}$ weakly increases in $i \in [0, 1]$, and strictly increases in the range $[0, h)$.

Moreover, for any $\beta \in (0, 1]$ we have $F_R^{B\beta} \succ_{FOSD} F_R^{G\beta}$: a good signal raises the likelihood that the challenger has a more moderate ideology. Indeed, the greater is β, i.e., the more accurate is the signal technology, the more the signals reveal about whether the challenger is a moderate or an extremist: for $1 \geq \beta' > \beta > 0$, $F_R^{B\beta'} \succ_{FOSD} F_R^{B\beta} \succ_{FOSD} F_R^{G\beta} \succ_{FOSD} F_R^{G\beta'}$. One can also show that the posteriors $F_R^{B\beta}$ and $F_R^{G\beta}$ inherit the MLRP ordering of $f_R^{G\beta}$ and $f_R^{B\beta}$ for each β. An analogous formulation describes beliefs about left-wing challengers.

[15] If we generalize equation (4) to $\pi_\beta(i) = \beta\pi(i) + (1-\beta)\tilde{\rho}$ for some exogenous constant $\tilde{\rho} \in (0, 1)$, then $\frac{\partial Pr(s=s_G)}{\partial \beta} = \rho - \tilde{\rho}$. The results that we establish in Proposition 3 extend to any $\tilde{\rho}$, and $\tilde{\rho} \geq \rho$ (so increasing β does not raise the unconditional probability of a good signal realization) is a sufficient condition for Lemma 1.

In summary, the signal structure Π_β is defined by an accuracy parameter β and a benchmark likelihood function π. A new challenger is drawn before each election. All voters see the new signal realization $s \in \{s_G, s_B\}$ about the challenger's ideology. Because the ideology of a challenger from a given party is an i.i.d. draw each period, and the signal generating process Π_β is independent across periods, knowledge of past signal realizations contains no information about the ideologies of current or future challengers. Therefore, we again focus on stationary strategies: after a politician takes office, her policy choices depend only on her ideology—it is independent of past signal realizations. So, too, voter choices in any given election are functions only of the incumbent's most recent policy choice, and the signal about the current challenger's ideology. The retrospective/prospective voting choices reflect voters' updated beliefs about a challenger after observing the binary signal: each voter x sets two signal-specific utility (voting) standards. In equilibrium, each voting standard corresponds to the expected discounted payoff from electing the challenger after observing that particular signal.

4.1 Analysis

Consider any benchmark likelihood function π that satisfies (A.2). If the signal Π_β is completely uninformative ($\beta = 0$), then the unique equilibrium is that established in Sections 2 and 3. When the signal is informative ($\beta > 0$), voters set two re-election cutoffs, w_G and w_B, where $0 < w_G < w_B < 1$. In equilibrium, the decisive median voter is indifferent between re-electing the incumbent and electing the challenger if the realized signal is s_G and the incumbent implemented policy w_G. The median voter is similarly indifferent if the realized signal is s_B and the incumbent implemented the more extreme policy w_B. Thus, a right-wing incumbent who implements policy $y \in [0, w_G]$ always wins re-election; an incumbent who implements policy $y \in (w_G, w_B]$ wins if and only if voters receive a bad signal about the challenger; and an incumbent who implements an extreme policy $y > w_B$ always loses.

Consider a right-wing incumbent with ideology $i > w_B$. Her choices reduce to deciding whether (1) to compromise to w_G in order to ensure re-election even when the signal about a challenger is good; (2) to compromise to w_B and win re-election only when the signal about a challenger's ideology is bad, which happens with probability $(1 - \rho)$; or (3) to implement her ideal policy i and lose re-election for sure. Thus, compromising politicians $i > w_B$ must decide whether to compromise all the way to w_G to ensure victory, or only to compromise partially to w_B and be ousted from office with probability ρ each period when she draws a challenger who generates a good signal realization.[16]

For politician $i \geq w_B$, the direct period payoff cost of implementing policy w_G instead of w_B is $-(i-w_B)^2+(i-w_G)^2 = 2i(w_B-w_G)+w_G^2-w_B^2$. Because $i \leq 1$ and $w_B > w_G$, this cost is strictly less than $2(w_B-w_G)$. Consequently, when $w_B - w_G$ is close to zero, this cost is even closer to zero. The direct benefit from choosing w_G instead of w_B is the increase in the probability of re-election from $(1-\rho)$ to one, that is, a probability increase of $\rho > 0$. Hence, if w_B is close enough to w_G, then any incumbent with ideology $i \in [w_B, 1]$ faces a small marginal cost of changing her policy choice from w_B to w_G, but a discrete positive benefit from guaranteeing re-election and avoiding replacement by a challenger. Thus, when the difference $(w_B - w_G)$ is small, the option to compromise to w_G strictly dominates the option to compromise to w_B, so that the relevant decision for incumbents $i \in [w_B, 1]$ becomes whether to compromise to w_G or to choose as policy their own ideology.

We focus on the case where all politicians who choose to compromise do so by compromising to w_G. A sufficient condition for no incumbent to compromise to w_B, i.e., for all compromising incumbents to compromise to w_G, is that the signal about the challenger be sufficiently noisy. This reflects the practical observation that when signals are noisy, w_G is not that much smaller than w_B so that an incumbent who finds it optimal to compromise, might as well compromise a little more, in order to ensure victory. We later investigate what "sufficiently noisy" signals mean in a setting where uncertainty over ideologies is described by linear densities.

Lemma 1 *Suppose F_R satisfies (A.1) and π satisfies (A.2). If the signal Π_β about the challenger is sufficiently noisy (β is sufficiently small) then no incumbent compromises to w_B.*

Thus, when the signal about the challenger is informative, but sufficiently noisy, equilibrium outcomes are characterized by the two re-election cutoffs described above, $0 < w_G < w_B < 1$, and *one* compromise cutoff c. Incumbents with ideologies $i \in (0, w_G]$ implement their

[16] Politicians with ideology $i \in (w_G, w_B)$, but close to w_G always prefer to compromise to w_G. If the distance between w_B and w_G is large (because the signal about the challenger is very informative), then incumbents with ideology $i \in (w_G, w_B)$ sufficiently close to w_B may prefer to implement their own preferred policies and win re-election if and only if the signal is s_B.

own ideologies as policy, while incumbents $i \in (w_G, c)$ compromise to w_G. Both groups are re-elected regardless of whether the signal about the challenger is good or bad. Extremist incumbents $i \in [c, 1]$ implement $p(i) = i$ and are always ousted from office. The indifference condition is that politician $i = c$ is indifferent between (a) compromising to $p(i) = w_G$ to guarantee re-election even when the signal about a challenger is good, and (b) implementing her own extreme ideology $p(i) = i$ and losing re-election for sure. Figure 4 depicts these thresholds.

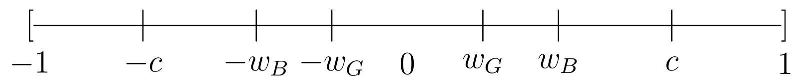


Figure 4: Equilibrium Thresholds

We now ask our second fundamental question: do voters value receiving an informative but noisy signal about a challenger's ideology?

To answer this question, we make a technical assumption that voters are sufficiently patient with discount factors $\delta > \delta^{**}(F_R)$, where given any ideology distribution F_R satisfying (A.1), $\delta^{**}(F_R) \geq 0$ is the minimum discount factor such that for all $\delta \in (\delta^{**}(F_R), 1)$,

$$\frac{2 - \delta + \sqrt{\delta(2-\delta)}}{2(1-\delta)} \geq \max_{i \in [0,c]} \left(\frac{f_R(i)}{f_R(c)} \right), \text{ for all } c \in [0, 1]. \tag{5}$$

As δ goes from zero to one, $\frac{2-\delta+\sqrt{\delta(2-\delta)}}{2(1-\delta)}$ strictly increases from 1 to infinity. Therefore, $\delta^{**}(F_R) < 1$. Moreover, if f_R is weakly increasing (e.g., F_R is uniform), then $\delta^{**}(F_R) = 0$, i.e., our results hold for all $\delta \in (0, 1)$.[17]

Proposition 3 *Suppose F_R satisfies (A.1), π satisfies (A.2), and voters are patient, $\delta \in (\delta^{**}(F_R), 1)$. Then voters strictly prefer not to receive signals about the challenger if the signal Π_β is sufficiently noisy (β is sufficiently small).*

Thus, learning about whether a challenger is more likely to be a moderate or an extremist *reduces* voter welfare. To understand the result, let w^* be the equilibrium re-election standard and c^* be the compromising cutoff when the signal is completely uninformative ($\beta = 0$). If β is positive, but small, politicians only compromise to w_G. The key is that re-election cutoff

[17]More generally, $\delta > \delta^{**}(F_R)$ is sufficient, but not necessary, for Proposition 3; we suspect that regardless of the level of $\delta \in (0, 1)$, voters strictly prefer not to receive sufficiently noisy signals about challengers.

w_G is stricter than w^*, $w_G < w^*$, reflecting that voters are more favorably disposed to a challenger with a good signal. That is, the presence of an informative signal forces incumbents to compromise by more to ensure re-election. As a result, fewer incumbents compromise, i.e., $c < c^*$, and more incumbents implement their own extreme ideologies. This hurts all voters. Moreover, the informative signal about the challenging candidate's ideological preferences only affects the winning vote margin and not who wins—it does *not* improve the selection of winning challenging candidates. This is because whether a challenging candidate wins or not does *not* depend on whether there is a good or bad signal about her ideology, but *only* on whether or not she faces an extremist incumbent who chose not to compromise.

To glean a deeper understanding for the result, recall that when signals are completely uninformative, i.e., when $\beta = 0$, the median voter is just indifferent between re-electing an incumbent who adopts the equilibrium standard w^* as policy, and trying the risky challenger. The median voter does not internalize that *if* he set a slacker standard, then more extreme incumbents would choose to compromise to win re-election, rather than locate extremely, which would raise voter welfare. The key is that the welfare gain from greater compromise is first-order, while the cost from slightly inefficient replacement is second-order.[18] Slightly informative signals induce the median voter to set a more demanding standard for re-election, i.e., to reduce w_G; and because all incumbents who compromise do so to w_G, this serves to set a more demanding standard for re-election, hurting voters. Moreover, voter welfare continues to decrease following any increase in the signals informativeness that decreases w_G without leading incumbents to compromise to w_B.

