B On-Line Appendix

This on-line appendix presents additional results to "Learning about Challengers" (Câmara and Bernhardt).

B.1 Ideology Moderation when Players are Impatient

Remark B1: Fix any distributions F_R and F'_R that satisfy (A.1) such that $F_R \succ_{FOSD} F'_R$. Define $\Delta f_R(i) \equiv f'_R(i) - f_R(i)$. Let $g(i)$ be any bounded, weakly decreasing function that is differentiable almost everywhere. Then $\int_0^1 g(i)\Delta f_R(i)\,di \geq 0$.

Proof: It follows from the same argument as Remark 2 in Section A.2. ■

Lemma B.1 *Consider party selection of challengers. Suppose the distribution of politicians' ideologies F_R first-order stochastically dominates F'_R, $F_R \succ_{FOSD} F'_R$, and assumption (A.1) holds for both distributions. If agents are sufficiently impatient, then all voters weakly prefer the more moderate distribution of ideologies F'_R.*

Proof: Assume (A.1) holds for distributions F_R and F'_R, where $F_R \succ_{FOSD} F'_R$ so that F'_R has more moderate ideologies. Symmetrically define F_L and F'_L. Let $\{c, w\}$ and $\{c', w'\}$ be the unique equilibrium cutoffs for distributions F and F', respectively, and let U_0 and U'_0 be the corresponding equilibrium expected payoffs of the median voter. Let $\Delta U \equiv U'_0 - U_0$, and $\Delta f_R(i) \equiv f'_R(i) - f_R(i)$. We want to show that $\Delta U \geq 0$. By contradiction, suppose that $\Delta U < 0$, which implies that $w < w'$. There are two cases to consider.

Case 1) Suppose $w < w'$ and $c \leq c'$. Use (12) and (13) to compute ΔU,

$$\begin{aligned}
\Delta U &= \int_0^w \frac{-i^2}{(1-\delta)}\Delta f_R(i)di + \int_w^{w'} \left[\frac{-i^2}{(1-\delta)} f'_R(i) - U_0 f_R(i)\right] di \\
&+ \int_{w'}^{c} \left[U'_0 f'_R(i) - U_0 f_R(i)\right] di + \int_c^{c'} \left[U'_0 f'_R(i) - (-i^2 + \delta U_0) f_R(i)\right] di \\
&+ \int_{c'}^{1} \left[(-i^2 + \delta U'_0) f'_R(i) - (-i^2 + \delta U_0) f_R(i)\right] di.
\end{aligned}$$

Substitute $\int_w^{w'} \frac{-i^2}{(1-\delta)} f_R'(i)di$ by the smaller number $\int_w^{w'} U_0' f_R'(i)di$, and substitute $\int_c^{c'} -(-i^2 + \delta U_0) f_R(i)di$ by the smaller number $\int_c^{c'} -U_0 f_R(i)di$ to obtain

$$\Delta U \geq \int_0^w \frac{-i^2}{(1-\delta)} \Delta f_R(i)di + \int_w^{c'} \left[U_0' f_R'(i) - U_0 f_R(i)\right] di$$
$$+ \int_{c'}^1 \left[(-i^2 + \delta U_0') f_R'(i) - (-i^2 + \delta U_0) f_R(i)\right] di. \tag{60}$$

Add and subtract $\int_w^{c'} U_0' f_R(i)di + \int_{c'}^1 \left[-i^2 + \delta U_0'\right] f_R(i)di$ to the RHS of (60). Next, rearrange terms to isolate ΔU on the LHS:

$$\Delta U \left[1 - \int_w^{c'} f_R(i)di - \delta \int_{c'}^1 f_R(i)di\right] \geq \int_0^w \frac{-i^2}{(1-\delta)} \Delta f_R(i)di + \int_w^{c'} U_0' \Delta f_R(i)di$$
$$+ \int_{c'}^1 (-i^2 + \delta U_0') \Delta f_R(i)di. \tag{61}$$

The term $\left[1 - \int_{w'}^{c} f_R(i)di - \delta \int_c^1 f_R(i)di\right]$ on the LHS of (61) is strictly positive and we have assumed that $\Delta U < 0$. Thus, the LHS is strictly negative. To yield a contradiction, it suffices to show that the RHS is weakly positive.

