On the Benefits of Party Competition*

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Abstract

We study the role of parties in a citizen-candidate repeated-elections model in which voters have incomplete information. We first identify a novel "party competition effect" in a setting with two opposing parties. Compared with "at large" selection of candidates, party selection makes office-holders more willing to avoid extreme ideological stands, and this benefits voters of all ideologies. We then allow for additional parties. With strategic voting, citizens benefit most when the only two parties receiving votes are more moderate. With sincere voting, even with three parties, extreme parties can thrive at the expense of a middle party; and whether most citizens prefer two or three parties varies with model parameters.

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1 Introduction

This paper studies the role of party competition in a citizen-candidate repeated-elections model in which voters have limited information about candidate's ideologies, and incumbents' past actions help voters predict their future choices if re-elected. We contrast party selection of candidates with at-large selection in absence of parties. For politicians, we ask: Would politicians be more representative of their constituency with party or at-large selection? Would more incumbents lose with party or at-large selection of challengers? For voters, we ask: Would party selection induce voters to set more demanding standards of representation on incumbents for them to win re-election? Would voters be better off with party selection or at-large selection?

Our model is as follows. Rational, forward-looking citizens, be they voters, electoral candidates or elected representatives, care about the policy choices of representatives in office. Each period, an incumbent faces an electoral challenge from one or more opposing candidates. Voters have limited information about the challengers' ideologies, but can use an incumbent's past actions in office to make inferences about her future actions if re-elected.[1] We maintain the basic tenet of citizen-candidate models that candidates cannot make credible promises. Citizens are fully-rational and forward-looking, and maximize future discounted utility.

In the first part of the paper, we contrast outcomes when a single challenger is drawn at random from the entire spectrum of ideologies "at large", with those that obtain when the challenger is drawn from a party representing the wing of the ideological space "opposite" from the incumbent.[2] Given standard assumptions the median voter is decisive and sets a simple retrospective voting rule: vote for the incumbent candidate if and only if the incumbent's policy is sufficiently moderate. An incumbent with a sufficiently moderate ideology can represent her own ideology and win re-election. All other winning incumbents take positions as close to their ideologies as they possibly can and

[1] Most models of candidate location ignore differences between incumbents and challengers. Yet, these differences matter—in practice, incumbents generally win re-election. Our model highlights one key difference: voters typically know far more about incumbents than challengers, who are usually untried in the office for which they are running.

[2] As discussed in the literature review, there is empirical evidence that party labels provide voters with information about candidates' ideologies (see, for example, Snyder and Ting, 2002).

still be re-elected. Voters use this information to update their beliefs about an incumbent's ideology and hence future actions: the location of the marginal re-elected incumbent leaves the median voter indifferent between re-electing the incumbent and selecting an untried challenger.

We identify a novel "party competition effect" in a setting with two opposing parties. Party competition makes office-holders more reluctant to take extreme ideological stands, and induces voters to set more demanding standards for re-election. The 'party competition effect' materializes because an incumbent fears being replaced by a challenger from the opposing party by more than she fears being replaced by a challenger from at large—the incumbent's own ideology is further from the likely ideology and location of a challenger selected by the opposing party. As a result, ceteris paribus, with party selection, an incumbent is more willing to adopt positions closer to the median voter's preferred position in order to win re-election. In turn, party selection raises the random value of an untried politician to the median voter, causing the median voter to set more demanding standards for re-election. The mere existence of a left-wing party discourages right-wing elected officials from drifting into extremism. Our "party competition effect" provides a fully endogenous theory of party discipline: office holders are willing to follow party lines, even though there is no party-controlled reward mechanism. Incumbents avoid extreme ideological stands and take the responsibility not to lose the elections.

We then show that party selection of challengers reduces turnover of incumbents. This result is far from immediate as the indirect effect of party selection on re-election standards—voters set more demanding standards—is to increase turnover. Effectively, we show that the direct effect on turnover of the increased willingness of extreme candidates to restrict location in the face of party selection dominates the indirect effect of tighter re-election standards. Due to this self-induced "party discipline", in equilibrium, more incumbent types are re-elected with party selection than with at-large selection.

Our welfare analysis collects the implications of these observations to show that the "party competition effect" benefits all voters of all ideologies. Hence, our analysis provides a new rationale for political parties.[3] The median voter values the greater willingness of office-holders to compromise

[3] We emphasize the parsimonious nature of our model of parties. Parties do not pool financial resources. They do not exercise party discipline, nor dictate party lines. Parties have no control on elected candidates' policy choices. There is no partisanship: citizens do not care about party identity per se, but only about the policies adopted by elected representatives. Rather, we only impute to parties the ability to aggregate citizens with like-minded political views, and show that the resulting "party competition effect" raises welfare, thus providing a minimal rationale for political parties.

and adopt moderate positions closer to the median policy to win re-election; and all other voters benefit from the reduction in the per-period variance in adopted policies, and the reduced turnover, which lowers the variability over time in policies adopted by elected representatives.

The second part of our paper considers what happens when there are more than two parties. We address this question in two ways. We first consider equilibrium with strategic voting, where coordination results in only two parties receiving votes in an election. In the simple environment with an even number of parties, ordered by ideology, and symmetrically placed around the median, there is an equilibrium where only candidates from the two most moderate parties win, and our previous characterizations extend. This equilibrium selection criterion is motivated by the observation that the winning candidate is always the Condorcet winner of the multi-party election. The moderate parties serve a role of "screening" out extremist candidates, who are in other parties. We find that the more moderate are the winning parties, the higher is the welfare of all voters.

We then turn to consider equilibrium outcomes with sincere voting and plurality rule. We focus on the provocative case where there is a centrist party and two extreme parties, and an incumbent faces challengers drawn from the other two parties. Now, the median voter is no longer decisive, but the set of voters supporting a given candidate is still given by an interval of ideologies. As with two parties, an incumbent may deviate from her optimal policy to gain reelection. However, in contrast to a two-party setting, an extreme-party incumbent may even implement a policy more *extreme* than her optimal policy, in order not to take away votes from the centrist party challenger, and ensure that the centrist defeats the opposing extreme party challenger.

Our numerical characterization focus on liner loss functions and uniformly distributed ideologies. In surprising contrast to a static citizen candidate model, we find that unless the centrist party is far larger than the extreme parties, then even the best incumbent from the center party—an incumbent with the median ideology—cannot win re-election. The reason is that even with discount factors as small as 0.1, most members of an extreme party, if elected, would moderate their location to win re-election; nearby members of the centrist party understand this and support challengers from the extreme parties. Strikingly, we show that extreme party incumbents win by adopting positions that centrist party incumbents would lose with.

Our comparative statics analysis compares equilibrium outcomes as we vary the size of the centrist party for a fixed discount factor of 0.3, and as we vary the discount factor, fixing equally-sized parties. The limit case where the center party consists of just the median candidate is very different from a two party system, as a tiny centrist party wins votes of moderates, but not enough to win re-election. The competition with the centrist party raises the measure of extreme party incumbent types willing to compromise. As a result, most voters prefer the three party system if the center party is small enough.

However, as the size of the centrist party grows, the winning policies of the extreme party grow initially more extreme, but a centrist incumbent still does not win reelection. As a result, more voters begin to prefer the two-party system. But as the size of the centrist party crosses a given (large) threshold, its incumbents start to win reelection; and because centrist incumbents must sharply constrain their policy toward the median to defeat extreme party challengers, most voters begin to prefer three-party competition to two. Hence, the welfare effects of multi-party competition are non-monotonic in the size of the centrist party.

With equally sized parties, we find that the centrist party incumbents never win reelection for any discount factor. Indeed, once the discount factor exceeds 0.6, all extreme party incumbent compromise to win re-election—eliminating all risk for the median voter. In contrast, with two parties, because voters might get "lucky" with a draw of a moderate challenger, equilibrium requires that some extreme ideologies not compromise. For low discount factors, the presence of a centrist party whose incumbent always loses reelection is detrimental to the electorate, as the winning platforms belong to the extreme parties and are more extreme than in a two-party setting. However, the fraction of voters who prefer three parties to two rises with the discount factor, and once the discount factor grows large all voters benefit from the greater willingness to compromise and eliminated risk.