This powerful negative result regarding the welfare costs of learning about challengers holds as long as no incumbent type compromises to w_B in equilibrium. Proposition 3 only assumes that the signal about the challenger is sufficiently noisy as a sufficient condition for

[18]In fact, Lemma A.1 in the Appendix proves the stronger result that were a social planner to impose *any* re-election standard w that is stricter than the equilibrium standard, $w < w^*$, this would strictly decrease the welfare of all voters. Indeed, voter welfare declines continuously with reductions in w. This result contrasts sharply with the recent papers by Ashworth et al. (2012) and Caselli et al. (2012), who argue that in equilibrium voters would benefit from stricter re-election thresholds. A key difference between these models and ours is that in our model politicians and voters are horizontally differentiated by ideology, while in these other papers politicians are not policy motivated—they are vertically differentiated by ability and choose costly effort levels that are not directly observed by voters.

no incumbent to compromise to w_B (Lemma 1). But how small does β have to be in practice?

The bound on the signal's informativeness for all compromising incumbents to choose w_G is an intricate function of model parameters. In particular, the signal's information content as captured by β is a measure of information about a challenger's fundamental *characteristics* (her ideology). However, how ideology translates into actual policy choices and hence the median voter's expected payoff from electing a challenger depends on the equilibrium of the game. For instance, a signal may be very informative about a challenger's ideology, but if most challengers compromise in equilibrium, the payoff relevant information content of the signal may be small. Hence, if the parameters of the model are such that many incumbents compromise, the result in Lemma 1 may hold even for very informative signals. In particular, one expects the result to hold for higher β when an incumbent's incentives to compromise are higher—when agents are very patient or the unconditional distribution of challengers' ideologies is more extreme. We now show that this intuition holds for our linear density parameterization.

4.2 Linear Densities with Signals

We focus on signal Π_β when the prior distribution f_R and the posterior distributions $f_R^{G\beta}$ and $f_R^{B\beta}$ (updated after observing the signal realization) are linear functions of the challenger's ideology. Formally, for all $i \in [0,1]$,

$$\begin{aligned} f_R(i) &= \alpha + 2(1-\alpha)i, \\ f_R^{G\beta}(i) &= \alpha^{G\beta} + 2(1-\alpha^{G\beta})i, \\ f_R^{B\beta}(i) &= \alpha^{B\beta} + 2(1-\alpha^{B\beta})i, \end{aligned}$$

where $0 < \alpha^{B\beta} < \alpha < \alpha^{G\beta} < 1$. Recall that α captures the degree of ideological moderation, where a higher α implies a more moderate ideology distribution. Hence, voters start with a prior belief α about the degree of the challenger's ideology moderation, then their beliefs rise to $\alpha^{G\beta}$ after observing a good signal s_G, and fall to $\alpha^{B\beta}$ after a bad signal s_B.

If the prior probability of a good signal is $\rho \in (0,1)$, then Bayes' rule yields:

$$\rho f_R^{G\beta}(0) + (1-\rho) f_R^{B\beta}(0) = f_R(0) \Rightarrow \rho\alpha^{G\beta} + (1-\rho)\alpha^{B\beta} = \alpha.$$

To guarantee that the posteriors $\alpha^{G\beta}$ and $\alpha^{B\beta}$ are always between 0 and 2, we assume that $0 < \alpha - \rho < 1$ and define the bounds $\overline{\alpha} = \alpha + (1-\rho)$ and $\underline{\alpha} = \alpha - \rho$. Conditional on the right-wing challenger's ideology i, the benchmark likelihood function π generates realization s_G with probability

$$\pi(i) = \frac{[\overline{\alpha} + 2(1-\overline{\alpha})i]\rho}{f_R(i)} = \rho + \rho(1-\rho)\frac{(1-2i)}{f_R(i)}. \tag{6}$$

Note that π satisfies (A.2),

$$\frac{\partial \pi(i)}{\partial i} = \rho(1-\rho)\left\{\frac{-2}{f_R(i)} - 2(1-\alpha)\frac{(1-2i)}{f_R(i)^2}\right\} = -\frac{2\rho(1-\rho)}{f_R(i)^2} < 0.$$

Substitute (6) into (4). Signal Π_β generates realization s_G with conditional probability

$$\pi_\beta(i) = (1-\beta)\rho + \beta\pi(i) = \rho + \rho(1-\rho)\beta\frac{(1-2i)}{f_R(i)}.$$

Integrating over ideologies $i \in [0,1]$, the overall probability of a good signal is ρ. After receiving a signal, voters update their beliefs using Bayes' rule:

$$f_R^{G\beta}(i) = \frac{\pi_\beta(i) f_R(i)}{\rho} = f_R(i) + (1-\rho)\beta(1-2i) = \alpha^{G\beta} + 2(1-\alpha^{G\beta})i,$$

where $\alpha^{G\beta} \equiv \alpha + (1-\rho)\beta$ captures the expected increased degree of ideological moderation, and

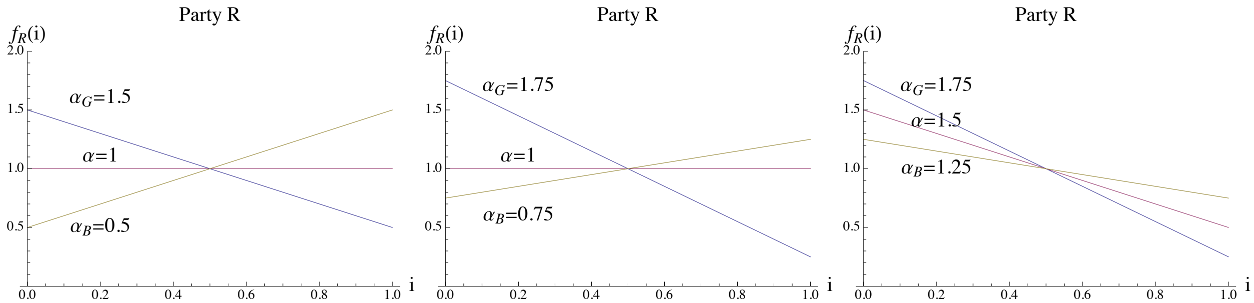
$$f_R^{B\beta}(i) = \frac{[1-\pi_\beta(i)]f_R(i)}{(1-\rho)} = f_R(i) - \rho\beta(1-2i) = \alpha^{B\beta} + 2(1-\alpha^{B\beta})i,$$

where $\alpha^{B\beta} \equiv \alpha - \rho\beta$ captures the expected decreased degree of ideological moderation.

Note that $\alpha^{G\beta} - \alpha^{B\beta} = \beta$. That is, the distance between the posterior beliefs strictly increases with the signal's informational content β, as the signal "rotates" the linear posterior around $i = 0.5$. Figure 5 illustrates prior and posterior beliefs for different parameter values.

We numerically solve the model when good and bad signals are equally likely ($\rho = \frac{1}{2}$), for different values of δ and α. Figure 6 shows that as long as agents are even modestly patient with a discount factor δ that exceeds 0.3, *no* incumbent compromises to w_B even when $\beta = 1$ so the signal is maximally informative.[19] More generally, the region where incumbents never compromise to w_B is greater when agents value the future more (δ is higher) and the overall

[19]When $\beta = 1$ and $\rho = 0.5$, condition $0 < \alpha - \rho < 1$ implies $\alpha \in (0.5, 1.5)$.



(a) $\alpha = 1,\ \beta = 1,\ \rho = 0.5$ (b) $\alpha = 1,\ \beta = 1,\ \rho = 0.25$ (c) $\alpha = 1.5,\ \beta = 0.5,\ \rho = 0.5$

Figure 5: Prior α and posterior beliefs α_G and α_B for different values of $\{\alpha, \beta, \rho\}$.

distribution of ideologies is more extreme (α is lower). Thus, for this linear density parameterization, voters necessarily strictly prefer not to receive information about challengers whenever $\delta \geq 0.3$. Moreover, in this parameter region where no incumbent compromise to w_B, every increase in the signal's informativeness (every increase in $\beta \in [0, 1]$) decreases w_G, which further decreases voter welfare. Consequently, voter welfare is maximized by an uninformative signal ($\beta = 0$), and is decreasing in β, minimized by the most informative signal ($\beta = 1$).

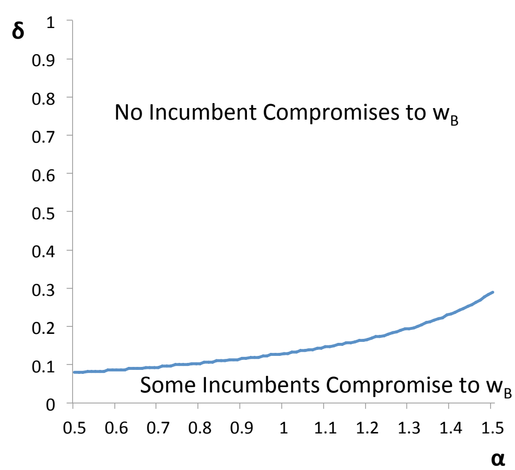


Figure 6: Party Selection with Linear Densities and Signal $\beta = 1$.

It is worthwhile to reflect on our two welfare results. We find that under plausible scenarios, more information about challengers impairs welfare. In contrast, while more moderate distributions of challenger ideologies impair voter welfare if concentrated sufficiently on centrists, they can enhance voter welfare if moderation is concentrated more on extremists.

To understand what underlies the difference, observe that with a more moderate pool of challengers, a challenger who replaces an incumbent is likely better—selection improves. In contrast, whenever compromising incumbents always do so by enough to ensure reelection, electoral outcomes only hinge on an incumbent's policy choices—more informative signals only serve to force incumbents to compromise by more to ensure re-election. Thus, the first-order effect of improving the distribution of challengers is to raise the value of replacing an incumbent with a new challenger. In contrast, the first-order effect of improving the information about challengers is to raise the cost of compromise to incumbents.

4.3 Endogenous Information Transmission

In the on-line Appendix we endogenize the information transmitted to voters about challengers, by examining cheap-talk messages sent by informed interest groups. Below we highlight some of these results.

Consider two symmetrically-situated interest groups that have the same utility functions as voters $\{-\gamma, +\gamma\}$. Before each election, the IGs costlessly gain access to information about whether the challenger is more likely to have a moderate or an extreme ideology. This learning process corresponds to the signal Π_β with possible realizations $s \in \{s_G, s_B\}$ described previously. The signal received by IGs is non-verifiable, but each IG can send a public (cheap-talk) message to voters about a challenger's ideology. One can interpret the IGs as newspapers or other political institutions with limited biases in their political preferences, and better access to political information than voters. The messages have the natural interpretation as political endorsements for a challenger or incumbent.