To see that the RHS is weakly positive, define $g(i) = \frac{i^2}{(1-\delta)}$ if $i \in [0, w]$, $g(i) = U_0'$ if $i \in [w, c']$, and $g(i) = (-i^2 + \delta U_0')$ if $i \in [c', 1]$. Thus the RHS equals $\int_0^1 g(i) \Delta f_R(i)di$. Since $g(i)$ is weakly decreasing,[1] the result then follows from Remark B1.

Case 2) Now suppose $w < w'$ and $c' < c$. We need to show that c' is not too much lower than c—that is, we need to show that the median voter's direct payoff benefit from the more moderate ideology distribution F_R' exceeds the indirect cost from a lower compromise cutoff c'. This is trivially true when $\delta = 0$, since no incumbent compromises—all politicians implement their preferred policies, and the more moderate ideology distribution F_R' *strictly increases* the median voter's expected payoff. By continuity, the result then follows for all sufficiently small δ. Together cases 1 and 2 imply that $w' \leq w$, hence the median voter's payoff is higher under the more moderate ideology distribution. All voters then prefer F_R'. ■

[1]Function g is strictly decreasing when $i \in [0, w]$ and $i \in [c', 1]$, and constant when $i \in [w, c']$. Moreover, g is decreasing at $i = w$ since $w < w'$ implies $\frac{-w^2}{(1-\delta)} > \frac{-w'^2}{(1-\delta)} = U_0'$. It is also decreasing at $i = c'$ since $U_0' > -c' + \delta U_0'$, which follows from $(1-\delta)U_0' = -w' > -c'$.

B.2 At-Large Selection of Challengers

Recall that with party competition, F_R represents the ideology distribution of challengers from the right-wing party $i \in [0,1]$. Without party selection, we say that a challenger's ideology is drawn from at-large. That is, ideology $i \in [-1,+1]$ of a challenger is distributed according to the probability density function f (with associated c.d.f. F), where f is symmetric around $i=0$, with $f(i) = f_R(i)/2$, for all $i \geq 0$. Thus, the overall distribution of political ideologies is unaffected by the mode of challenger selection.

We start by describing the equations that characterize equilibrium outcomes for at-large selection of challenger. The full equilibrium derivation is in Bernhardt et al. (2009).

(C.1) The median voter must be indifferent between electing the challenger and re-electing an incumbent who adopts policy w;

(C.2) The incumbent with ideology c must be indifferent between compromising to w to win re-election and adopting as policy her own ideology, hence losing to the challenger.

The first condition defines the re-election standard $w \in (0,1)$: an incumbent politician is re-elected if and only if she implements policy $y \in [-w,w]$. The second condition defines the compromise cutoff $c \in (w,1)$: politicians with ideology $i \in (w,c)$ compromise to w in order to win re-election, while politicians with ideology $i \in (-c,-w)$ compromise to $-w$.

Given cutoffs $0 < w < c < 1$, let U_i be the discounted expected payoff of a voter with ideology i from electing an untried challenger. With a symmetric equilibrium, quadratic utility implies that for voter i, the expected discounted payoff of electing an untried challenger drawn from at-large distribution F is related to the median voter's payoff according to $U_i = U_0 - \frac{i^2}{(1-\delta)}$. To see this, note that

$$\begin{aligned} U_i \;=\; & \int_0^w \frac{-(i-y)^2}{(1-\delta)} f(y)dy + \int_w^c \frac{-(i-w)^2}{(1-\delta)} f(y)dy + \int_c^1 \left[-(i-y)^2 + \delta U_i\right] f(y)dy \\ & + \int_{-w}^0 \frac{-(i-y)^2}{(1-\delta)} f(y)dy + \int_{-c}^{-w} \frac{-(i+w)^2}{(1-\delta)} f(y)dy + \int_{-1}^{-c} \left[-(i-y)^2 + \delta U_i\right] f(y)dy. \end{aligned}$$