The paper proceeds as follows. After a literature review, Section 2 identifies the party competition effect, by comparing selection of challengers at large, with the outcome of two-party competition. Section 3 explores multiparty competition and Section 4 concludes. Most proofs are in an appendix.

1.1 Literature Review

Our analysis is most closely related to Duggan (2000), who develops the basic theory of repeated citizen-candidate election with incomplete information about candidates' policy preferences. Bernhardt et al. (2004) study related issues when politicians face term limits, and more senior politicians can obtain "pork transfers" for their districts from districts with less senior politicians.[4] Banks and Duggan (2001) extend Duggan's (2000) analysis to allow for multiple ideological dimensions. Bernhardt and Ingberman (1985) is the first paper to consider informational differences between incumbents and challengers. Most of the literature on informational differences between incumbents and challengers focuses on legislative ability rather than ideology. Mattozzi and Merlo (2008) study a model in which parties screen their candidates, certifying their valences to the electorate in exchange for rents provided by the representatives in office. [5]

Our analysis within a repeated citizen-candidate election with incomplete information framework formalizes the original intuition by Downs (1957) that party labels provide voters with information about candidates. Fiorina (1981) and Lindbeck and Weibull (1993) document that voters learn about the policy positions of candidates from party labels. More recently, Snyder and Ting (2002) find that party dummies explain much of the variation in the voters' placements of candidates on a left-right scale. These empirical findings have fostered recent theoretical analyses. Snyder and Ting (2002) study the relationship between platform choices and the information power of party labels. Ashworth and Bueno de Mesquita (2004) provide a formal analysis in which party discipline, candidate affiliation, and ideological homogeneity are all determined endogenously within a strategic electoral-legislative setting. In contrast to these papers, we adopt a minimalistic approach to parties—our parties do not dictate party lines. Our 'party competition' effect is driven solely by the ability of parties to aggregate individuals with similar political ideologies.

Our analysis of the benefits of the party system is also related to the literature on endogenous

[4]Relatedly, Reed (1994) solves an example featuring moral hazard and adverse selection in the provision of a public good with a term limit of two. Banks and Sundaram (1998) and Reed (1989) consider related adverse selection models.

[5]Banks and Sundaram (1993) develop a dynamic model in which representatives exert effort to represent their constituency. Over time, voters learn which representatives are lazy, and vote them out, so a smaller fraction of more senior incumbents lose. In a similar vein, Austen-Smith and Banks (1989) and Ferejohn (1986) consider dynamic games in which representatives dislike exerting effort.

party formation. Feddersen (1993) explains party formation as coalitions of voters, whereas in Osborne and Tourky (2003), parties arise in legislatures due to economies of scale. Persson, Roland and Tabellini (2003) study a model in which electoral institutions endogenously determine party fragmentation. Morelli (2004) studies a model in which parties facilitate coordination among voters and allow candidates to commit to policies. He finds conditions under which proportional representation gives rise to more parties than plurality rule. Jackson and Moselle (2001) study a legislative voting game in which decisions are made over both ideological and distributive dimensions. Parties form because it is assumed that elected representatives can commit to enforce party agreements. Levy (2004) focuses on the role of parties in insuring the credibility of policy commitments. She supposes that individual candidates must adopt their ideal policy. If, instead, candidates are grouped into parties, she assumes that they can commit to any policy in the Pareto set of the party. In her static model, this commitment function of parties changes the policy outcome only when the policy space is multi-dimensional. In contrast, we consider a uni-dimensional dynamic model, we endow parties no commitment powers, but show that the presence of an opposing party serves indirectly as a commitment device, by inducing an incumbent to moderate policies out of fear of being replaced by someone from the opposite side of the political spectrum.

2 The 'Party Competition Effect'

There is an interval $[-a, +a]$ of citizen candidates, each indexed by her private ideology x, where x has support $[-a, +a]$. Ideologies are private information to candidates. Ideologies are distributed across society according to the c.d.f. F, with an associated single-peaked density f that is symmetric about the median voter's ideology, $x = 0$. At any date t, an office holder with ideology x selects a policy $p(x) \equiv y \in [-a, +a]$. The time-$t$ utility of a citizen x depends on the implemented policy y, according to a symmetric, single-peaked loss function $L_x(y) = l(|x - y|)$, where l is $\mathcal{C}^2$, and $l' < 0$, $l'' \leq 0$. We normalize $l(0) = 0$ without loss of generality. Note that L satisfies the single-crossing property: L'_x is increasing in x. Period utilities are discounted by factor $\delta < 1$.[6]

We focus on symmetric, stationary and stage-undominated perfect Bayesian equilibrium (PBE).

[6] To ease presentation, we abstract from ego rents from holding office. Our results extend qualitatively with ego rents.

A stationary policy strategy p prescribes that at any time t, the policy y selected by an office holder depends only on her ideology y. The policy strategy is symmetric if $p(x) = -p(-x)$. If representatives adopt symmetric stationary strategies, stage-undominated PBE voters' strategies are as follows. If the date-t incumbent office-holder adopts platform y, then a voter x votes to re-elect the incumbent if and only if $L_x(y) \geq U_x$, where U_x is the equilibrium expected continuation utility from selecting a new representative at random.[7] In a PBE, the median voter is said to be decisive whenever an incumbent office-holder who adopts policy y is re-elected if and only if $L_0(y) \geq U_0$, i.e., the incumbent is re-elected if and only if the median voter prefers him to the challenger.

2.1 At-large selection of challengers

With at-large selection of challengers, at the beginning of any period $t \geq 1$, the incumbent runs for re-election against a challenger drawn at random from $f(\cdot)$. At date 0, there is an election between untried challengers. We show in Theorem A1 in the appendix that as long as the citizens' loss functions do not display too much risk aversion, and their risk aversion does not increase too fast, there is a unique symmetric, stage-undominated, stationary perfect Bayesian equilibrium. The equilibrium is completely summarized by thresholds $\{w, c\}$, where $0 < w < c < a$. Candidates with centrist ideology $x \in [0, w]$ and extremist candidates $x \in [c, a]$ adopt their preferred policy $y = x$ in office. Centrists are re-elected and extremists are ousted from office. Moderate candidates $x \in [w, c]$ do not adopt their preferred policy, as they would then lose office. They compromise and adopt the most extreme ideology that allows them to win re-election, i.e., they locate at w. The characterization is symmetric for $x < 0$.

The equilibrium obeys the following recursive equations:

$$L_0(w) = U_0(c, w), \tag{1}$$

$$L_c(w) = (1 - \delta) L_c(c) + \delta U_c(c, w) = \delta U_c(c, w). \tag{2}$$

The median voter is decisive: she is indifferent between re-electing a candidate who implements policy w and electing the random challenger. Candidate c is indifferent between implementing policy

[7] We see symmetry and stage undomination as natural equilibrium requirements. Stationarity permits a tractable representation of equilibrium that highlights the features of party competition.

w forever, or policy c once and then being replaced by a random challenger.

For any citizen x, the PBE continuation expected value from electing the challenger is:

$$
\begin{aligned}
U_x(w,c) &= \int_{-a}^{-c} \left(L_x(y)(1-\delta)+\delta U_x\right) dF(y) + \int_{-c}^{-w} L_x(-w) dF(y) \\
&\quad + \int_{-w}^{w} L_x(y) dF(y) + \int_{w}^{c} L_x(w) dF(y) + \int_{c}^{a} \left(L_x(y)(1-\delta)+\delta U_x\right) dF(y).
\end{aligned} \tag{3}
$$

Throughout our analysis, we assume that the parameters characterizing the economy are such that the median voter is decisive and candidates with more extreme ideologies are less willing to compromise, so that equilibrium is described by the set $\{w, c\}$. This amounts to assuming that citizens are not too risk averse, and their risk aversion does not increase too fast. Theorem A1 in the appendix provides sufficient conditions for this to hold, extending Theorem 1 in Duggan (2000), which proves the result for linear loss $L_x(y) = -|x-y|$, when ideologies are uniformly distributed.