We prove that the set of equilibrium cutoffs w_G and w_B in the model without IGs, also characterize the equilibrium in the model where information is endogenously transmitted by IGs—as long as the political biases of the IGs are not too large (γ is small enough). When the IGs are only slightly politically biased, there exists an informative equilibrium in which voters rely on IG endorsements to learn about a challenger's expected ideology. However, voters must account for incentive misalignments. There is a range of intermediate policies such that a right-wing incumbent is always supported by the right-wing IG, so that its endorsement reveals no information to voters. Voters then rely on the information transmitted by the opposing IG, which turns out to be informative. Conversely, in a more extreme range of policies implemented by the right-incumbent, the left-wing IG always supports the left-challenger; now the informative endorsement comes from the right IG.[20]

An interesting implication is that if interest groups are sufficiently biased (γ is large enough), then the incentive misalignment between the median voter and the IGs is so large that the median voter ceases to learn in any cheap-talk equilibrium about the signals received by the IGs. Consequently, in an economy where the results of Proposition 3 hold, having very extreme IGs is better than having very centrist IGs. Extreme IGs guarantee that voters will not learn about the challenger, when the value of this information is negative.

5 Conclusion

Our paper focus on two questions. First, how do changes in the ideology distribution of candidates affect the equilibrium behavior of incumbent politicians? Voters can only credibly threaten to replace an incumbent who implements policies that are too extreme if they expect the challenging candidate to implement better policies. However, improving this outside option—having challengers with preferences closer to those of most voters—need not benefit voters. This is because more moderate challengers reduce the incentives generated by party competition for an incumbent to compromise, since losing to a more moderate challenger is less costly.

Our second question asks: do voters benefit from learning about challengers during the general election? Although ideology information conveyed by party labels always benefits voters, additional information about particular challengers reduces welfare under a robust set of circumstances. We show that if voters receive sufficiently noisy signals about a challenger's ideological preferences, then incumbents who compromise do so to the stricter reelection standard required to defeat a challenger who receives a good signal. This stricter re-election standard causes more incumbents to adopt extreme policies. We show that this

[20] These prediction are consistent with Chiang and Knight (2011). They find that newspaper endorsements are influential in the sense that voters are more likely to support a recommended candidate after publication of an endorsement. Moreover, endorsements of Democratic candidates by left-leaning newspapers are less influential than those from neutral or right-leaning newspapers; and likewise for endorsements of Republicans.

reduced willingness to compromise *always* reduces voter welfare—learning about a challenger during the general election harms voters. The key is that the welfare cost from compromisers becoming extremists is first-order, while the welfare gain from the remaining compromisers becoming marginally more moderate is second-order.

We conjecture that analogous results would obtain if politicians also have heterogeneous valences — as in Bernhardt et al. (2011)— and voters receive noisy signals about a challenger's valence rather than about her ideology—very noisy learning about a challenger's ability hurts voters for precisely the same reason as noisy learning about her ideology. Such noisy learning forces incumbents to moderate policy to defeat a challenger who is more likely to be able, causing more extreme incumbents to cease compromising, without an accompanying improvement in the selection of challengers. Even more interesting to contemplate is the possibility of more accurate signals about challenger valences that increase the separation between re-election thresholds by enough that not all compromising incumbents do so by enough to ensure victory. As a result, the signals improve the distribution of elected officials. The question that we leave for future research is: when would such significant learning improve selection by enough to raise voter welfare?

A Appendix

A.1 Equilibrium in the Basic Model

In this section we briefly describe the equations that characterize equilibrium outcomes. Given the fundamentals δ and F_R, equilibrium behavior is fully characterized by a unique pair of cutoffs w and c. We start by summarizing the two indifference conditions that pin down these equilibrium cutoffs — the full equilibrium derivation can be found in Bernhardt et al. (2009).

(C.1) The median voter must be indifferent between electing the challenger and re-electing an incumbent who adopts policy w;

(C.2) The incumbent with ideology c must be indifferent between compromising to w to win re-election and adopting as policy her own ideology, hence losing to the challenger.

The first condition defines the re-election standard $w \in (0, 1)$: an incumbent politician is re-elected if and only if she implements policy $y \in [-w, w]$. The second condition defines the compromise cutoff $c \in (w, 1)$: politicians with ideology $i \in (w, c)$ compromise to w in order to win re-election, while politicians with ideology $i \in (-c, -w)$ compromise to $-w$.

It is useful to define the following auxiliary equilibrium values. Given the fundamentals δ and F_R and the unique equilibrium cutoffs $0 < w < c < 1$, define the following:

Expected Payoff: Let U_i^R be the discounted expected payoff of a voter with ideology i from electing an untried challenger from party R, and similarly define U_i^L for party L.

$$U_i^R = \int_0^w \frac{-(i-y)^2}{(1-\delta)} f_R(y)dy + \int_w^c \frac{-(i-w)^2}{(1-\delta)} f_R(y)dy + \int_c^1 \left[-(i-y)^2 + \delta U_i^L\right] f_R(y)dy, \quad (7)$$

$$U_i^L = \int_{-w}^0 \frac{-(i-y)^2}{(1-\delta)} f_L(y)dy + \int_{-c}^{-w} \frac{-(i+w)^2}{(1-\delta)} f_L(y)dy + \int_{-1}^{-c} \left[-(i-y)^2 + \delta U_i^R\right] f_L(y)dy. \quad (8)$$

Expected Policy: Define the auxiliary values E^R and E^L as the unique solutions to the following system of equations [21]

$$E^R = \int_0^w \frac{y}{(1-\delta)} f_R(y)dy + \int_w^c \frac{w}{(1-\delta)} f_R(y)dy + \int_c^1 \left[y + \delta E^L\right] f_R(y)dy, \quad (9)$$

$$E^L = \int_{-w}^0 \frac{y}{(1-\delta)} f_L(y)dy + \int_{-c}^{-w} \frac{-w}{(1-\delta)} f_L(y)dy + \int_{-1}^{-c} \left[y + \delta E^R\right] f_L(y)dy, \quad (10)$$

where by symmetry $E^R = -E^L$. We can then rewrite (9) as

$$E^R = \int_0^w \frac{y}{(1-\delta)} f_R(y)dy + \int_w^c \frac{w}{(1-\delta)} f_R(y)dy + \int_c^1 \left[y - \delta E^R\right] f_R(y)dy. \quad (11)$$

We now derive the remaining equilibrium equations in three steps.

Step 1) In any symmetric equilibrium the median voter is indifferent between a left- and a right-wing challenger, $U_0^R = U_0^L \equiv U_0$, so we can rewrite (7) as

$$U_0 = \int_0^w \frac{-y^2}{(1-\delta)} f_R(y)dy + \int_w^c \frac{-w^2}{(1-\delta)} f_R(y)dy + \int_c^1 \left[-y^2 + \delta U_0\right] f_R(y)dy. \quad (12)$$

[21] E^R and E^L can be interpreted as the discounted expected equilibrium policies, following the associated elections of right-wing and left-wing challengers. Note that equations (9) and (10) are similar to equations (7) and (8), were we substitute $-(i-y)^2$ by y.

In equilibrium, the median voter indifference condition **(C.1)** implies that

$$U_0 \;=\; \frac{-w^2}{(1-\delta)}. \tag{13}$$

Together (12) and (13) imply

$$-\frac{w^2}{(1-\delta)} = \int_0^w \frac{-i^2}{(1-\delta)} f_R(i)di + \int_w^c \frac{-w^2}{(1-\delta)} f_R(i)di + \int_c^1 \left[-i^2 - \delta\frac{w^2}{(1-\delta)}\right] f_R(i)di. \tag{14}$$

Step 2) We show that the expected discounted payoff to voter i of electing an untried challenger is related to the median voter's payoff according to

$$U_i^R \;=\; U_0 - \frac{i^2}{(1-\delta)} + 2iE^R, \tag{15}$$

$$U_i^L \;=\; U_0 - \frac{i^2}{(1-\delta)} + 2iE^L. \tag{16}$$

To see this, add $\frac{i^2}{(1-\delta)} - 2iE^R$ to both sides of (7). In the RHS of the new equation, substitute E^R by the RHS of (9),

$$\begin{aligned}
&\left(U_i^R + \frac{i^2}{(1-\delta)} - 2iE^R\right) \\
=\;& \int_0^w \frac{-(i-y)^2 + i^2 - 2iy}{(1-\delta)} f_R(y)dy + \int_w^c \frac{-(i-w)^2 + i^2 - 2iw}{(1-\delta)} f_R(y)dy \\
&+ \int_c^1 \left[-(i-y)^2 + i^2 - 2iy + \delta\left(U_i^L + \frac{i^2}{(1-\delta)} - 2iE^L\right)\right] f_R(y)dy.
\end{aligned}$$

Simplify to obtain

$$\begin{aligned}
\left(U_i^R + \frac{i^2}{(1-\delta)} - 2iE^R\right) \;=\;& \int_0^w \frac{-y^2}{(1-\delta)} f_R(y)dy + \int_w^c \frac{-w^2}{(1-\delta)} f_R(y)dy \\
&+ \int_c^1 \left[-y^2 + \delta\left(U_i^L + \frac{i^2}{(1-\delta)} - 2iE^L\right)\right] f_R(y)dy. \qquad (17)
\end{aligned}$$

Similarly, add $\frac{i^2}{(1-\delta)} - 2iE^L$ to both sides of (8) and use (10). Simplify to obtain

$$\begin{aligned}
\left(U_i^L + \frac{i^2}{(1-\delta)} - 2iE^L\right) \;=\;& \int_{-w}^0 \frac{-y^2}{(1-\delta)} f_L(y)dy + \int_{-c}^{-w} \frac{-w^2}{(1-\delta)} f_L(y)dy \\
&+ \int_{-1}^{-c} \left[-y^2 + \delta\left(U_i^R + \frac{i^2}{(1-\delta)} - 2iE^R\right)\right] f_L(y)dy. \qquad (18)
\end{aligned}$$

Using the symmetry between f_L and f_R, note that equations (12), (17) and (18) all have the exact same structure (as a function of the expression on the left-hand side). Together they imply that in equilibrium we must have

$$U_0 \;=\; U_i^R + \frac{i^2}{(1-\delta)} - 2iE^R \;=\; U_i^L + \frac{i^2}{(1-\delta)} - 2iE^L.$$

Therefore (15) and (16) must hold, concluding this step.