Expand all the quadratic terms, $-(i-y)^2 = -i^2 + 2iy - y^2$. By symmetry $f(y) = f(-y)$, so for each $y > 0$ the term $2iy$ cancels out with the corresponding term $2iy'$, where $y' = -y$. Add $\frac{i^2}{(1-\delta)}$ to both sides of the equation. Because $f(i) = f(-i) = f_R(i)/2$ for $i \geq 0$, substitute

f by f_R and simplify

$$\begin{aligned}
&\left(U_i + \frac{i^2}{(1-\delta)}\right) \\
&= \int_0^w \frac{-y^2}{(1-\delta)} f_R(y)dy + \int_w^c \frac{-w^2}{(1-\delta)} f_R(y)dy + \int_c^1 \left[-y^2 + \delta\left(U_i + \frac{i^2}{(1-\delta)}\right)\right] f_R(y)dy.
\end{aligned}$$

Consequently, $U_i + \frac{i^2}{(1-\delta)} = U_0$, where

$$U_0 = \int_0^w \frac{-y^2}{(1-\delta)} f_R(y)dy + \int_w^c \frac{-w^2}{(1-\delta)} f_R(y)dy + \int_c^1 \left[-y^2 + \delta U_0\right] f_R(y)dy. \tag{62}$$

In equilibrium, the median voter indifference condition (C.1) implies that

$$U_0 = \frac{-w^2}{(1-\delta)}. \tag{63}$$

Therefore, in equilibrium, $U_i = U_0 - \frac{i^2}{(1-\delta)} = -\frac{(w^2+i^2)}{(1-\delta)}$. The compromise cutoff c is defined by indifference condition (C.2),

$$-\frac{(c-w)^2}{(1-\delta)} = 0 + \delta U_i \tag{64}$$

$$\Rightarrow (c-w)^2 = \delta(w^2+c^2). \tag{65}$$

Solve this quadratic equation for the relevant $c > w > 0$ solution:

$$c = \frac{w}{1-\delta} + \sqrt{\frac{\delta w^2}{(1-\delta)^2} + \delta\frac{w^2}{(1-\delta)}} = \theta(\delta)w, \tag{66}$$

where

$$\theta(\delta) = \frac{1+\sqrt{\delta(2-\delta)}}{(1-\delta)}. \tag{67}$$

Note that for any $\delta \in (0,1)$ we have $\theta(\delta) > 1$ and

$$\frac{\partial\theta(\delta)}{\partial\delta} = \frac{1}{\sqrt{\delta(2-\delta)}} + \frac{\theta(\delta)}{(1-\delta)} > 0. \tag{68}$$

Substitute (63) into (62). Equilibrium is then summarized by w^* and c^* such that $c^* = \theta(\delta)w^*$ and w^* solves

$$-\frac{w^2}{(1-\delta)} = \int_0^w \frac{-i^2}{(1-\delta)} f_R(i)di + \int_w^{\theta(\delta)w} \frac{-w^2}{(1-\delta)} f_R(i)di + \int_{\theta(\delta)w}^1 \left[-i^2 - \frac{\delta w^2}{(1-\delta)}\right] f_R(i)di,$$

or simply

$$\int_0^w \left[i^2 - w^2\right] f_R(i)di + \int_{\theta(\delta)w}^1 (1-\delta)\left[i^2 - w^2\right] f_R(i)di = 0. \tag{69}$$

The LHS of (69) is strictly positive when $w = 0$, and strictly negative when $w = \frac{1}{\theta(\delta)}$. Because (A.1) holds for f_R, the LHS of (69) is a continuous, strictly decreasing function of $w \in \left(0, \frac{1}{\theta(\delta)}\right)$. Therefore, there exists a unique $w^* \in \left(0, \frac{1}{\theta(\delta)}\right)$ that solves (69).

Lemma B.2 *Fix ideology distribution F_R satisfying (A.1). For $\delta \in (0,1)$, let $w^*(\delta)$ be the unique solution to* (69)*, and $c^*(\delta) = \theta(\delta)w^*(\delta)$. Then $\frac{\partial w^*(\delta)}{\partial \delta} < 0$, $\frac{\partial c^*(\delta)}{\partial \delta} > 0$, $\lim_{\delta \to 1} w^*(\delta) = 0$, and $\lim_{\delta \to 1} c^*(\delta) = 1$.*

Proof: For $\delta \in (0,1)$, use the LHS of (69) to define

$$g(\delta) = \int_0^{w^*(\delta)} \left[i^2 - w^*(\delta)^2\right] f_R(i)di + \int_{\theta(\delta)w^*(\delta)}^1 (1-\delta)\left[i^2 - w^*(\delta)^2\right] f_R(i)di.$$