The sufficient conditions must address two issues. One deals with the decisiveness of the median voter. If the median voter prefers the incumbent, then so do all voters "closer" to the incumbent, so the incumbent must win the election. However, if the median voter prefers the challenger, she need not be part of the winning coalition. In particular, if voters exhibit increasing risk aversion, so that the marginal disutility is increasing in the distance between a voter and a representative's location, an incumbent may be able to cobble together a winning coalition of voters with opposing extreme ideologies. In this case, the median voter is not decisive. For example, an incumbent who adopts a intermediate right-wing platform, may win the votes of all sufficiently right-wing voters, lose the median's vote, yet win the votes from very risk-averse left-wing extremists who fear that the incumbent may be replaced by an even more extreme right-wing ideologue, thereby winning re-election. What underlies this is that the median voter effectively faces less risk than do voters with more extreme ideologies because representatives cannot locate as far away from the median voter. Formally, the condition that may be violated if voters are too risk averse is that $L_x(0) - U_x(c,w)$ is weakly decreasing in x, for any w, c.

The second issue is that if voters' risk aversion increases too fast, the office holder's choice of location may not be determined by a cutoff c. That is, it may not be the case that if an incumbent with ideology $x > 0$ prefers not to compromise to win re-election then all incumbents with more extreme ideologies $x' > x$ also prefer not to compromise. An extremist incumbent may fear being replaced by an opposing extremist by so much, that she is more willing to compromise and adopt a policy in $\{-w, w\}$ to win re-election than is a more moderate incumbent. Formally, the condition that may be violated if voters' risk aversion increases too fast is that $L_x(w) - \delta U_x(x, w)$ be concave in $x > w$, for any w.

2.2 Two Parties

We contrast outcomes in the repeated election model with at-large selection of candidates, with those that obtain when challenging candidates are chosen by opposing parties, A and B. We initially assume that party A includes all citizen-candidates with ideology $x < 0$, and party B includes all possible candidates with ideology $x > 0$. At date zero, there is an election between untried challengers from each party. In any subsequent election, the incumbent faces a challenger from the opposite party. That is, incumbents are always endorsed by their parties. Equivalently, we could assume that if the party does not endorse "its" incumbent, then voters who are indifferent between untried challengers from the two parties select the opposing party's candidate.[8]

We prove in Theorem A2 in the appendix that as long as voters are not too risk-averse, then the symmetric, stage-undominated, stationary perfect Bayesian equilibrium is characterized by two thresholds: v and k. A representative with a centrist ideology $x \in [-v, v]$ and those with extreme ideologies $x \in [-a, -k] \cup [k, a]$ adopt their preferred policy $y = x$ in office. Moderates are re-elected and extremists are ousted. Moderates office-holders with $x \in [-k, -v]$, and $x \in [v, k]$, adopt policies $-v$ and v respectively, and are re-elected.

For an office-holder with ideology x, we must distinguish her expected continuation payoff when next period's office-holder is selected from x's own party, denoted by $\overline{U}_x$, from her continuation

[8] Paradoxically, the party competition effect disappears if (i) parties did not automatically endorse their incumbents and (ii) voters indifferent between untried challengers split their votes evenly. The competition internal to the party for re-endorsement turns out to perfectly offset the virtuous forces of competition between parties.

payoff when next period's office-holder is selected from the opposing party, $\underline{U}_x$. For $x > 0$, we have

$$\underline{U}_x(v,k) = 2\Big[\int_{-a}^{-k} \big(L_x(y)(1-\delta) + \delta\overline{U}_x(v,k)\big)\, dF(y) + \int_{-k}^{-v} L_x(-v) dF(y) + \int_{-v}^{0} L_x(y) dF(y)\Big] \tag{4}$$

$$\overline{U}_x(v,k) = 2\Big[\int_{0}^{v} L_x(y) dF(y) + \int_{v}^{k} L_x(v) dF(y) + \int_{k}^{a} \left(L_x(y)(1-\delta) + \delta\underline{U}_x(v,k)\right) dF(y)\Big]. \tag{5}$$

The recursive equations characterizing the equilibrium are

$$L_0(v) = \overline{U}_0(v,k) = \underline{U}_0(v,k), \tag{6}$$

$$L_k(v) = (1-\delta) L_k(k) + \delta\underline{U}_k(v,k) = \delta\underline{U}_k(v,k). \tag{7}$$

2.3 Equilibrium and Welfare Comparison

We now show that the introduction of parties makes candidates more willing to compromise. We proceed in separate lemmata. We first show that an office-holder prefers to be replaced by a candidate randomly selected from at large to being replaced by a candidate from the opposing party.

Lemma 1 *Any incumbent x prefers to be replaced by a candidate from her own party to being replaced by a candidate from at large to being replaced by a candidate from the opposing party. That is, equilibrium payoffs are ranked as follows: $\underline{U}_x(w,c) < U_x(w,c) < \overline{U}_x(w,c)$.*

Proof. Suppose that $x > 0$; the case for $x < 0$ is analogous by symmetry. Subtracting equation (4) from equation (3), and using the symmetry of f, yields:

$$\begin{aligned} U_x - \underline{U}_x = &\int_{-a}^{-c} \delta\left(U_x - \overline{U}_x\right) dF(y) - \int_{-a}^{-c} \left(L_x(y)(1-\delta) + \delta\overline{U}_x\right) dF(y) + \int_{0}^{w} L_x(y) dF(y) \\ &+ \int_{c}^{a} \left(L_x(y)(1-\delta) + \delta U_x\right) dF(y) - \int_{-c}^{-w} L_x(-w) dF(y) + \int_{w}^{c} L_x(w) dF(y) - \int_{-w}^{0} L_x(y) dF(y) \\ = &\int_{c}^{a} \left[L_x(y) - L_x(-y)\right](1-\delta)\, dF(y) + \int_{w}^{c} \left[L_x(w) - L_x(-w)\right] dF(y) \\ &+ \int_{0}^{w} \left[L_x(y) - L_x(-y)\right] dF(y) + 2\int_{c}^{a} \delta\left(U_x - \overline{U}_x\right) dF(y). \end{aligned} \tag{8}$$

Thus,

$$\begin{aligned} \left(U_x - \underline{U}_x\right)\left(1 - \left(2\delta[F(a) - F(c)]\right)^2\right) = &\Big[\int_{c}^{a} \left[L_x(y) - L_x(-y)\right](1-\delta)\, dF(y) \\ &+ \int_{w}^{c} \left[L_x(w) - L_x(-w)\right] dF(y) + \int_{0}^{w} \left[L_x(y) - L_x(-y)\right] dF(y)\Big]\left(1 - 2\delta[F(a) - F(c)]\right). \end{aligned}$$

The result then follows because (a) $L_x(y) > L_x(-y)$ for any $y > 0$, (b) $\delta \leq 1$, and (c) $F(0) = 1/2$ and $c > 0$ imply that $[F(a) - F(c)] < 1/2$. The proof that $\overline{U}_x - U_x > 0$ is analogous. ■

We next prove that when comparing compromise sets under party competition $[v, k]$ and at-large selection $[w, c]$, it must be either that $v < w$ and $k > c$, or that $v > w$ and $k < c$. That is, *the compromise set is either enlarged or reduced at both extremes.*

Lemma 2 *When comparing the at-large selection compromise set* (w, c) *and the party competition compromise set* (v, k) *it must be the case that* $(w - v)(c - k) < 0$.