Step 3) From (13), (16) and symmetry $E^L = -E^R$, we have

$$U_i^L \;=\; U_0 - \frac{i^2}{(1-\delta)} + 2iE^L = -\frac{w^2}{(1-\delta)} - \frac{i^2}{(1-\delta)} - 2iE^R. \tag{19}$$

Using (19), compromise cutoff c is then defined by the indifference condition **(C.2)**,

$$\begin{aligned} -\frac{(c-w)^2}{(1-\delta)} &= 0 + \delta U_i^L \\ \iff -\frac{(c-w)^2}{(1-\delta)} &= \delta\left[-\frac{w^2}{(1-\delta)} - \frac{c^2}{(1-\delta)} - 2cE^R\right]. \end{aligned} \tag{20}$$

We solve quadratic equation (20) for the relevant $c > w > 0$ solution,

$$c \;=\; \frac{w}{(1-\delta)} + \delta E^R + \sqrt{\left(\frac{w}{(1-\delta)} + \delta E^R\right)^2 - w^2}. \tag{21}$$

Equilibrium is then defined by the unique cutoffs w^* and c^* and auxiliary value E^{R*} that solve the system of equations (11), (14) and (21).

A.2 Auxiliary Results

We now provide two useful results.

Remark 2: Fix any $\hat{k} \in (0,1)$. Take any distributions F_R and F'_R that satisfy (A.1) such that $F_R \succ_{\hat{k}} F'_R$. That is, $F_R \succ_{FOSD} F'_R$ and $f_R(i) = f'_R(i)$ for all $i \geq \hat{k}$. Define $\Delta f_R(i) \equiv f'_R(i) - f_R(i)$. Let $g(i)$ be any bounded, weakly increasing function in $i \in [0,\hat{k}]$, differentiable almost everywhere. Then $\int_0^{\hat{k}} g(i)\Delta f_R(i)\,di \leq 0$. The inequality is strict if there exists $i^* < \hat{k}$ such that $\frac{dg(i)}{di}\Big|_{i=i^*} > 0$ and $F'_R(i^*) > F_R(i^*)$.

Proof: Condition $f_R(i) = f'_R(i)$ for all $i \geq \hat{k}$ implies $\Delta f_R(i) = 0$ for all $i \geq \hat{k}$. Consequently, $\int_0^{\hat{k}} g(i)\Delta f_R(i)\,di = \int_0^1 g(i)\Delta f_R(i)\,di \leq 0$, where the inequality follows immediately from $F_R \succ_{FOSD} F'_R$ and the fact that g is weakly increasing. That is, $\int_0^1 g(i)\Delta f_R(i)\,di = g(i)[F'_R(i) - F_R(i)]|_0^1 - \int_0^1 \frac{dg(i)}{di}[F'_R(i) - F_R(i)]di = -\int_0^1 \frac{dg(i)}{di}[F'_R(i) - F_R(i)]di \leq 0$, where the inequality follows from $\frac{dg(i)}{di} \geq 0$ and $F'_R(i) \geq F_R(i)$. The inequality is then strict when there exists $i^* < \hat{k}$ such that $\frac{dg(i)}{di}\Big|_{i=i^*} > 0$ and $F'_R(i^*) > F_R(i^*)$. ■

Remark 3: Fix any pair of ideology distributions F_R and F'_R that satisfies (A.1), such that $F_R \succ_k F'_R$ for some $k \in (0,1)$. Using (A.1), define $\underline{f} > 0$ as $\underline{f} = \min_{i\in[0,1]}\{f_R(i), f'_R(i)\}$, and $\overline{f} < \infty$ as $\overline{f} = \max_{i\in[0,1]}\{f_R(i), f'_R(i)\}$. Let $\{w(\delta), c(\delta)\}$ and $\{w'(\delta), c'(\delta)\}$ be the corresponding equilibrium cutoffs when the discount factor is δ. Then there exists a cutoff $\delta^*(F_R, F'_R, k) < 1$ such that, for all $\delta > \delta^*(F_R, F'_R, k)$, the following inequalities hold:

$$k \; < \; \min\{c(\delta), c'(\delta)\}, \tag{22}$$

$$\max\{w(\delta), w'(\delta)\} \; < \; \min\{c(\delta), c'(\delta)\}, \tag{23}$$

$$\max\{w(\delta), w'(\delta)\} \; < \; \frac{\delta \underline{f}}{\left[4 + 2\delta\overline{f}\,\right]} \min\left\{\left[c^2(\delta) - w^2(\delta)\right], \left[c'^2(\delta) - w'^2(\delta)\right]\right\}. \tag{24}$$

Proof: Intuitively, the result follows from the fact that, if δ is sufficiently large, then compromise cutoffs c and c' are sufficiently close to one, while w and w' are sufficiently close to zero. A formal proof is given by Lemma B.3 in the on-line Appendix. ■

A.3 Proof of Proposition 1

Consider distributions F_R and F'_R as described by the proposition. Hence F'_R has a strictly more moderate distribution of ideologies in $[0, k)$, and the same distribution as F_R in $[k, 1]$. Using (A.1), define $\underline{f} > 0$ as $\underline{f} = \min_{i\in[0,1]}\{f_R(i), f'_R(i)\}$, and $\overline{f} < \infty$ as $\overline{f} = \max_{i\in[0,1]}\{f_R(i), f'_R(i)\}$. Let $\delta^*(F_R, F'_R, k)$ be the cutoff described by Remark 3 in Section A.2. Suppose agents are sufficiently patient with discount factor $\delta > \delta^*(F_R, F'_R)$, so that inequalities (22), (23) and (24) hold.

Let $\{w, c\}$ be the equilibrium cutoffs with fundamentals $\{\delta, F_R\}$, where we omit the cutoffs' implicit dependence on the fundamentals since we are holding fundamentals fixed. Let U_0 be the median voter's payoff defined by (12), and E^R as defined by (11). To simplify presentation, define

$$\Gamma \; = \; \sqrt{\left(\frac{w}{(1-\delta)} + \delta E^R\right)^2 - w^2} = \sqrt{\frac{w^2\delta(2-\delta)}{(1-\delta)^2} + \frac{2w\delta E^R}{(1-\delta)} + (\delta E^R)^2},$$

and rewrite (21) as $c = \frac{w}{(1-\delta)} + \delta E^R + \Gamma$. Since $w > 0$ and $E^R > 0$, it follows that $\Gamma > 0$. Moreover, $c < 1$ implies $\Gamma < 1$.

Let w', c', U'_0, E'^R, and Γ' be the corresponding values for the equilibrium with fundamentals $\{\delta, F'_R\}$. Define $\Delta f_R(i) \equiv f'_R(i) - f_R(i)$, $\Delta U \equiv U'_0 - U_0$, $\Delta w \equiv w' - w$, $\Delta\Gamma \equiv \Gamma' - \Gamma$, and $\Delta E^R \equiv E'^R - E^R$. Thus, we can write

$$\Delta c \equiv c' - c = \frac{\Delta w}{(1-\delta)} + \delta \Delta E^R + \Delta \Gamma. \tag{25}$$

We want to show that voters (weakly) prefer F_R, that is, $\Delta U \leq 0$. By contradiction, suppose that $\Delta U > 0$, which implies that $w' < w$, or simply $\Delta w < 0$. Define $\hat{k} = \max\{k, w\}$. Then $\Delta w < 0$ implies that $w' < w \leq \hat{k} < \min\{c, c'\}$ by (22) and (23). We have two cases.

Case 1) Suppose $w' < w \leq \hat{k} < c \leq c'$, i.e., $\Delta w < 0$, but $\Delta c \geq 0$. Equation (25) then implies that $\delta \Delta E^R + \Delta \Gamma > 0$. From the definition of Γ, for $\Delta \Gamma \geq 0$ to hold when $\Delta w < 0$, it must be that $\Delta E^R > 0$. We now derive a contradiction to $\Delta E^R > 0$. Since $k \leq \hat{k}$, we have $f'_R(i) = f_R(i)$ for $i \in [\hat{k}, 1]$. Using this fact and (11) we have

$$\begin{aligned}\Delta E^R = & \int_0^{w'} \frac{i}{(1-\delta)} \Delta f_R(i) di + \int_{w'}^{w} \left[\frac{w'}{(1-\delta)} f'_R(i) - \frac{i}{(1-\delta)} f_R(i) \right] di \\ & + \int_w^{\hat{k}} \left[\frac{w'}{(1-\delta)} f'_R(i) - \frac{w}{(1-\delta)} f_R(i) \right] di \\ & + \int_{\hat{k}}^{c} \left[\frac{w'}{(1-\delta)} - \frac{w}{(1-\delta)} \right] f'_R(i) di + \int_c^{c'} \left[\frac{w'}{(1-\delta)} - (i - \delta E^R) \right] f'_R(i) di \\ & + \int_{c'}^{1} -\delta \Delta E^R f'_R(i) di. \end{aligned} \tag{26}$$

Add and subtract $\int_{w'}^{w} \frac{i}{(1-\delta)} f'_R(i) di + \int_w^{\hat{k}} \frac{w}{(1-\delta)} f'_R(i) di$ from the RHS of (26) to obtain

$$\begin{aligned}\Delta E^R = & \int_0^{w} \frac{i}{(1-\delta)} \Delta f_R(i) di + \int_w^{\hat{k}} \frac{w}{(1-\delta)} \Delta f_R(i) di + \int_{w'}^{w} \left[\frac{w'}{(1-\delta)} - \frac{i}{(1-\delta)} \right] f'_R(i) di \\ & + \int_w^{c} \left[\frac{w'}{(1-\delta)} - \frac{w}{(1-\delta)} \right] f'_R(i) di + \int_c^{c'} \left[\frac{w'}{(1-\delta)} - (i - \delta E^R) \right] f'_R(i) di \\ & + \int_{c'}^{1} -\delta \Delta E^R f'_R(i) di. \end{aligned} \tag{27}$$

The LHS of (27) is strictly positive by assumption. Next we show that the RHS is strictly negative, which yields a contradiction. Note that

$$\int_0^{w} \frac{i}{(1-\delta)} \Delta f_R(i) di + \int_w^{\hat{k}} \frac{w}{(1-\delta)} \Delta f_R(i) di \leq 0, \tag{28}$$

where the inequality follows from Remark 2 in Section A.2.[22] The third and fourth integrals

[22] To see this, define the weakly increasing function g as $g(i) = i$ for $i = [0, w]$ and $g(i) = w$ for $i = [w, \hat{k}]$. The result then follows from Remark 2 since $F_R \succ_{FOSD} F'_R$ and $f_R(i) = f'_R(i)$ for $i \in [\hat{k}, 1]$.

are strictly negative since $w' < w$. Moreover, $w' < w$ and $c \leq c'$ imply

$$\int_c^{c'} \left[\frac{w'}{(1-\delta)} - (i - \delta E^R) \right] f'_R(i)di \;\; \leq \;\; \int_c^{c'} \left[\frac{w}{(1-\delta)} - c + \delta E^R \right] f'_R(i)di \;\leq\; 0,$$

where the last inequality follows from $\frac{w}{(1-\delta)} - c + \delta E^R = -\Gamma < 0$. Finally, the remaining integral on the RHS of (27) is negative, i.e., $-\delta \Delta E^R < 0$, by assumption. Thus, each of the terms on the RHS is negative, a contradiction.