By the definition of w^*, it must be the case that $\frac{\partial g(\delta)}{\partial \delta} = 0$, that is,

$$\begin{aligned} 0 &= \int_0^{w^*(\delta)} -2w^*(\delta)\frac{\partial w^*(\delta)}{\partial \delta} f_R(i)di \\ &\quad - \left(\frac{\partial \theta(\delta)}{\partial \delta} w^*(\theta) + \theta(\delta)\frac{\partial w^*(\delta)}{\partial \delta}\right)(1-\delta)[\theta(\delta)^2 w^*(\theta)^2 - w^*(\theta)^2] f_R(\theta(\delta)w^*(\theta)) \\ &\quad + \int_{\theta(\delta)w^*(\theta)}^1 \left[-(i^2 - w^*(\theta)^2) - 2(1-\delta)w^*(\theta)\frac{\partial w^*(\delta)}{\partial \delta}\right] f_R(i)di. \end{aligned}$$

Rewrite to isolate $\frac{\partial w^*(\delta)}{\partial \delta}$,

$$\frac{\partial w^*(\delta)}{\partial \delta} A(\delta) = -\frac{\partial \theta(\delta)}{\partial \delta} B(\delta) + C(\delta), \tag{70}$$

where

$$\begin{aligned} A(\delta) &\equiv \int_0^{w^*(\delta)} 2w^*(\delta) f_R(i)di + \theta(\delta)(1-\delta)[\theta(\delta)^2 w^*(\theta)^2 - w^*(\theta)^2] f_R(\theta(\delta)w^*(\theta)) \\ &\quad +2(1-\delta)w^*(\theta)\int_{\theta(\delta)w^*(\theta)}^1 f_R(i)di, \\ B(\delta) &\equiv w^*(\theta)(1-\delta)[\theta(\delta)^2 w^*(\theta)^2 - w^*(\theta)^2] f_R(\theta(\delta)w^*(\theta)), \\ C(\delta) &\equiv \int_{\theta(\delta)w^*(\theta)}^1 \left[-(i^2 - w^*(\theta)^2)\right] f_R(i)di. \end{aligned}$$

Because $\theta(\delta) > 1$ and $\frac{\partial \theta(\delta)}{\partial \delta} > 0$, both $A(\delta)$ and $B(\delta)$ are strictly positive, while $C(\delta)$ is strictly negative. Therefore, the RHS of (70) is strictly negative, which implies $\frac{\partial w^*(\delta)}{\partial \delta} < 0$.

Because $\frac{\partial c^*(\delta)}{\partial \delta} = \frac{\partial \theta(\delta)}{\partial \delta} w^*(\delta) + \theta(\delta)\frac{\partial w^*(\delta)}{\partial \delta}$, the result $\frac{\partial c^*(\delta)}{\partial \delta} > 0$ requires the increase in $\theta(\delta)$ to be proportionally larger than the decrease in $w^*(\delta)$. To prove this, substitute $\frac{\partial w^*(\delta)}{\partial \delta}$ by

$\frac{1}{\theta(\delta)}\frac{\partial c^*(\delta)}{\partial\delta} - \frac{w^*(\delta)}{\theta(\delta)}\frac{\partial\theta(\delta)}{\partial\delta}$ in the LHS of (70). Rewrite to obtain

$$\frac{\partial c^*(\delta)}{\partial\delta}A(\delta) = \frac{\partial\theta(\delta)}{\partial\delta}\left(-B(\delta) + \frac{w^*(\delta)}{\theta(\delta)}A(\delta)\right) + C(\delta). \tag{71}$$

This simplifies to

$$\frac{\partial c^*(\delta)}{\partial\delta}A(\delta) = \frac{\partial\theta(\delta)}{\partial\delta}\left(\frac{2w^*(\delta)^2}{\theta(\delta)}\right)\left(\int_0^{w^*(\theta)} f_R(i)di + (1-\delta)\int_{\theta(\delta)w^*(\delta)}^1 f_R(i)di\right) + C(\delta).$$