Proof. Consider at-large selection first. Substituting the continuation utility (3) into the Bellman equation (1) yields:

$$0 = -L_0(w) + 2\int_c^a (L_0(y)(1-\delta) + \delta L_0(w))\, dF(y) + 2\int_w^c L_0(w) dF(y) + 2\int_0^w L_0(y) dF(y).$$

With party competition, substituting the continuation utility (4) in the Bellman equation (6) yields:

$$0 = -L_0(v) + 2\int_k^a (L_0(y)(1-\delta) + \delta L_0(v))\, dF(y) + 2\int_v^k L_0(v) dF(y) + 2\int_0^v L_0(y) dF(y).$$

The two equations have the same form. Hence, letting $\phi(w, c)$ equal the relevant right-hand side, the result follows because:

$$\begin{aligned}
\frac{dw}{dc} &= -\frac{\phi_2(w,c)}{\phi_1(w,c)} = -\frac{-2(L_0(c)(1-\delta) + \delta L_0(w)) f(c) + 2L_0(w) f(c)}{-L_0'(w) + 2\int_c^a \delta L_0'(w) dF(y) - 2L_0(w) f(w) + 2\int_w^c L_0'(w) dF(y) + 2L_0(w) f(w)} \\
&= \left(-\frac{1}{L_0'(w)}\right) \frac{2(1-\delta)[L_0(w) - L_0(c)] f(c)}{2\delta[F(a) - F(c)] + 2[F(c) - F(w)] - 1} < 0 \text{ for } 0 \leq w \leq c,
\end{aligned}$$

and the inequality follows because $L_0'(w) < 0$, $L_0(w) > L_0(c)$, $f(c) > 0$ and $2[F(a) - F(c)]\delta + 2[F(c) - F(w)] - 1 < -2[F(a) - F(w)] - 1 < 0$, as $F(a) = 1$ and $F(w) > 1/2$ because $w > 0$. ■

Lemma 2 uses the stationarity of the electoral environment to prove that party selection cannot cause voters to set such more demanding standards for re-election that they move in by more than the increased willingness of extreme candidates to compromise. This result is central to our equilibrium and welfare comparison. Below we prove that the compromise interval is larger with party competition than with at-large selection. Combined with Lemma 2, this means both that with party competition more office-holders are willing to compromise, and that when they compromise, they take more centrist positions.

Proposition 1 *The comparison of the compromise set under party competition* $[v, k]$ *and at-large selection* $[w, c]$ *is such that* $v < w$ *and* $k > c$.

Proposition 1 has immediate turnover and hence welfare implications. We now show that the introduction of parties makes all voters better off. Unlike most welfare analyses in this literature we do not only consider the effect on the median voter's welfare. Our welfare concept is Pareto efficiency.

Theorem 1 *All voters prefer party competition to at-large selection of candidates.*

The intuition for this result is simple. All citizens like insurance because they are weakly risk averse, and they discount utilities. Parties provide ex-ante insurance against extremist policies, because (a) $k > c$, i.e., there is less expected turn-over, and (b) $v < w$, i.e., positions are more moderate over all.

Remark (General Parties) Our results do not depend on the stark left-right division of potential candidates into parties. In particular, our results extend to a more general notion of parties that preserves symmetry. Now, each party is identified by a distribution of candidates where G_B first-order stochastically dominates G_A and the associated densities, g_A and g_B, are symmetric in the sense that $g_A(x) = g_B(-x)$, for all x. When parties overlap, so the support of G_B is $[-\hat{a}, a]$, with $-\hat{a} < 0$, the median voter is still decisive and policy holders are still retained in office if and only if their adopted policy y belongs to a symmetric interval around the median policy of 0. How are strategic outcomes affected? Most transparently, suppose that the left-most member of the right-wing B party, $-\hat{a}$, is not so far to the left that she would cease to compromise to win re-election. Then, the effect of 'broadening' the ideological membership of each party enters through the reduced incentives of the marginal representative to compromise. In particular, right-wing members of party B are less willing to compromise because if they represent their own ideology, they may be replaced by a right-wing member of the opposing party. As a result, the equilibrium is described by a set $\{\hat{v}, \hat{k}\}$, where $k > \hat{k} > c$ and $v < \hat{v} < w$, i.e., the equilibrium moves toward that associated with at-large selection. Somewhat counter-intuitively, it follows that all voters are made better off if the 'ideological distance' between parties is raised so there is less overlap of ideologies (i.e., a rightward shift of G_B on $[-\hat{a}, 0]$).[9]

[9] If $\hat{a}$ is sufficiently large and close to a, the equilibrium characterization becomes more involved. Whether an office-holder compromises or not depends on her party identity. Consider a party A office holder with ideology $x > 0$. If ousted

3 More than two parties

3.1 Strategic voting

We now explore the possibility of introducing additional parties. Despite the restriction to stage-undominated stationary symmetric equilibrium, it is easy to appreciate that there are multiple equilibria. We pursue two different equilibrium selection approaches. We first consider equilibrium outcomes with strategic voters, in which the electorate always selects the candidate who corresponds to the Condorcet winner of the multiparty election. This equilibrium selection criterion corresponds to a hypothesis of hyper-rationality by voters: They coordinate so as not to waste votes on candidates who would lose the election in a two-party race. The equilibrium selection approach is motivated by the following observation. It follows from the single-crossing property of voters' preferences that an equilibrium with a Condorcet winner always exists, that in such an equilibrium the median voter is decisive, and that only the two most moderate parties capture any votes.

The simplest way to represent this equilibrium is in a setting with an even number of parties, symmetrically ordered with respect to the median voter. We label the two most moderate such parties, $M_A = [-a, 0)$, and $M_B = (0, a]$. A moment of reflection allows one to appreciate that our previous analysis extends. Only parties M_A and M_B place candidates in power, so one can safely ignore the other parties in the analysis. The median voter is willing to retain an incumbent if and only if her platform is sufficiently moderate: i.e., if $|p_x| \leq v$. An incumbent from party M_B chooses platform $p_x = x$ if $x \leq v$, compromises and takes $p_x = v$ if $x \in [v, k]$, and chooses $p_x = x$ to lose reelection to a challenger from M_A if $x \in [k, a]$. A symmetric characterization holds for party M_A.

The equilibrium is identical to that with two parties, save that the two competing parties are now more moderate. However, all voters need not benefit from this increased moderation. While the "direct" effect of restricting attention to moderate candidates (i.e., holding (v, k) fixed) clearly raises the welfare of all voters, restricting selection to moderate candidates could make incumbents

from office, she will be replaced from a challenger from party B, and such a candidate's ideology is likely to the right of the median voter. Hence, the party A office holder has a smaller incentive to compromise than under at-large selection. In sum, we must introduce thresholds $\underline{k}_i < 0 < \overline{k}_i$, for A, B. Party-i office holders compromise and adopt policy $-v$ if and only if $x \in [\underline{k}_i, -v]$, and adopt policy v if and only if $x \in [v, \underline{k}_i]$. Because g_A is symmetric to g_B around zero, and G_B dominates G_A, we obtain that $\underline{k}_A < \underline{k}_B$ and $\overline{k}_A < \overline{k}_B$. Because the equilibrium is symmetric across parties, we have $\underline{k}_A = -\overline{k}_B$, $\underline{k}_B = -\overline{k}_A$, and $-\underline{k}_A = \overline{k}_B$, $-\underline{k}_B = \overline{k}_A$. We conjecture that $v < w$, and that $\underline{k}_A < -c < \underline{k}_B$ and $\overline{k}_A < c < \overline{k}_B$.

less willing to compromise, raising the likelihood of turnover, and hurting extreme voters. We now provide sufficient conditions for this not to occur. These conditions immediately imply that all voters prefer nature to select more moderate candidates, which, in turn, implies that voters prefer a multi-party system to a two-party selection.