Case 2) Now suppose suppose $w' < w \leq \hat{k} < c' < c$, which implies $\Delta w \leq 0$ and $\Delta c < 0$. Using (12) and the fact that $f'_R(i) = f_R(i)$ for $i \in [\hat{k}, 1]$, we have

$$\begin{aligned} \Delta U \;\; = \;\; & \int_0^{w'} \frac{-i^2}{(1-\delta)} \Delta f_R(i)di + \int_{w'}^{w} \left[\frac{-w'^2}{(1-\delta)} f'_R(i) + \frac{i^2}{(1-\delta)} f_R(i) \right] di \\ & + \int_w^{\hat{k}} \left[\frac{-w'^2}{(1-\delta)} f'_R(i) + \frac{w^2}{(1-\delta)} f_R(i) \right] di + \int_{\hat{k}}^{c'} \left[\frac{-w'^2}{(1-\delta)} + \frac{w^2}{(1-\delta)} \right] f'_R(i)di \\ & + \int_{c'}^{c} \left[-i^2 + \delta U'_0 + \frac{w^2}{(1-\delta)} \right] f'_R(i)di + \int_c^1 \delta \Delta U f'_R(i)di. \end{aligned}$$

Add and subtract $\int_{w'}^{w} \left[\frac{i^2 + w^2}{(1-\delta)} \right] f'_R(i)di + \int_w^{\hat{k}} \left[\frac{w^2}{(1-\delta)} \right] f'_R(i)di + \int_{c'}^{c} U'_0 f'_R(i)di$ from the RHS,

$$\begin{aligned} \Delta U \;\; = \;\; & \int_0^{w} \frac{-i^2}{(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{-w^2}{(1-\delta)} \Delta f_R(i)di \\ & + \int_{w'}^{w} \left[\frac{-w'^2 + w^2 - w^2 + i^2}{(1-\delta)} \right] f'_R(i)di + \int_w^{c'} \left[\frac{-w'^2 + w^2}{(1-\delta)} \right] f'_R(i)di \\ & + \int_{c'}^{c} \left[-i^2 - (1-\delta)U'_0 + U'_0 + \frac{w^2}{(1-\delta)} \right] f'_R(i)di + \int_c^1 \delta \Delta U f'_R(i)di. \end{aligned}$$

On the RHS, substitute $\Delta U = \frac{-w'^2 + w^2}{(1-\delta)}$ and bring all ΔU terms to the LHS,

$$\begin{aligned} \Delta U \left[1 - \int_{w'}^{c} f'_R(i)di - \delta \int_c^1 f'_R(i)di \right] \;\; = \;\; & \int_0^{w} \frac{-i^2}{(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{-w^2}{(1-\delta)} \Delta f_R(i)di \\ & + \int_{w'}^{w} \frac{(-w^2 + i^2)}{(1-\delta)} f'_R(i)di + \int_{c'}^{c} \left[-i^2 + w'^2 \right] f_R(i)di. \end{aligned} \quad (29)$$

The LHS of (29) is positive since $\Delta U > 0$. Since the RHS term $\int_{w'}^{w} \frac{(-w^2+i^2)}{(1-\delta)} f'_R(i)di$ is negative, for the RHS of (29) to be positive it must be the case that

$$\int_0^{w} \frac{-i^2}{(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{-w^2}{(1-\delta)} \Delta f_R(i)di \;\; > \;\; \int_{c'}^{c} \left[i^2 - w'^2 \right] f'_R(i)di,$$

which implies

$$\int_0^{w} \frac{-i^2}{(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{-w^2}{(1-\delta)} \Delta f_R(i)di \;\; > \;\; \int_{c'}^{c} \left[c'^2 - w'^2 \right] f'_R(i)di. \quad (30)$$

The LHS of (30) captures the direct (beneficial) impact on the median voter's payoff, when we change from F_R to the more moderate ideology distribution F'_R. The RHS captures the indirect impact on the median's payoff, caused by the (harmful) decrease in the compromise cutoff from c to c'. Next we derive a contradiction by showing that the payoff impact of the compromise cutoff change is larger. Loosely speaking, the RHS of (30) is larger when either the decrease to c' is large, the compromise set $(c' - w')$ is large, or when the p.d.f. $f'_R(i)$ in the range $[c, c']$ is large. These are usually associated with more patient agents, or more probability mass on politicians with extreme ideologies.

Rewrite (27) for the case $c' < c$,

$$\begin{aligned}\Delta E^R &= \int_0^w \frac{i}{(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{w}{(1-\delta)}\Delta f_R(i)di + \int_{w'}^{w}\left[\frac{w'}{(1-\delta)} - \frac{i}{(1-\delta)}\right] f'_R(i)di \\ &+ \int_w^{c'}\left[\frac{w'}{(1-\delta)} - \frac{w}{(1-\delta)}\right] f'_R(i)di + \int_{c'}^{c}\left[i - \delta E'^R - \frac{w}{(1-\delta)}\right] f'_R(i)di \\ &+ \int_c^1 -\delta\Delta E^R f'_R(i)di.\end{aligned}$$

Add and subtract $\int_{c'}^{c}\delta E^R f'_R(i)di$ from the RHS, and move the ΔE^R terms to the LHS,

$$\begin{aligned}\Delta E^R\left[1 + \delta\int_{c'}^1 f'_R(i)di\right] &= \int_0^w \frac{i}{(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{w}{(1-\delta)}\Delta f_R(i)di \\ &+ \int_{w'}^{w}\left[\frac{w'}{(1-\delta)} - \frac{i}{(1-\delta)}\right] f'_R(i)di + \int_w^{c'}\left[\frac{w'}{(1-\delta)} - \frac{w}{(1-\delta)}\right] f'_R(i)di \\ &+ \int_{c'}^{c}\left[i - \delta E^R - \frac{w}{(1-\delta)}\right] f'_R(i)di.\end{aligned}$$

Since the third and fourth integrals are negative, and $c' < c$, this implies

$$\begin{aligned}\Delta E^R\left[1 + \delta\int_{c'}^1 f'_R(i)di\right] &< \int_0^w \frac{i}{(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{w}{(1-\delta)}\Delta f_R(i)di \\ &+ \int_{c'}^{c}\left[c - \delta E^R - \frac{w}{(1-\delta)}\right] f'_R(i)di.\end{aligned}$$

Using the definition of Γ we have

$$\Delta E^R\left[1 + \delta\int_{c'}^1 f'_R(i)di\right] < \int_0^w \frac{i}{(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{w}{(1-\delta)}\Delta f_R(i)di + \int_{c'}^{c}\Gamma f'_R(i)di. \quad (31)$$

There are two sub-cases to consider.

Subcase 2a) Suppose $\Delta E^R \geq 0$. From (31) this implies that

$$\int_{c'}^{c}\Gamma f'_R(i)di > \int_0^w \frac{-i}{(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{-w}{(1-\delta)}\Delta f_R(i)di.$$

Multiply both sides of this inequality by the strictly positive number $(c'^2 - w'^2)$ to obtain

$$\int_{c'}^{c} (c'^2 - w'^2)\Gamma f'_R(i)di > \int_0^w \frac{-i(c'^2 - w'^2)}{(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{-w(c'^2 - w'^2)}{(1-\delta)} \Delta f_R(i)di. \quad (32)$$

Since $\Gamma < 1$, the LHS of (32) is strictly smaller than the RHS of (30). Therefore,

$$\begin{aligned} &\int_0^w \frac{-i^2}{(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{-w^2}{(1-\delta)} \Delta f_R(i)di \\ > &\int_0^w \frac{-i(c'^2 - w'^2)}{(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{-w(c'^2 - w'^2)}{(1-\delta)} \Delta f_R(i)di. \end{aligned}$$

Rewrite

$$\int_0^w \frac{i(c'^2 - w'^2) - i^2}{(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{w(c'^2 - w'^2) - w^2}{(1-\delta)} \Delta f_R(i)di \quad > \quad 0. \quad (33)$$

However, the LHS of (33) is weakly negative, a contradiction. To see this, proceed as in footnote 22 and define the function g as $g(i) = \frac{i(c'^2 - w'^2) - i^2}{(1-\delta)}$ for $i = [0, w]$ and $g(i) = \frac{w(c'^2 - w'^2) - w^2}{(1-\delta)}$ for $i = [w, \hat{k}]$. The derivative of g w.r.t. i is zero in the interval $[w, \hat{k}]$, and it is strictly positive in the interval $[0, w]$, since $\frac{(c'^2 - w'^2) - 2i}{(1-\delta)} \geq \frac{(c'^2 - w'^2) - 2w}{(1-\delta)} \geq 0$, where the last inequality follows since $\frac{1}{2}(c'^2 - w'^2) > \frac{\delta \underline{f}}{[4 + 2\delta \overline{f}]}(c'^2 - w'^2) > w$ from (24). Consequently, g is weakly increasing in the interval $[0, \hat{k}]$. The result then follows from Remark 2 since $F_R \succ_{FOSD} F'_R$ and $f_R(i) = f'_R(i)$ for $i \in [\hat{k}, 1]$.