Since $A(\delta) > 0$ and $\frac{\partial\theta(\delta)}{\partial\delta}\left(\frac{2w^*(\delta)^2}{\theta(\delta)}\right)(1-\delta)\int_{\theta(\delta)w^*(\delta)}^1 f_R(i)di > 0$, to prove $\frac{\partial c^*(\delta)}{\partial\delta} > 0$ it suffices to show that

$$\frac{\partial\theta(\delta)}{\partial\delta}\left(\frac{2w^*(\delta)^2}{\theta(\delta)}\right)\int_0^{w^*(\theta)} f_R(i)di \quad > \quad -C(\delta). \tag{72}$$

From equilibrium condition (69) and the definition of $C(\delta)$,

$$-C(\delta) \quad = \quad \frac{1}{(1-\delta)}\int_0^{w^*(\delta)}\left[-i^2 + w^*(\delta)^2\right] f_R(i)di < \frac{1}{(1-\delta)}\int_0^{w^*(\delta)} w^*(\delta)^2 f_R(i)di. \tag{73}$$

From (72) and (73), it suffices to show that

$$\begin{aligned}\frac{\partial\theta(\delta)}{\partial\delta}\left(\frac{2w^*(\delta)^2}{\theta(\delta)}\right)\int_0^{w^*(\theta)} f_R(i)di \quad &> \quad \frac{w^*(\delta)^2}{(1-\delta)}\int_0^{w^*(\delta)} f_R(i)di,\\ \iff 2\frac{\partial\theta(\delta)}{\partial\delta} \quad &> \quad \frac{\theta(\delta)}{(1-\delta)},\end{aligned}$$

which holds from (68).

The result $\lim_{\delta\to 1} w^*(\delta) = 0$ follows immediately from (69), while $\lim_{\delta\to 1} c^*(\delta) = 1$ follows from substituting $w^*(\delta)$ by $\frac{c^*(\delta)}{\theta(\delta)}$ in (69). ■

Lemma B.3 *Fix any pair of ideology distributions F_R and F'_R that satisfies (A.1), such that $F_R \succ_k F'_R$ for some $k \in (0,1)$. Using (A.1), define $\underline{f} > 0$ as $\underline{f} = \min_{i\in[0,1]}\{f_R(i), f'_R(i)\}$, and $\overline{f} < \infty$ as $\overline{f} = \max_{i\in[0,1]}\{f_R(i), f'_R(i)\}$. Let $\{w(\delta), c(\delta)\}$ and $\{w'(\delta), c'(\delta)\}$ be the corresponding equilibrium cutoffs when the discount factor is δ. Then there exists a cutoff $\delta^*(F_R, F'_R, k) < 1$ such that, for all $\delta > \delta^*(F_R, F'_R, k)$, the following inequalities hold for both at-large and party selection of candidates:*

$$\begin{aligned} k \quad &< \quad \min\{c(\delta), c'(\delta)\}, &(74)\\ \max\{w(\delta), w'(\delta)\} \quad &< \quad \min\{c(\delta), c'(\delta)\}, &(75)\\ \max\{w(\delta), w'(\delta)\} \quad &< \quad \frac{\delta\underline{f}}{\left[4 + 2\delta\overline{f}\,\right]}\min\left\{\left[c^2(\delta) - w^2(\delta)\right], \left[c'^2(\delta) - w'^2(\delta)\right]\right\}. &(76)\end{aligned}$$

Proof: **Case 1)** First consider at-large selection of challengers. The result that there exists a $\delta^* < 1$ such that inequalities (74) and (75) hold for all $\delta \in (\delta^*, 1)$ follows immediately from Lemma B.2, since as we increase δ to the limit of one, each w strictly decreases to zero and each c strictly increases to one.

As we increase δ, the LHS of (76) decreases to zero. The RHS of (76) is strictly positive for any $\delta \in (0, 1)$. Hence, it only remains to show that the RHS of (76) increases with δ. From Lemma B.2, both $[c^2(\delta) - w^2(\delta)]$ and $[c'^2(\delta) - w'^2(\delta)]$ increase with δ. Moreover, the derivative of $\frac{\delta \underline{f}}{[4+2\delta\overline{f}\,]}$ with respect to δ equals $\left(\frac{\underline{f}}{[4+2\delta\overline{f}]} - \frac{2\overline{f}\delta\underline{f}}{[4+2\delta\overline{f}]^2}\right)$, which simplifies to $\left(\frac{4\underline{f}}{[4+2\delta\overline{f}]^2}\right) > 0$. Hence the RHS of (76) increases with δ.