Proposition 2 *Suppose that F is uniform and the loss function is homogeneous, i.e.* $l(m|x-y|) = g(m)\, l(|x-y|)$, *where* $g(m) > 0$ *for all* $m \neq 0$. *Then the threshold function is linearly homogeneous* $w(a) = aw(1)$ *and* $c(a) = ac(1)$. *Hence the turnover probability is constant in* a, *whereas the welfare* $[\underline{U}_x + \overline{U}_x]/2$ *strictly decreases in* a *for every* x.

This result sheds light on how outcomes would be affected were parties to have an imperfect informational advantage over voters, so that parties can partially identify ideologies of members, distinguishing between moderates and extremists, and can choose the population from which they draw challenging candidates. In an extended version of our paper we show that parties select moderate candidates. It immediately follows that under the regularity conditions of Proposition 2, all voters would benefit from this "screening " in two-party competition. In this way, introducing 'party screening' by parties complements and does not substitute for the 'party-competition effect' that we analyze.

3.2 Sincere voting

One issue with this strategic voting is that it demands significant coordination by voters, and, in practice, third party candidates (e.g., Ross Perot, Ralph Nader) often win substantial vote shares. This leads us to investigate equilibrium outcomes where citizens vote sincerely for their most preferred candidates. That is, each citizen votes for the candidate he expects to deliver the highest utility given rational expectations over future electoral outcomes, i.e., over both the possibility that the candidate then wins re-election, and the ideology of a replacement if the candidate loses re-election bid.

Most provocatively, we modify our basic framework to allow for three parties, L, M and R. We normalize the support of ideologies to $[-1, 1]$. The left-wing party L draws its members from the interval $[-1, m)$, the middle party M draws its members from $[-m, m]$, and the right-wing party R

has members with ideologies in $(m, 1]$.[10] In the resulting three-way competition, an incumbent faces challengers drawn from each of the other parties. Thus, an incumbent from party L faces challengers drawn from both parties M and R. To obtain quantitative characterizations, we assume that voters have Euclidean preferences, $L_x(y) = -|x - y|$, and that ideologies are uniformly distributed.

While the median voter is no longer decisive, in equilibrium, the citizens who support a candidate are characterized by an interval of ideologies: with a uniform distribution over ideologies, the winner of the election is determined by the largest interval. As before, an incumbent may deviate from her optimal policy to gain reelection. However, unlike with two parties, an extreme party incumbent may implemenent a policy that is *more* extreme than her optimal policy, in order not to take away votes from the party M challenger and ensure M's victory over the opposing extreme party challenger.

We fully characterize the equilibrium configuration as a function of parameters. The nature of the equilibrium takes one of three general forms, depending on parameters, and two equilibria exist for limited regions of the parameter space. Rather than exhaustively detail the equations describing the equilibrium for each scenario, we describe their salient features. We then solve the model numerically and illustrate the key substantive features.

Equilibrium 1: Middle party incumbent loses re-election. When δ and m are small to moderate, then even the best incumbent from party M cannot win re-election, but an untried challenger from party M may win against incumbents from parties L or R who locate too extremely. We only characterize party R, because the characterization of party L is symmetric.

- There is a win set $W_r = [m, w_r]$. Party R incumbents with ideology x in the win set choose policy $p(x) = x$ and win reelection.
- There is a compromise set $C = (w_r, c_r)$, with $c_r > w_r$. Here, c_r is the ideal policy of an incumbent who is indifferent between compromising or implementing her best "losing" policy (either her ideal policy, in which case she is replaced by a challenger from party L, or the least more extreme policy that leads a challenger from party M to win).
- There can be a non-empty set (c_r, q_r). Party R incumbents with ideal policies in this set

[10] Characterizations are similar if, for example, four parties are symmetrically situated as in Section 3.1.

implement their ideal policies and are then replaced by challengers from L.

- There can be a non-empty pooling set $[q_r, p_r]$. Incumbents with ideal policies in this set implement p_r, and are then replaced by a challenger from M. The "pooling position" p_r is just extreme enough that the M party challenger wins enough votes at the expense of the incumbent to defeat the L party challenger. Here q_r is the ideal policy of an incumbent indifferent between implementing her own ideal policy and then being replaced by a challenger from L, and implementing p_r in order to be replaced by a challenger from party M.
- Finally there can be a non-empty set $(p_r, 1]$. Such incumbents are so extreme that they can implement their extreme ideal policies and win such a small vote share that they are replaced by a challenger from party M.

The threshold w_r is such that the vote share of a party R incumbent who locates at w_r equals the vote share of a party L challenger; the threshold p_r is such that if a party R incumbent locates at p_r, then the vote shares of the two challengers are equal. The thresholds c_r and q_r are pinned down by the indifference conditions discussed above. By recursive substitution, one can solve for the expected continuation payoff $U_x(M)$ to x from electing a party M challenger in terms of parameters, and then use this to calculate the continuation payoffs $U_x(R)$ and $U_x(L)$. This yields a system of equations that we solve numerically.

Equilibrium 2: Compromising middle party incumbents win re-election. When both δ is small and m is large enough, incumbents from party M who locate moderately can win re-election. This is because once the size of the middle party grows sufficiently large, parties L and R only have challenger candidates with relatively extreme ideologies that are less attractive to voters.

- The middle party M is divided in three sets. An incumbent with ideology x in the win set $W_m = [-w_m, w_m]$ adopts policy $p(x) = x$ and wins reelection. An incumbent with ideology $x \in (w_m, c_m]$, where $c_m > w_m$, adopts platform w_m and wins re-election. Finally, a party M member with $x > c_m$ adopts her ideal policy and then loses to a challenger from L. A symmetric characterization holds for incumbents in party M with ideologies $x < 0$.

- Party R comprises a (possibly empty) win set $W_r = [m, w_r]$ and a compromise set $C_r = (w_r, c_r]$, with $c_r > w_r > w_m$. Again, an incumbent with ideology x in the win set adopts policy $p(x) = x$ and win reelection, an incumbent with ideology in the compromise set adopts policy w_r and wins reelection, and an incumbent with ideology $x > c_r$ adopts policy $p(x) = x$ and loses reelection. An analogous symmetric partition characterizes party L.

This equilibrium has two sub-cases depending on the party affiliation of the challenger who defeats an extreme party incumbent. Suppose that the incumbent is from party R. As m increases, the affiliation of the winning challenger changes from L to M. For m intermediate, there are two equilibria, depending on the party of the winning challenger. These two equilibria reflect that an extremist incumbent is less willing to compromise if she will be replaced by a party M challenger when she loses. The reduced compromising expected from a party L challenger is unattractive to moderate voters; as a result the party M challenger wins, and party M incumbents who locate sufficiently moderately win re-election. If, instead, the winning challenger is from L, the consequences to a party R incumbent of losing are worse, so that c_r is larger as a result. In turn, the greater compromising expected from a party L challenger attracts moderate voters, so the party L challenger defeats the party M challenger, so that beliefs are consistent.

Equilibrium 3: All extremes compromise. When δ is large enough, all party R incumbents compromise and win reelection i.e., $c_r \geq 1$. Party M challengers would adopt their own ideologies and lose re-election if elected, but they are never elected. Remarkably, in equilibrium, whenever $\delta \geq 0.6$ and party M is the largest, i.e., $m \geq \frac{1}{3}$, all party R incumbents adopt a platform w_r that is more moderate than m, the most moderate party R ideology. That is, even the most extreme incumbent cares enough about the future that she compromises to win re-election, and due to the relatively extreme ideologies in the extremist parties, she must locate more moderately than m to win reelection. Equilibrium is pinned down by the voter $-b$ indifferent between party M and L challengers, and the voter a who is indifferent between a party R incumbent who locates at w_r and a party M challenger: in equilibrium, a party R incumbent locating at w_r wins just enough votes to defeat the party L challenger, $1-a = 1-b$.

3.2.1 Strategic Comparisons of Two and Three Party Systems

When $m = 0$, almost all candidates are members of two parties. Nonetheless, the strategic environment with three parties and $m = 0$ is very different from a two party competition between L and R. In particular, a middle party incumbent, with ideology 0 wins the votes of moderates in L and R, just not enough votes to win re-election. One can show that this implies that $w_r > v$ (with two parties, an incumbent must compromise by more to win), but $c_r - k > w_r - v$, (with two parties, fewer incumbents compromise).