Subcase 2b) Suppose $\Delta E^R < 0$. Because $1 + \delta \int_{c'}^1 f'_R(i) < 2$, we can rewrite (31) as

$$2\Delta E^R \quad < \quad \int_0^w \frac{i}{(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{w}{(1-\delta)} \Delta f_R(i)di + \int_{c'}^{c} \Gamma f'_R(i)di. \quad (34)$$

Because $\Gamma < 1$, $f'_R(i) \leq \overline{f}$, and $c' < c$, we have $\int_{c'}^{c} \Gamma f'_R(i)di \leq \overline{f}[c - c'] = -\overline{f}\Delta c$. Multiply both sides by $\frac{\delta}{2}$ to obtain

$$\delta \Delta E^R \quad < \quad \int_0^w \frac{\delta i}{2(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{\delta w}{2(1-\delta)} \Delta f_R(i)di - \frac{\delta \overline{f} \Delta c}{2}. \quad (35)$$

Since $\Delta E^R < 0$ and $\Delta w < 0$, it must be that $\Delta\Gamma < 0$. Since $\Delta w < 0$ and $\Delta\Gamma < 0$, it must be that $\Delta c < \delta \Delta E^R$ (see (25)). Together $\Delta c < \delta \Delta E^R$ and (35) imply

$$\Delta c \quad < \quad \int_0^w \frac{\delta i}{2(1-\delta)} \Delta f_R(i)di + \int_w^{\hat{k}} \frac{\delta w}{2(1-\delta)} \Delta f_R(i)di - \frac{\delta \overline{f} \Delta c}{2}. \quad (36)$$

Bring the Δc terms to the LHS and divide both sides by $\left[1+\frac{\delta \bar{f}}{2}\right]$ to obtain

$$\Delta c \quad < \quad \frac{1}{\left[1+\frac{\delta \bar{f}}{2}\right]}\left\{\int_0^w \frac{\delta i}{2(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{\delta w}{2(1-\delta)}\Delta f_R(i)di\right\}. \tag{37}$$

Since $f_R'(i) \geq \underline{f}$ and $c' < c$, we have $\int_{c'}^{c} f_R(i) \geq \underline{f}[c-c'] = -\underline{f}\ \Delta c$. Rewrite (30) as

$$\int_0^w \frac{-i^2}{(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{-w^2}{(1-\delta)}\Delta f_R(i)di \quad > \quad -\underline{f}\ \Delta c\left[c'^2 - w'^2\right]. \tag{38}$$

Rewrite (38) as

$$\Delta c \quad > \quad \frac{1}{\underline{f}\left[c'^2 - w'^2\right]}\left\{\int_0^w \frac{i^2}{(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{w^2}{(1-\delta)}\Delta f_R(i)di\right\}. \tag{39}$$

Together (37) and (39) imply

$$\begin{aligned} &\frac{1}{\left[1+\frac{\delta \bar{f}}{2}\right]}\left\{\int_0^w \frac{\delta i}{2(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{\delta w}{2(1-\delta)}\Delta f_R(i)di\right\} \\ > \quad &\frac{1}{\underline{f}\left[c'^2 - w'^2\right]}\left\{\int_0^w \frac{i^2}{(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{w^2}{(1-\delta)}\Delta f_R(i)di\right\}. \end{aligned} \tag{40}$$

Define $b = \frac{\delta \underline{f}}{\left[4+2\delta \bar{f}\right]}$ and rewrite (40) as

$$\int_0^w \frac{\left[2bi(c'^2 - w'^2) - i^2\right]}{(1-\delta)}\Delta f_R(i)di + \int_w^{\hat{k}} \frac{\left[2bw(c'^2 - w'^2) - w^2\right]}{(1-\delta)}\Delta f_R(i)di \quad > \quad 0. \tag{41}$$

However, the LHS of (41) is weakly negative, a contradiction. To see this, proceed as in Subcase 2a and define the function g as $g(i) = \frac{2bi(c'^2-w'^2)-i^2}{(1-\delta)}$ for $i = [0, w]$ and $g(i) = \frac{2bw(c'^2-w'^2)-w^2}{(1-\delta)}$ for $i = [w, \hat{k}]$. The derivative of g w.r.t. i is zero in the interval $[w, \hat{k}]$, and it is strictly positive in the interval $[0, w]$, since $2b(c'^2 - w'^2) - 2i \geq 2b(c'^2 - w'^2) - 2w \geq 0$, where the last inequality follows from $b = \frac{\delta \underline{f}}{\left[4+2\delta \bar{f}\right]}$ and (24). Consequently, g is weakly increasing in the interval $[0, \hat{k}]$. The result then follows from Remark 2 since $F_R \succ_{FOSD} F_R'$ and $f_R(i) = f_R'(i)$ for $i \in [\hat{k}, 1]$. This concludes the first part of the Proposition.

To prove the strict welfare result in the second part of the Proposition, we repeat the above steps and use the strict inequality result from Remark 2. Suppose that $f_R(i) \neq f_R'(i)$ for some $i < w$. This assumption together with continuity of the densities and the FOSD ordering imply that there exists $i^* \in [0, w]$ such that $F_R'(i^*) > F_R(i^*)$. We want to show that $\Delta U < 0$. By contradiction, suppose $\Delta U \geq 0$, so that $w' \leq w$. Repeat the steps from Case 1 to arrive at (27) . Now the LHS of (27) is weakly positive, $\Delta E^R \geq 0$. To arrive at a contradiction, we prove that the RHS of (27) is strictly negative. This follows because inequality (28) is now strict. This is so because function g defined in footnote 22 is strictly increasing at i^* (since $i^* < w$) and $F_R'(i^*) > F_R(i^*)$, so we can use the strict inequality result from Remark 2. Similarly repeat the steps in Case 2, and arrive at equations (33) and (41), which now feature weak inequalities. However, the contradiction is that the LHS of (33) and (41) are now strictly negative. This follows since in each case the constructed function g strictly increases at i^* and $F_R'(i^*) > F_R(i^*)$, so we can apply Remark 2. ■

A.4 Proof of Proposition 2

Fix any discount $\delta \in (0,1)$ and any ideology distribution F_R' that satisfies (A.1). Let $\{w', c'\}$ be the equilibrium cutoffs given $\{\delta, F_R'\}$. Define $\underline{f} = \min_{i\in[0,1]} f_R'(i)$, $\overline{f} = \max_{i\in[0,1]} f_R'(i)$, and

$$k^* \;=\; \min\left\{w', \frac{\delta \underline{f}\,[c'^2 - w'^2]}{\left[4 + 2\delta\overline{f}\,\right]}, \sqrt{[c'^2 - w'^2]\,[c' - w']\,\underline{f}}\right\}. \tag{42}$$

Recall that for any F_R' and $\delta > 0$, it must be that $0 < w' < c' < 1$, and by (A.1) we have $\underline{f} > 0$. Consequently, k^* is strictly positive.

Take any distribution F_R that satisfies (A.1), such that $F_R \succ_k F_R'$ for some $k \in (0, k^*]$. The proof is very similar to that of Proposition 1. Following the steps in the proof of Proposition 1, we want to show that $U_0' < U_0$, i.e., $w' > w$.

By contradiction suppose $w' \leq w$. By the definition of k and k^* this implies $k \leq k^* \leq w' \leq w$. There are three cases to consider.

Case 1) Suppose $w' \leq w < c \leq c'$;

Case 2) Suppose $w' \leq w \leq c' < c$;

Cases 1 and 2 follow from the same steps in the proof of Proposition 1. Since $k \leq k^*$, $f_R(i) = f_R'(i)$ for all $i \in [k^*, 1]$. Moreover, $k^* \leq w' \leq w$ implies that inequality (22), used in the proof of Proposition 1, continues to hold. Definition (42) states $k^* \leq \frac{\delta \underline{f}\left[c'^2 - w'^2\right]}{\left[4 + 2\delta\overline{f}\,\right]}$, which substitutes inequality (24) used in the proof of Proposition 1.

Case 3) Suppose $w' < c' < w < c$. Only Case 3 was not covered in Proposition 1, since inequality (23) precludes this case for patient voters. To prove this remaining case, we use the fact that $k^* \leq \sqrt{[c'^2 - w'^2]\,[c' - w']\,\underline{f}}$ by definition (42).

Since $f_R(i) = f'_R(i)$ for all $i \in [k^*, 1]$, and $k^* \leq w'$, rewriting (29) for this case yields

$$\Delta U \left[1 - \int_{w'}^{c} f'_R(i)di - \delta \int_{c}^{1} f'_R(i)di\right] = \int_0^{k^*} \frac{-i^2}{(1-\delta)} \Delta f_R(i)di + \int_{w'}^{c'} \frac{(-w^2 + i^2)}{(1-\delta)} f'_R(i)di. \quad (43)$$

Assume by contradiction that $\Delta U \geq 0$. Then, the RHS must also be positive, and hence,

$$\int_0^{k^*} \frac{-i^2}{(1-\delta)} \Delta f_R(i)di \;\; \geq \;\; \int_{w'}^{c'} \frac{(w^2 - i^2)}{(1-\delta)} f'_R(i)di. \quad (44)$$

Note that $\int_0^{k^*} -i^2 \Delta f_R(i)di = \int_0^{k^*} \left[-i^2 f'_R(i) + i^2 f_R(i)\right] di < \int_0^{k^*} i^2 f_R(i)di < \int_0^{k^*} (k^*)^2 f_R(i)di < \int_0^{1} (k^*)^2 f_R(i)di = (k^*)^2$. Moreover, $c' < w$ implies $\int_{w'}^{c'} (w^2 - i^2) f'_R(i)di > \int_{w'}^{c'} (c'^2 - i^2) f'_R(i)di > \int_{w'}^{c'} (c'^2 - w'^2) f'_R(i)di = (c'^2 - w'^2) \int_{w'}^{c'} f'_R(i)di \geq (c'^2 - w'^2)(c' - w')\underline{f}$, where the last inequality follows from $f'_R \geq \underline{f}$. Therefore, (44) implies

$$(k^*)^2 \;\; > \;\; (c'^2 - w'^2)(c' - w')\underline{f}, \quad (45)$$

which contradicts $k^* \leq \sqrt{[c'^2 - w'^2]\,[c' - w']\,\underline{f}}$, concluding the proof.