Therefore, under at-large selection of challengers, there exists a $\delta^* < 1$ such that inequalities (74), (75) and (76) hold for all $\delta \in (\delta^*, 1)$, concluding this step of the proof.

Case 2) Now consider party-selection of challengers. From Bernhardt et al. (2009, Proposition 1), compromise cutoffs $\{c, c'\}$ are always larger and reelection cutoffs $\{w, w'\}$ are always lower with party selection than at-large selection. Therefore, if inequalities (74), (75) and (76) hold for at-large selection, then they also hold for party selection of challengers. Consequently, these inequalities hold with party selection for all $\delta \in (\delta^*, 1)$, where $\delta^* < 1$ is the cutoff defined in the previous step, which concludes the proof. ■

Proposition B.1 *Consider at-large selection of challengers. Suppose the distribution of politicians' ideologies F_R first-order stochastically dominates F'_R, $F_R \succ_{FOSD} F'_R$, and assumption (A.1) holds for both distributions. Then all voters weakly prefer the more moderate distribution of ideologies F'_R.*

Proof: Assume (A.1) holds for distributions F_R and F'_R, where $F_R \succ_{FOSD} F'_R$ so that F'_R has more moderate ideologies. Symmetrically define F_L and F'_L. Let $\{c, w\}$ and $\{c', w'\}$ be the unique equilibrium cutoffs for distributions F and F', respectively, and let U_0 and U'_0 be the corresponding equilibrium expected payoffs of the median voter.

Let $\Delta U \equiv U'_0 - U_0$, and $\Delta f_R(i) \equiv f'_R(i) - f_R(i)$ for all $i \in [0, 1]$. By contradiction, suppose that $\Delta U < 0$, which implies that $w' > w$. From (66), $c' = \theta(\delta)w'$ and $c = \theta(\delta)w$, so it must be the case that $c' > c$. The same steps of the proof of Lemma B.1, "Case 1", yield a contradiction, concluding the proof. Note that Case 2 in Lemma B.1 is irrelevant, since it considers $w' > w$ and $c' < c$, which cannot happen with at-large selection because of (66). ■

Proposition B.1 shows that with at-large selection, given any ideology distributions F_R and F'_R, if $F_R \succ_{FOSD} F'_R$, then all voters weakly prefer the more moderate distribution of ideologies. That is, independently of δ, the direct positive effect of more moderate ideologies always dominates the negative impact of reduced compromise. In other words, c does not shrink toward the median by too much—losing to a challenger becomes less costly to an incumbent, but not too much so. This reflects that with quadratic utilities, in a symmetric equilibrium, *the change in the median voter's expected payoff from electing a challenger exactly equals the change in the incumbent's expected payoff from losing re-election to a challenger*. This result sharply contrasts with Proposition 1 because, with party selection, the median voter and right-wing incumbents benefit differently from changes in the ideology distribution of left-wing challengers.[2]

Interestingly, all the results concerning the welfare consequences of noisy signals about challengers hold for both at-large and party selection. This is because the proofs only rely on the median voter's indifference condition; they do not rely on the indifference conditions of compromising politicians, which is the main difference between at-large and party selection models.

B.3 Endogenous Information Transmission

In this section we endogenize the information transmitted to voters about challengers. To do this, we introduce two symmetrically-situated interest groups that have the same utility functions as voters $\{-\gamma, +\gamma\}$. Before each election, the IGs costlessly observe a common signal about whether the challenger is more likely to have a moderate or an extreme ideology. This learning process corresponds to the signal Π_β with possible realizations $s \in \{s_G, s_B\}$.

[2] If we consider at-large selection of challengers and Euclidean preferences instead of quadratic, one can construct examples where ideology moderation hurts voters. We focus on quadratic utility to highlight the sharp contrast in the incentives generated by at-large and two-party selection.

The signal received by IGs is non-verifiable, but each IG can send a public (cheap-talk) message to voters about a challenger's ideology.