To understand why $w_r > v$, first note that when $m = 0$, if the standards for re-election are the same in the two settings, i.e., $v = w_r$, then the compromise sets are also equal, $k = c_r$. When an extremist challenger compromises, the party M challenger stands no chance of winning. The issue is simply whether with three parties, the party M challenger steals more votes from a party R incumbent who locates at v or from the party L challenger.

To answer this, first note that the party M challenger is valued equally by voters a and $-a$. In contrast, voter a gains more from the party R incumbent who locates at v than $-a$ gains by voting for the party L challenger. In fact, the party L challenger could be extreme, in which case he is replaced by a distant party R challenger. Relative to voter 0 in a two party system, a's utility from the party R incumbent is higher by the amount a; while voter $-a$'s utility is not similarly increased by the party L challenger. It follows that the party M challenger takes away fewer votes from the party R incumbent. Hence, a party R incumbent need not moderate by as much to win re-election: w_r is raised past v.

To understand why $w_r > v$, suppose by contradiction that $c_r - w_r = k - v$, i.e., that incumbents are as willing to compromise to win re-election with three parties as two. But since $w_r > v$, the expected location of party L challenger is more extreme, and since $c_r > k$, a party L challenger is more likely to stay in office. Hence, losing an election is far worse with three parties: we must have $c_r - w_r > v - k$.

These findings mean that welfare comparisons of two and three party systems are not a priori clear. In particular, the median voter both prefers incumbents to compromise by more (i.e., w_r smaller), and for more extreme incumbents to compromise (i.e., c_r larger), so that the regimes have offsetting effects. But our numerical simulations show that almost all voters prefer the three party system if the center party is small enough. That is, the increased willingness to compromise ($c_r - k > w_r - v$) dominates the extremization of winning thresholds ($w_r > v$) from a welfare perspective.

3.2.2 Effect of size m of middle party

Figure 1 graphs how key endogenous variables vary with the size $m = 0, 0.05, 0.1, \ldots$ of the middle party M when the discount factor is $\delta = 0.3$. Figure 2 graphs the fraction of voters whose expected welfare is higher in a three party system than a two party system. It helps to decompose the discussion according to which equilibrium form characterizes outcomes.

Equilibrium 1: Middle party incumbent loses. For $m \leq 0.5$, there is an equilibrium where the middle party incumbent loses, and indeed for $m < 0.5$, this is the unique equilibrium outcome. One's intuition from a static setting might be that middle party candidates would be successful, especially if $m = 0$, in which case the party M candidate locates at the median. The locations of w_r and c_r reveal why the middle party is, in fact, so singularly unsuccessful.[11] In particular, a party M incumbent who locates at 0 must win the vote of a citizen with ideology 1/3 to win re-election. However, in equilibrium, most party R challengers are prepared to compromise ($c_r(m)$ is large), and restrict their location to platform w_r, which is much smaller than 2/3, to win re-election. The voter with ideology 1/3 is closer to platform w_r than to the median 0, and hence he votes for the party R challenger, so that even a party M incumbent who adopts platform 0 loses reelection. In particular, a party R incumbent can win by taking the same position that a party M incumbent would lose with, because the party R incumbent can count on all the votes of extreme citizens.

Figure 1 reveals that for $m \leq 0.5$, w_r grows larger as m increases—extreme party incumbents do not need to restrict location by as much to win. This is because the likely ideologies of challengers from the other two parties *both* grow more extreme. The increase in w_r has two direct consequences: (i) because $w_l = -w_r$ becomes more extreme, a party R incumbent is more willing to compromise (c_r rises with m) and (ii) eventually for $m \in [0.2, 0.4]$ the likely party L candidates locate so extremely that party R incumbents with extreme ideologies $x \in [q_r, 1]$ locate at $p_r >> 1$ in order to lose to a party M challenger rather than a party L challenger. That is, $q_r < 1$ for $m \in [0.2, 0.4]$, and these

[11]Most generally, if $\delta > 0.1$ (so that politicians have at least minimal incentives to compromise to win re-election), party M must be at least 33% larger than an extreme party for its best candidate, $x = 0$, to win re-election.

incumbents take positions that are more extreme than those held by *anyone* to avoid being replaced by a candidate with an opposing ideology.

Figure 2 reveals that when $m = 0$, the greater willingness of extreme party incumbents to compromise when there is a center party dominates from a welfare perspective: indeed, all voters prefer the three party system. However, as m rises, this fraction drops sharply because of the increase in w; and it drops further at $m = 0.2$ for extreme voters due to the extreme location of incumbents $x \in [q_r, 1]$ who locate at $p_r > 1$. Indeed, fluctuations in the fraction of voters who prefer the three party system for $m \in [0.2, 0.5]$ largely reflect the fluctuations in q_r (for example, the large spike up at $m = 0.45$ in the fraction preferring the three party system is because $q_r = 1$ for $m \geq 0.45$).

Equilibrium 2: Compromising middle party incumbents win. For $m \geq 0.5$, there is an equilibrium where the compromising party M incumbents win, and for $m \geq m^* \sim 0.52$, this is the unique equilibrium. When an incumbent from M who locates at 0 just wins re-election, this introduces a discontinuity in the strategic calculus of party M incumbents: If all incumbents from M lose re-election, then each of them adopts her own ideology; but if an incumbent who locates at 0 wins re-election, then a strictly positive measure of party M incumbents would compromise at 0 to win re-election. As a result, at $m \in [0.5, m^*]$, there are two equilibria, depending on whether or not a positive measure of party M incumbents compromise and gain reelection. [12] When this is the case, party R incumbents do not mind losing election as much as in the case where party M incumbents are never in power. As a result, c_r drops sharply and w_r grows more extreme, allowing a party M incumbent who locates at 0 to win re-election. Figure 2 reveals that this shift out in w_r and reduction in the willingness of party R incumbents to compromise sharply lowers the fraction of voters who prefer the three party system to the two-party system when $m = 0.55$.

For $m \in [0.5, 0.65]$, there is an equilibrium in which a losing party R incumbent is replaced by a party L challenger. For $m \in [0.65, 1)$, there is an equilibrium where a losing party R incumbent is replaced by a party M challenger. What supports the multiple equilibria for $m = 0.65$ is that an extreme party incumbent is less willing to compromise if she will be replaced by a party M challenger than if she will be replaced by a challenger from the opposing extreme party. This reduced willingness to compromise

[12] To avoid clutter, Figure 1 graphs the equilibrium outcome for $m = 0.5$ where no centrists win re-election.

raises the vote share of the party M challenger. This result manifests itself by the sharp drop in c_r at $m = 0.7$, where the sole equilibrium has a party M challenger winning — a party R incumbent is far less willing to compromise if she believes she will be replaced by a party M challenger than by a party L challenger. Figure 2 reveals that once m is this large, all voters prefer the three party system, simply because the party M incumbent wins the initial election, and is always willing to compromise so that extreme party challengers never win. This moderating effect keeps the policy chosen by the M incumbent moderate even in the limit case where the extreme parties are of negligible size.

3.2.3 Effect of discount factor δ

Figure 3 illustrates how equilibrium outcomes vary with δ when $m = \frac{1}{3}$ so that the three parties are the same size, and Figure 4 reveals the fraction of voters that prefer the three party system. For all values of m, every party M incumbent loses reelection. For $\delta \in (0, 0.3]$, the party R incumbent may locate extremely at p_r to lose reelection to a party M challenger (who then loses re-election). But for $\delta > 0.3$, an extreme party incumbent may only lose to the opposite extreme party challenger. As δ increases, we see that the winning threshold w_r decreases and the compromise threshold c_r increases to the extent that for $\delta \geq 0.6$ every member of party R restricts location to $w_r < m$ in order to win re-election. As a result, every extreme party incumbent wins reelection. Because extreme party politicians choose such moderate policies, any party M incumbent would lose reelection to an extreme party challenger. In fact, for $\delta \geq 0.7$, *no one* votes for a party M challenger. The median voter is then indifferent between a party R incumbent who locates at w_r and a party L challenger who compromises and locates at $w_l = -w_r$: All risk is eliminated for the median voter.