A corollary of Proposition 2 is that there always exists a more extreme distribution of ideologies that benefits voters. That is, since $k^* > 0$ and $\underline{f} > 0$, there always exists a cumulative distribution function F_R that satisfies (A.1), such that $F_R \succ_{k^*} F'_R$. [23] ∎

A.5 Proof of Lemma 1

Suppose assumptions (A.1)-(A.2) hold. Because utilities are quadratic, in any symmetric, stationary perfect Bayesian equilibrium with simple strategies it must be the case that the median voter is decisive. Hence, re-election cutoffs are determined by his indifference conditions. Moreover, assumption (A.2) implies that for any $\beta > 0$, the median voter's expected

[23] For example, take any f'_R satisfying (A.1) such that $f'_R > \underline{f} > 0$ for some $\underline{f}$. Compute $k^* > 0$ and define the auxiliary function z as follows: $z(i) = -\left(\frac{k^*}{4}\right)^2 + \left(i - \frac{k^*}{4}\right)^2$ for $i \in [0, \frac{2k^*}{4}]$, and $z(i) = \left(\frac{k^*}{4}\right)^2 - \left(i - \frac{3k^*}{4}\right)^2$ for $i \in [\frac{2k^*}{4}, \frac{4k^*}{4}]$. Define the new density f_R by: $f_R(i) = f'_R(i)$ for $i \in [k^*, 1]$, and $f_R(i) = f'_R(i) + \underline{f}z(i)$ for $i \in [0, k^*)$. One can verify that f_R is an absolutely continuos density function and $f_R \geq \underline{f}\left(\frac{k^*}{4}\right)^2 > 0$. Therefore, it satisfies (A.1). Moreover $F_R \succ_{FOSD} F'_R$, as $z(i)$ shifts probability mass from $[0, \frac{k^*}{2}]$ to $[\frac{k^*}{2}, k^*]$.

payoff from electing an untried challenger following a good signal strictly exceeds his expected payoff after observing a bad signal. Therefore, if w_G and w_B are the re-election standards after observing good and bad signals, then $0 < w_G < w_B < 1$.

The median voter's expected payoff from electing a challenger after a good signal is U_0^G. The expected payoff from re-electing an incumbent who adopts policy w_G and is always re-elected is $-\frac{w_G^2}{(1-\delta)}$. Thus, the median voter's indifference condition can be written as

$$U_0^G = -\frac{w_G^2}{(1-\delta)}. \tag{46}$$

The median voter's expected payoff from electing a challenger after a bad signal is U_0^B. When the incumbent implements policy w_B, she is re-elected when the signal about the challenger is bad, which happens with probability $(1-\rho)$; the incumbent then implements w_B again next period. She loses re-election when the signal is good (probability ρ), in which case she is replaced by the challenger, who delivers expected payoff U_0^G to the median voter. Hence, the median voter's indifference condition implies

$$U_0^B = -w_B^2 + \delta\left[\rho U_0^G + (1-\rho)U_0^B\right].$$

Solve this median voter indifference condition for

$$U_0^B = -\frac{w_B^2}{[1-\delta(1-\rho)]} + \frac{\delta\rho U_0^G}{[1-\delta(1-\rho)]}. \tag{47}$$

Subtract (47) from (46) and then solve for

$$U_0^G - U_0^B = \frac{w_B^2 - w_G^2}{[1-\delta(1-\rho)]}. \tag{48}$$

In our stationary equilibrium, an incumbent's policy choice is only a function of her ideology, and not past signal realizations. Therefore, the only difference between U_0^G and U_0^B in the LHS is the probability distribution over ideologies. Given our assumptions about the signal, this difference goes to zero as β goes to zero. Therefore, if the signal is sufficiently noisy, the difference in re-election standards $(w_B^2 - w_G^2)$ must also be small.

For politician $i \geq w_B$, the direct period payoff cost of implementing policy w_G instead of w_B is $-(i-w_B)^2+(i-w_G)^2 = 2i(w_B-w_G)+w_G^2-w_B^2$. Because $i \leq 1$ and $w_B > w_G$, this cost is strictly less than $2(w_B - w_G)$. Consequently, if $w_B - w_G$ is close to zero, then this cost is even closer to zero. The direct benefit from choosing w_G instead of w_B is the increase in the probability of re-election from $(1-\rho)$ to one, that is, a probability increase of $\rho > 0$. Hence, if w_B is close enough to w_G, then any incumbent with ideology $i \in [w_B, 1]$ faces a small marginal cost of changing her policy choice from w_B to w_G, but a discrete positive benefit from guaranteeing re-election and avoiding replacement by a challenger. Thus, when the difference $(w_B - w_G)$ is small, the option to compromise to w_G strictly dominates the option to compromise to w_B, so that the relevant decision for incumbents $i \in [w_B, 1]$ is whether to compromise to w_G or to choose as policy their own ideology, which concludes the proof. ■

A.6 A stricter re-election cutoff

Before proving Proposition 3, we ask: would voters benefit if they could commit to a more demanding re-election cutoff w?

Lemma A.1 *Suppose F_R satisfies (A.1) and let δ^{**} be such that* (5) *holds for all $\delta \geq \delta^{**}$. Fix any $\delta \in (\delta^{**}, 1)$. Let w^* be the equilibrium re-election standard given fundamentals $\{\delta, F_R\}$, and let $w \leq w^*$ be any exogenous re-election standard. Voters' expected payoff strictly increases in $w \in [0, w^*]$. Thus, any more demanding re-election cutoff $w < w^*$ strictly reduces the expected payoff of all voters.*

Proof: Suppose F_R satisfies (A.1) and $\delta \in (\delta^{**}, 1)$. Let w^* be the unique equilibrium re-election standard and c^* the unique equilibrium compromise cutoff.

We now show that voter welfare decreases if we impose an exogenous[24] reelection standard $w < w^*$. Given w, the incumbent is reelected if she adopts a policy in the set $[-w, w]$, and loses to the challenger otherwise. As before, centrist politicians $[-w, w]$ choose to implement their preferred policies and are reelected. Politicians with non-centrist ideologies must choose whether to compromise to be reelected or not. To compute the new optimal strategy of politicians, we must simultaneously solve for a new compromise cutoff $\tilde{c}$ and new expected payoffs from electing challengers. In particular, a new median voter payoff $\tilde{U}_0$. That is, although the ideology of a challenger is drawn from the same distribution F_R (or F_L) as before, the endogenous behavior of politicians is affected by the stricter reelection cutoff w.

[24]This exogenous reelection cutoff w will be the endogenous reelection cutoff w_G in Proposition 3.

In Section A.1 we use the system of equations (11), (12), (13) and (21) to solve for U_0^*, w^*, c^* and E^{R*}. However, given an exogenous reelection cutoff w, indifference condition (13) no longer holds. Therefore, we rewrite the remaining equations as functions of $w \in [0, w^*]$,

$$\tilde{U}_0(w) = \int_0^w \frac{-i^2}{(1-\delta)} f_R(i)di + \int_w^{\tilde{c}(w)} \frac{-w^2}{(1-\delta)} f_R(i)di + \int_{\tilde{c}(w)}^1 \left[-i^2 + \delta\tilde{U}_0(w)\right] f_R(i)di, \quad (49)$$

$$\tilde{E}^R(w) = \int_0^w \frac{i}{(1-\delta)} f_R(i)di + \int_w^{\tilde{c}(w)} \frac{w}{(1-\delta)} f_R(i)di + \int_{\tilde{c}(w)}^1 \left[i - \delta\tilde{E}^R(w)\right] f_R(i)di, \quad (50)$$

$$\tilde{c}(w) = \frac{w}{(1-\delta)} + \delta\tilde{E}^R(w) + \sqrt{\left(\frac{w}{(1-\delta)} + \delta\tilde{E}^R(w)\right)^2 - \frac{w^2}{(1-\delta)} - \delta\tilde{U}_0(w)}. \quad (51)$$

Together conditions (49) to (51) characterize the equilibrium for an exogenous w. Define

$$\tilde{\Gamma}(w) \equiv \sqrt{\left(\frac{w}{(1-\delta)} + \delta\tilde{E}^R(w)\right)^2 - \frac{w^2}{(1-\delta)} - \delta\tilde{U}_0(w)},$$

so that we can rewrite (51) as $\tilde{c}(w) = \frac{w}{(1-\delta)} + \delta\tilde{E}^R(w) + \tilde{\Gamma}(w)$.

We want to show that $\frac{\partial\tilde{U}_0(w)}{\partial w} > 0$ for all $w \in [0, w^*]$. From (49), the derivative of $\tilde{U}_0(w)$ is

$$\frac{\partial\tilde{U}_0(w)}{\partial w}\left[1 - \delta\int_c^1 f_R(i)di\right] = \frac{\partial\tilde{c}(w)}{\partial w}\left[\tilde{c}(w)^2 - \frac{w^2}{(1-\delta)} - \delta\tilde{U}_0(w)\right] f_R(\tilde{c}(w)) - \frac{2w}{(1-\delta)}\left[F_R(\tilde{c}(w)) - F_R(w)\right]. \quad (52)$$

By contradiction, suppose that $\frac{\partial\tilde{U}_0(w)}{\partial w} \leq 0$ for some $w \leq w^*$, which implies that

$$\frac{2w}{(1-\delta)}\left[F_R(\tilde{c}(w)) - F_R(w)\right] \geq \frac{\partial\tilde{c}(w)}{\partial w}\left[\tilde{c}(w)^2 - \frac{w^2}{(1-\delta)} - \delta\tilde{U}_0(w)\right] f_R(\tilde{c}(w)). \quad (53)$$

Step 1) We first provide some basic inequalities. When the exogenous cutoff is $w = 0$ we have $\tilde{U}_0(0) < \frac{-0^2}{1-\delta}$. When the exogenous cutoff is $w = w^*$ we have $\tilde{U}_0(w^*) = \frac{-w^{*2}}{1-\delta}$, since (13) holds at w^*. From Theorem A2 in Bernhardt et al. (2009), there exists a unique equilibrium cutoff w^*, therefore it must be the case that $\tilde{U}_0(w) < \frac{-w^2}{1-\delta}$ for all $w < w^*$ (otherwise all original equilibrium conditions (11), (12), (13) and (21) would be satisfied for some $w \neq w^*$, a contradiction to uniqueness). Consequently, $-\delta\tilde{U}_0(w) \geq \frac{\delta w^2}{(1-\delta)}$ for all $w \leq w^*$.