One can interpret the IGs as newspapers or other political institutions with limited biases in their political preferences, and better access to political information than voters. The message has the natural interpretation as a political endorsement for a challenger or incumbent. Reflecting the left-right preference misalignment between IGs, and the fact that party selection is the norm, we focus our analysis on party selection of challengers. However, our results extend to at-large selection.

Due to the intrinsic nature of the cheap-talk game, many equilibria exist. We focus on the equilibrium in which, in the cases where voters will not believe a message sent by a particular IG (leaving the IG indifferent to its message choice), the IG selects its message as if there is an infinitesimally small positive probability that a majority of voters will follow the IG's advice.

Proposition B.2 *The set of equilibrium cutoffs described by Proposition 3, in the model without IGs, is also an equilibrium in the model where information is endogenously transmitted by IGs, as long as their political bias is not too large (γ is small enough).*

Proof: Suppose all conditions of Proposition 3 hold, and let w_G and w_B be the equilibrium cutoffs established by the median voter. Now consider the left-wing IG with ideology $-\gamma \leq 0$, and define $Lw_G(\gamma)$ and $Lw_B(\gamma)$ as the re-election standards of the IG. It follows from the quadratic utility function that $Lw_G(0) = w_G$ and that $Lw_G(\gamma)$ is a continuous strictly decreasing function of γ. The same holds for $Lw_B(\gamma)$. The opposite is true for the right-wing IG with ideology $\gamma \geq 0$. The IG's re-election standards $Rw_G(\gamma)$ and $Rw_B(\gamma)$ are continuous increasing functions of γ.

Therefore, there is a $\overline{\gamma} > 0$ such that $Rw_G(\overline{\gamma}) = Lw_B(\overline{\gamma}) \in (w_G, w_B)$. Suppose $\gamma < \overline{\gamma}$. When a right-incumbent implements a policy $y \in [0, w_G]$, the median voter always prefers to re-elect the incumbent, independently of the true signal about the challenger, so endorsements are irrelevant for his voting decision. When the incumbent implements a policy $y \in (w_G, Rw_G]$, the right-wing IG always wants to re-elect the right-wing incumbent—because of this incentive misalignment, the right-wing IG cannot truthfully reveal the signal to the median voter. However, the incentives of the median voter and the left IG are aligned, and the median re-elects the right-wing incumbent if and only if she is endorsed by the left-wing interest group. When a right-wing incumbent implements policy $y \in (Rw_G, Lw_B]$, the incentives of the median voter and both IGs are aligned, so both groups can truthfully communicate the signal to the median voter. When the right-wing incumbent implements a more extreme policy $y \in (Lw_B, w_B]$, the left-wing IG always supports the left-challenger, so this endorsement reveals no information. Now the informative endorsement is that of the right IG. Finally, the median voter never re-elects an incumbent who implements radical policies $y \in (w_B, 1]$, so messages from the IGs do not affect the median's voting choices.

In summary, for any $\gamma < \overline{\gamma}$ at least one IG is willing to truthfully communicate the signal to the median voter when the incumbent chooses a policy in the relevant range $y \in (w_G, w_B]$, where information about the challenger defines the median voter's vote. Hence, there exists a cheap-talk equilibrium that is informative and yields the same equilibrium outcomes as the equilibrium in Proposition 3. ■

Consider the equilibrium described by Proposition 3. For concreteness, consider a right-wing incumbent running against a left-challenger. In equilibrium, independently of any information possibly transmitted by IGs, the median voter always re-elects an incumbent who adopts policy $y \in [0, w_G]$, and never re-elects an incumbent who adopts policy $y \in (w_B, 1]$. The median voter would like to re-elect an incumbent who adopts policy $y \in (w_G, w_B]$ if and only if the *true* signal received by the IGs is bad, s_B. In this range the median voter must rely on the endorsement by the different interest groups, but he must take the incentives misalignments into account. Consequently, Proposition 2 holds if voters are able to infer the true realized signal $s \in \{s_G, s_B\}$ from the messages transmitted by the IGs when the incumbent implements policy $y \in (w_G, w_B]$.

The result in Proposition 2 holds trivially when $\gamma = 0$, in which case the interest groups' incentives are perfectly aligned with the median voter's. The IGs can truthfully report their findings, and the median voter will believe (and follow) their advice. In this case, an incumbent who implements policy $y \in (w_G, w_B]$ wins re-election if and only if she receives the endorsement of the interest groups, who endorse the incumbent if and only if the signal about the challenger is bad.