The contrast with static settings is stark. When $\delta = 0$, incumbents always adopt their ideologies. Then, for example, when $m = 0.5$, any party M incumbent with $x < 0.25$ wins re-election. The contrast with two party settings is as stark. Most transparently, even when $\delta \geq 0.6$, with two parties, equilibrium requires that some extreme ideologies not compromise. In fact, when selecting the challenger, the median voter might get a "lucky" draw of a moderate challenger $|x| < v$. To keep the median voter indifferent between an incumbent at v and a challenger, the median voter must face a risk that a replacement challenger will locate extremely.

Whether the two-party or three-party configuration party structure is better from a welfare perspective depends on the parameter values. This is because we have two counteracting effects. On the one hand, $w_r > v$ (with two parties, an incumbent must compromise by more to win), on the other hand $c_r - k > w_r - v$, (with two parties, fewer incumbents compromise). The first result makes the two-party configuration better from a welfare perspective, the second result makes the three-party configuration better.

When δ is close to zero, incumbents are only willing to compromise marginally, implying that the first effect dominates from a welfare perspective. The presence of a centrist party whose incumbent always loses reelection is detrimental to the electorate, as the winning platforms belong to the extreme parties and are more extreme than in a two-party setting. However, as δ grows, incumbents become more willing to compromise, and the second effect dominates from a welfare perspective. Hence, the fraction of voters who prefer three parties to two rises with the discount factor, and once the discount factor grows large all voters benefit from the greater willingness to compromise and the eliminated risk we discussed above.

4 Conclusion

We study the role of parties in a citizen-candidate repeated-elections model in which voters have incomplete information. By comparing selection of challengers from the population at large with two party competition, we identify a novel "party competition effect." Compared with "at large" selection of candidates, party selection makes office-holders more willing to avoid extreme ideological stands. Incumbents would like to minimize the chance of being replaced by a challenger from the opposite party. Politicians follow party discipline, even in absence of a party-controlled reward mechanism. Voters of all ideologies benefit from the party-competition effect. Our analysis thus provides a novel rationale for political parties: a key benefit of party competition is that it provides choice tied to clear ideological positions. The mere existence of a left-wing party discourages right-wing elected officials from drifting into extremism.

We explore how outcomes are affected by introducing additional parties. With strategic voting where only two parties receive votes in the equilibrium to an election, we find that the more moderate are the winning parties, the higher is the expected welfare of all citizens in the polity. With sincere voting in a three party system with a middle party, we show that for reasonable parameterizations, middle party incumbents are singularly unsuccessful: enough candidates from extreme parties compromise that they squeeze the vote share of a center party incumbent causing him to lose. Interestingly, the presence of a small, apparently ineffectual, middle party can be welfare enhancing: their competition induces extreme incumbents to moderate their platform by more to win re-election. Qualitatively, we find that when the middle party is small, most voters prefer the three party system, while the reverse is true for plausible intermediate sizes of the middle party. We also find that with equal-sized parties, the fraction of voters who prefer three parties to two rises with the discount factor, as extremists are more willing to compromise in a three party system than a two party system, and higher discount factors make incumbents more willing to compromise.

There are several extensions that one could consider to the analysis presented in this paper. An interesting theme is whether a given party configuration is stable, in the sense that citizens do not want to change party affiliations to improve their ex-ante welfare. We have conducted some analysis on this theme, using the following simple stability concept: A party configuration is stable if it is not the case that a small measure of citizens lying close to the boundary between two adjacent parties increase their ex-ante utility by switching party allegiance. In constructing our stability test, we allow marginal citizens from more than one party to simultaneously switch allegiance.

The multi-party configurations we study turn out to be unstable. Consider a multi-party configuration with even number of parties, symmetrically placed relative to the median voter. Suppose that in equilibrium, only the two most moderate parties receive votes, so that the Condorcet winner is selected in equilibrium. Then, in light of our Proposition 2, the smaller in measure are the most moderate parties, the higher is the welfare of all citizens. So, if a small measure of the most extreme citizens in the most moderate parties simultaneously switch allegiance to the adjacent parties, then the welfare of all citizens (including the switchers) increases.

A numerical investigation of our sincere voting framework with three parties, reveals that it, too, is not stable. For example, one can establish that for $\delta = 0.3$ for all $m < 0.5$, citizens with ideologies $[m - \epsilon, m]$ and $[-m, -m + \epsilon]$ on the boundary of party M, $\epsilon < m$ would be better off switching their allegiance to the extreme parties: the welfare of these marginal center party members would be higher if they joined the extreme parties. This result reflects that for $m < 0.5$, ideologies $[m-\epsilon, m]$ cannot win re-election as members of party M, but they can win as members of party R, and a challenger from an extreme party wins the initial election.

Finally, it would be worthwhile to extend our analysis along the lines of Bernhardt *et al.* (2004), to consider how term limits affect outcomes. In simple versions of our model with a term limit of two, and two parties, all sufficiently moderate voters continue to benefit from party competition, but, for example with Euclidean preferences, one can construct parameterizations in which party competition raises turnover of incumbents (i.e., lemma 2 may cease to hold as we have $v < w$, but $c > k$), in which case voters with extreme ideologies prefer at large selection of candidates.

Appendix: Proofs

Theorem A1. *There exist uniform bounds* $N'' < 0$ and $0 < N'''$ *such that if* $N'' < l'' \leq 0$ and $|l'''| < N'''$ *then there is a unique symmetric, stationary, stage-undominated equilibrium, and this equilibrium is determined by the thresholds* $0 < w < c < a$. Furthermore, for every voter $x \in [-a, a]$,

$$\frac{L_x(w) + L_x(-w)}{2} \quad \geq \quad U_x(w, c). \tag{9}$$

Proof. The proof of Theorem A1 (at large selection) follows the lemmas of the proof of Theorem A2 (two parties) below, with the necessary adjustments, such as replacing the lower retrospective set $\underline{R}_x$ by the at large retrospective set R_x, and the expected utility functions conditional on incumbent's party, $\underline{U}_x(\cdot)$ and $\overline{U}_x(\cdot)$, by the at large expected utility $U_x(\cdot)$. The last step of Lemma A10 is unnecessary for at large selection. Because the proof follows that of Theorem A2, we omit it. The proof is available from the authors upon request. ■

Theorem A2. *There exist uniform bounds* $M'' < 0$ *and* $0 < M'''$ *such that if* $M'' < l'' \leq 0$ and $|l'''| \leq M'''$ *then there is a unique symmetric, stationary, stage-undominated equilibrium, and this equilibrium is determined by the thresholds* $0 < v < k < a$. Furthermore, for every voter $x \in [-a, a]$,

$$\frac{L_x(v) + L_x(-v)}{2} \quad \geq \quad \left[\frac{\overline{U}_x(v,k) + \underline{U}_x(v,k)}{2}\right] \equiv U_x^*(v,k). \tag{10}$$

Proof. In any stationary, stage-undominated equilibrium the policy choice of elected officials is described by a win set W, a compromise set C and an extremist set E. Let $\overline{W}$, $\overline{C}$ and $\overline{E}$ be the win, compromise and extremist sets of the right wing party and $\underline{W}$, $\underline{C}$ and $\underline{E}$ be the win, compromise and extremist sets of the left wing party. In any symmetric equilibrium, $\overline{W} = -\underline{W}$, $\overline{C} = -\underline{C}$ and $\overline{E} = -\underline{E}$. Let $W = \overline{W} \cup \underline{W}$, $C = \overline{C} \cup \underline{C}$ and $E = \overline{E} \cup \underline{E}$. If a politician's ideology x belongs to the win set W, then she adopts as policy her own ideology, $p(x) = x$, and wins reelection. If x belongs to C, then she does not adopt her own ideology as policy — she compromises to policy $p(x) = \arg\min_{w \in W} |x - w|$, which is the least costly policy that allows her to win re-election. Define the compromise function $c(x) = \arg\min_{w \in W} |x - w|$. From symmetry, $-c(x) = c(-x)$. If the politician's ideology x belongs to E, then she implements her own policy $p(x) = x$ and loses reelection. Notice that $W \cup C \cup E = [-a, a]$ and $W \cap C \cap E = \emptyset$.