For every $w \in [0, w^*]$ we have $\tilde{c}(w) \leq c^* < 1$ (if $\tilde{c}(w) > c^*$ and $w \leq w^*$, then the previously indifferent politician c^* now strictly prefers to adopt her preferred policy and lose, a contradiction to $\tilde{c}(w) > c^*$). This implies $\tilde{E}^R(w) > 0$. Therefore, $\tilde{\Gamma}(w) > \sqrt{\frac{w^2}{(1-\delta)^2} - \frac{w^2}{(1-\delta)} - \delta\tilde{U}_0(w)} \geq$

$\frac{w\sqrt{\delta(2-\delta)}}{(1-\delta)}$, where the last inequality follows from $-\delta\tilde{U}_0(w) \geq \frac{\delta w^2}{(1-\delta)}$. This implies $\tilde{\Gamma}(w) > 0$. Moreover, $\tilde{E}^R(w) > 0$ and $\tilde{\Gamma}(w) > \frac{w\sqrt{\delta(2-\delta)}}{(1-\delta)}$ imply

$$\tilde{c}(w) \quad > \quad \frac{w}{1-\delta} + \frac{w\sqrt{\delta(2-\delta)}}{(1-\delta)} = w\left(\frac{1+\sqrt{\delta(2-\delta)}}{(1-\delta)}\right). \tag{54}$$

Step 2) We now show that $\frac{\partial \tilde{E}^R(w)}{\partial w} > 0$. Using (50), the derivative of $\tilde{E}^R(w)$ is

$$\begin{aligned}\frac{\partial \tilde{E}^R(w)}{\partial w} \quad = \quad & \frac{w}{(1-\delta)} f_R(w) + \frac{\partial \tilde{c}(w)}{\partial w}\frac{w}{(1-\delta)} f_R(\tilde{c}(w)) - \frac{w}{(1-\delta)} f_R(w) + \int_w^{\tilde{c}(w)} \frac{1}{(1-\delta)} f_R(i)di \\ & -\frac{\partial \tilde{c}(w)}{\partial w}\left[\tilde{c}(w) - \delta\tilde{E}^R(w)\right] f_R(\tilde{c}(w)) - \int_{\tilde{c}(w)}^1 \delta\frac{\partial \tilde{E}^R(w)}{\partial w} f_R(i)di.\end{aligned}$$

Simplify

$$\begin{aligned}\frac{\partial \tilde{E}^R(w)}{\partial w}\left[1+\delta(1-F_R(\tilde{c}(w)))\right] \quad = \quad & \frac{\partial \tilde{c}(w)}{\partial w}\left\{\frac{w}{(1-\delta)} - \tilde{c}(w) + \delta\tilde{E}^R(w)\right\} f(\tilde{c}(w)) \\ & + \frac{F_R(\tilde{c}(w)) - F_R(w)}{(1-\delta)}.\end{aligned} \tag{55}$$

Note that $\frac{w}{(1-\delta)} - \tilde{c}(w) + \delta\tilde{E}^R(w) = -\Gamma(w) < 0$. There are two cases to consider. First, if $\frac{\partial \tilde{c}(w)}{\partial w} \leq 0$, then $\frac{\partial \tilde{E}^R(w)}{\partial w} > 0$, proving Step 2. Now consider the case $\frac{\partial \tilde{c}(w)}{\partial w} > 0$. Because $-\delta\tilde{U}_0(w) \geq \frac{\delta w^2}{(1-\delta)}$, inequality (53) then implies

$$\frac{2w}{(1-\delta)}\left[F_R(\tilde{c}(w)) - F_R(w)\right] \quad \geq \quad \frac{\partial \tilde{c}(w)}{\partial w} f_R(\tilde{c}(w))\left[\tilde{c}(w)^2 - w^2\right].$$

Rearrange this inequality to bound $\frac{\partial \tilde{c}(w)}{\partial w}$:

$$\frac{2w}{(1-\delta)}\frac{\left[F_R(\tilde{c}(w)) - F_R(w)\right]}{f_R(\tilde{c}(w))\left[\tilde{c}(w)^2 - w^2\right]} \quad \geq \quad \frac{\partial \tilde{c}(w)}{\partial w}. \tag{56}$$

Because the term multiplying $\frac{\partial \tilde{c}(w)}{\partial w}$ in (55) is strictly negative, substitute the upper-bound from (56) to rewrite

$$\begin{aligned}\frac{\partial \tilde{E}^R(w)}{\partial w}\left[1+\delta(1-F_R(\tilde{c}(w)))\right] \quad \geq \quad & \frac{2w}{(1-\delta)}\frac{\left[F_R(\tilde{c}(w)) - F_R(w)\right]}{\left[\tilde{c}(w)^2 - w^2\right]}\left\{\frac{w}{(1-\delta)} - \tilde{c}(w) + \delta\tilde{E}^R(w)\right\} \\ & + \frac{F_R(\tilde{c}(w)) - F_R(w)}{(1-\delta)} \\ = \quad & \frac{\left[F_R(\tilde{c}(w)) - F_R(w)\right]}{(1-\delta)(\tilde{c}(w)^2 - w^2)}\left\{\frac{2w^2}{(1-\delta)} - 2\tilde{c}(w)w\right. \\ & \left. + 2\delta w\tilde{E}^R(w) + \tilde{c}(w)^2 - w^2\right\} \\ = \quad & \frac{\left[F_R(\tilde{c}(w)) - F_R(w)\right]}{(1-\delta)(\tilde{c}(w)^2 - w^2)}\left\{\frac{2\delta w^2}{(1-\delta)} + 2\delta w\tilde{E}^R(w) + \left[\tilde{c}(w) - w\right]^2\right\}.\end{aligned}$$

Each of the terms in the brackets is strictly positive. Therefore, $\frac{\partial E^R(w)}{\partial w} > 0$, proving Step 2.

Step 3) The derivative of $\tilde{c}(w)$ is

$$\frac{\partial \tilde{c}(w)}{\partial w} = \frac{1}{(1-\delta)} + \delta \frac{\partial \tilde{E}^R(w)}{\partial w} + \frac{\partial \tilde{\Gamma}(w)}{\partial w}.$$

We know from Step 2 that $\frac{\partial \tilde{E}^R(w)}{\partial w} > 0$. Moreover, $\tilde{\Gamma}(w) > 0$ and

$$\frac{\partial \tilde{\Gamma}(w)}{\partial w} = \frac{1}{2\tilde{\Gamma}(w)} \left\{ 2 \left(\frac{w}{(1-\delta)} + \delta \tilde{E}^R(w) \right) \left(\frac{1}{(1-\delta)} + \delta \frac{\partial \tilde{E}^R(w)}{\partial w} \right) - \frac{2w}{(1-\delta)} - \delta \frac{\partial \tilde{U}(w)}{\partial w} \right\} > 0,$$

where the inequality follows from $\tilde{E}^R(w) > 0$, the assumption $\frac{\partial \tilde{U}(w)}{\partial w} \leq 0$, and the fact that the term $\frac{-2w}{(1-\delta)}$. Consequently, $\frac{\partial \tilde{c}(w)}{\partial w} > \frac{1}{(1-\delta)}$.

Step 4) Substitute $\frac{\partial \tilde{c}(w)}{\partial w} > \frac{1}{(1-\delta)}$ into inequality (56)

$$\begin{aligned} \frac{2w}{(1-\delta)} \frac{[F_R(\tilde{c}(w)) - F_R(w)]}{f_R(\tilde{c}(w)) \left[\tilde{c}(w)^2 - w^2\right]} &> \frac{1}{(1-\delta)} \\ \Rightarrow \frac{[F_R(\tilde{c}(w)) - F_R(w)]}{f_R(\tilde{c}(w)) \left[\tilde{c}(w) - w\right]} &> \frac{\tilde{c}(w) + w}{2w}. \end{aligned} \tag{57}$$

Using (54), substitute $\tilde{c}(w)$ by the smaller number $w \left(\frac{1+\sqrt{\delta(2-\delta)}}{(1-\delta)} \right)$, so inequality (57) implies

$$\frac{[F_R(\tilde{c}(w)) - F_R(w)]}{f_R(\tilde{c}(w)) \left[\tilde{c}(w) - w\right]} > \frac{2 - \delta + \sqrt{\delta(2-\delta)}}{2(1-\delta)}. \tag{58}$$

The term $\frac{[F_R(\tilde{c}(w)) - F_R(w)]}{[\tilde{c}(w) - w]}$ equals the *average* $f_R(i)$ in the range $i \in [w, \tilde{c}(w)]$, hence the term is less than the maximum $f_R(i)$ in this range. Consequently, the LHS of (58) is less than $\max_{i \in [0, \tilde{c}(w)]} \left(\frac{f_R(i)}{f_R(\tilde{c}(w))} \right)$. This implies

$$\max_{i \in [0, \tilde{c}(w)]} \left(\frac{f_R(i)}{f_R(\tilde{c}(w))} \right) > \frac{2 - \delta + \sqrt{\delta(2-\delta)}}{2(1-\delta)}. \tag{59}$$

Inequality (59) violates assumption $\delta \in (\delta^{**}, 1)$, a contradiction—see equation (5). ■

A.7 Proof of Proposition 3

Suppose F_R satisfies (A.1), π satisfies (A.2), and $\delta \in (\delta^{**}, 1)$. Suppose the signal Π_β about the challenger is sufficiently noisy (β is sufficiently small), so that no incumbent compromises to w_B (Lemma 1). Let w^* and U_0^* be the equilibrium re-election standard and median voter's expected payoff from electing an untried challenger, in the economy without signals. Let w_G and $\hat{U}_0 = \rho U_0^G + (1-\rho)U_0^B$ be the equilibrium re-election standard (following a good signal) and median voter's expected payoff from electing an untried challenger, in the economy with signals. The assumptions on the signal imply that for any $\beta > 0$ we have $U_0^G > \hat{U}_0 > U_0^B$. First suppose that the re-election standard is less strict, $w_G \geq w^*$. This implies $-\frac{w^{*2}}{(1-\delta)} \geq -\frac{w_G^2}{(1-\delta)}$, and from equilibrium conditions $U_0^* \geq U_0^G$, that is, $U_0^* > \hat{U}_0$ and all voters strictly prefer the economy without signals. Now suppose that the re-election standard is stricter, $w_G < w^*$. The result then follows from Lemma A.1, concluding the proof. Note that if β is sufficiently small, then it must be the case that $w_G < w^*$. Moreover, Lemma A.1 also implies that voter welfare continues to decrease following any increase in the signals informativeness (β) that decreases w_G without leading incumbents to compromise to w_B. ■

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