A more interesting scenario is when the misalignment γ between the preferences of the median voter and the IGs is positive, but small. The left-wing IG disagrees with the median voter's re-election cutoffs, since it has biased preferences ($-\gamma < 0$) toward the left-challenger. The left-wing IG has its own re-election cutoffs $Lw_G < w_G$ and $Lw_B < w_B$. Similarly, the right-wing IG has biased preferences against the left-challenger, and its re-election cutoffs are $w_G < Rw_G$ and $w_B < Rw_B$. See Figure 7.

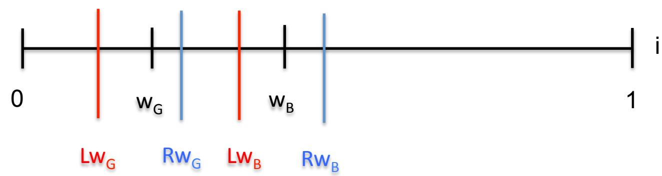

Figure 7: Re-election cutoffs when IGs transmit information to voters

When a right-incumbent implements a policy $y \in [0, w_G]$, the median voter always prefers to re-elect the incumbent, independently of the true signal about the challenger, so endorsements are irrelevant for his voting decision.[3] When the incumbent implements a policy $y \in (w_G, Rw_G]$, the right-wing IG always wants to re-elect the right-wing incumbent. This incentive misalignment results in the median voter not using the right-wing IG's message to update his beliefs about the challenger's ideology. However, the incentives of the median voter and the left IG are aligned, and the median re-elects the right-wing incumbent if and only if she is endorsed by the left-wing interest group. That is, with slight misalignment, *right-wing* incumbents are re-elected if and only if the *left-wing* interest group receives a bad signal about the left-challenger's ideology, which induces the left IG to endorse the right-incumbent.

When a right-wing incumbent implements policy $y \in (Rw_G, Lw_B]$, the incentives of the median voter and both IGs are aligned. Both groups support the incumbent if and only

[3]Even when endorsements do not affect who wins the election, they can affect the margin of victory. For $y \in [0, Lw_G]$ both IGs always support the incumbent so voters do not learn about the challenger, so vote margins are unaffected. When $y \in [Lw_G, w_G]$, the right-wing IG always supports the right-incumbent, while the left-wing IG supports the right-incumbent if and only if the signal about the left-challenger is bad. Therefore, voters use the left-wing IG's endorsement to learn about the challenger, which changes the identity of the marginal voter indifferent between the incumbent and challenger. So too, an incumbent who adopts more radical policies $y \in (w_B, Rw_B]$ always loses re-election, but the endorsement or not by the right-wing IG changes the vote margin.

if they receive a bad signal about the challenger, and the median votes for the candidate endorsed by the IGs. When the right-wing incumbent implements more extreme policy $y \in (Lw_B, w_B]$, the left-wing IG always supports the left-challenger, so this endorsement reveals no information. Now the informative endorsement is that of the right IG. This more extreme right-wing incumbent wins re-election if and only if she is supported by the right IG—that is, if and only if the true signal about the challenger's ideology is bad. Finally, the median never re-elects an incumbent who implements radical policies $y \in (w_B, 1]$.

An interesting implication is that if interest groups are sufficiently biased (γ is large enough), then the incentive misalignment between the median voter and the IGs is so large that the median ceases to learn in any cheap-talk equilibrium about the signals received by the IGs.[4] Consequently, in an economy where the results of Proposition 3 hold, having very extreme IGs is better than having very centrist IGs. That is, when the value of information is negative, the median voter would like to commit to not learn about the challenger, in order to slacken re-election cutoffs and thereby provide incentives to extreme incumbents to compromise. When IGs are the ones receiving signals about a challenger, a sufficiently large bias guarantees that they cannot transmit non-verifiable information to voters via cheap-talk.

[4]To be precise, the median voter ceases to learn when the incumbent implements a policy such that access to the signal would actually swing his vote. The median voter may still learn the true signal in states where it does not affect his voting decision. In such a siutation, the votes of voters with more extreme ideologies may be affected by the IGs' endorsements.