Define the lower retrospective set of voter x as the set of positions y implemented by an incumbent from x's party that x will re-elect over a random challenger from the opposite party, i.e., $\underline{R}_x = \{y | L_x(y) - \underline{U}_x(W, C) \geq 0\}$. Analogously, define the upper retrospective set of voter x as the set of positions y implemented by an incumbent from the opposite party that x will re-elect over a random challenger from x's party, i.e., $\overline{R}_x = \{y | L_x(y) - \overline{U}_x(W, C) \geq 0\}$. In the subsequent Lemmata, we show that the retrospective set of the median voter R_0 coincides with the win set W.

For any voter $x > 0$,

$$\begin{aligned}
\underline{U}_x(W, C) &= 2\int_{\underline{W}} L_x(y)dF(y) + 2\int_{\underline{C}} L_x(c(y))dF(y) + 2\int_{\underline{E}} [(1-\delta)L_x(y) + \delta\overline{U}_x(W, C)]dF(y), \\
\overline{U}_x(W, C) &= 2\int_{\overline{W}} L_x(y)dF(y) + 2\int_{\overline{C}} L_x(c(y))dF(y) + 2\int_{\overline{E}} [(1-\delta)L_x(y) + \delta\underline{U}_x(W, C)]dF(y).
\end{aligned}$$

Define the probability of turnover of an untried candidate discounted by δ as $\beta \equiv 2\delta \int_{\underline{E}} dF(y)$, and note that $\beta \in [0, 1)$. Substituting $\overline{U}_x$ into $\underline{U}_x$ and exploiting symmetry,

$$\begin{aligned}
\underline{U}_x(W, C) &= 2\int_{\underline{W}} [L_x(y) + \beta L_x(-y)]dF(y) + 2\int_{\underline{C}} [L_x(c(y)) + \beta L_x(-c(y))]dF(y) \\
&\quad +2\int_{\underline{E}} [(1-\delta)[L_x(y) + \beta L_x(-y)] + \delta\beta\underline{U}_x(W, C)]dF(y).
\end{aligned}$$

Subtracting $\beta^2\underline{U}_x(W, C)$ from both sides and dividing both sides by $(1-\beta)(1+\beta)$ yields

$$\begin{aligned}
\underline{U}_x(W, C) &= \frac{1}{1-\beta}\left\{2\int_{\underline{W}} \left[\frac{L_x(y) + \beta L_x(-y)}{1+\beta}\right] dF(y) + 2\int_{\underline{C}} \left[\frac{L_x(c(y)) + \beta L_x(-c(y))}{1+\beta}\right] dF(y)\right. \\
&\quad \left. +(1-\delta)2\int_{\underline{E}} \left[\frac{L_x(y) + \beta L_x(-y)}{1+\beta}\right] dF(y)\right\}. \qquad (11)
\end{aligned}$$

For any voter $x > 0$, the expected utility $\underline{U}_x$ from electing an untried candidate from the *opposing* party is a weighted average of the instantaneous utility of the policy chosen by an incumbent with *negative* ideology y and its symmetric *positive* counterpart $-y$, where more weight is on politicians with *negative* ideology, reflecting that positive positions are only taken *if* the left wing incumbent first loses. Moreover, politicians in the extremist set E are discounted by $(1-\delta)$ since they only stay in office for one period. Similarly,

$$\begin{aligned}\overline{U}_x(W,C) &= \frac{1}{1-\beta}\left\{2\int_{\overline{W}}\left[\frac{L_x(y)+\beta L_x(-y)}{1+\beta}\right]dF(y)+2\int_{\overline{C}}\left[\frac{L_x(c(y))+\beta L_x(-c(y))}{1+\beta}\right]dF(y)\right.\\ &\left.+(1-\delta)2\int_{\overline{E}}\left[\frac{L_x(y)+\beta L_x(-y)}{1+\beta}\right]dF(y)\right\},\end{aligned}$$

where more weight is given to positive ideologies y. Notice that

$$\frac{1}{1-\beta}\left\{2\int_{\overline{W}}dF(y)+2\int_{\overline{C}}dF(y)+(1-\delta)2\int_{\overline{E}}dF(y)\right\} = 1. \tag{12}$$

Lemma A1. *For any $x\in[-a,a]$, $0\in\underline{R}_x$, i.e. zero is in the lower retrospective set of all agents.*

Proof. Let $x\in[0,a]$. We need to show that $L_x(0)-\underline{U}_x(W,C)\geq 0$. Using equation (12), we substitute $L_x(0)$ into equation (11) to obtain

$$\begin{aligned}L_x(0)-\underline{U}_x(W,C) &= \frac{1}{1-\beta}\left\{2\int_{\underline{W}}\left[L_x(0)-\frac{L_x(y)+\beta L_x(-y)}{1+\beta}\right]dF(y)\right.\\ &+2\int_{\underline{C}}\left[L_x(0)-\frac{L_x(c(y))+\beta L_x(-c(y))}{1+\beta}\right]dF(y)\\ &\left.+(1-\delta)2\int_{\underline{E}}\left[L_x(0)-\frac{L_x(y)+\beta L_x(-y)}{1+\beta}\right]dF(y)\right\}.\end{aligned}$$

For all $y\leq 0\leq x$, concavity of L implies

$$(1+\beta)L_x(0)-L_x(y)-\beta L_x(-y) = l(x)-l(x-y)+\beta[l(x)-l(|x+y|)]\geq 0.$$

Therefore, the inequality holds and $0\in\underline{R}_x$ for all $x\in[0,a]$. An analogous argument establishes $0\in\underline{R}_x$ for all $x\in[-a,0]$. ■

Lemma A1 implies that $0\in W$. Hence, no incumbent with ideology $x>0$ will compromise to policy $y<0$ since she could compromise to 0 to win reelection. A symmetric argument applies to $x<0$.

Lemma A2. *The more moderate is a citizen's ideology, the higher is her expected utility from a challenger from the opposing party, i.e., $\frac{\partial\underline{U}_x(W,C)}{\partial x}\geq 0$, for $x\in[-a,0]$ and $\frac{\partial\underline{U}_x(W,C)}{\partial x}\leq 0$, for $x\in[0,a]$.*

Proof. Consider $x>0$. Then

$$\begin{aligned}\frac{\partial\underline{U}_x(W,C)}{\partial x} &= \frac{1}{1-\beta}\left\{2\int_{\underline{W}}\left[\frac{l'(x-y)+\beta l'(|x+y|)\frac{\partial(|x+y|)}{\partial x}}{1+\beta}\right]dF(y)\right.\\ &+2\int_{\underline{C}}\left[\frac{l'(x-c(y))+\beta l'(|x+c(y)|)\frac{\partial(|x+c(y)|)}{\partial x}}{1+\beta}\right]dF(y)\\ &\left.+(1-\delta)2\int_{\underline{E}}\left[\frac{l'(x-y)+\beta l'(|x+y|)\frac{\partial(|x+y|)}{\partial x}}{1+\beta}\right]dF(y)\right\}.\end{aligned}$$

Since $x > 0$ and $y \leq 0$, concavity of l implies that $0 \geq l'(|x+y|) \geq l'(x-y)$. Moreover,

$$\frac{\partial(|x+y|)}{\partial x} = \begin{cases} 1, & \text{if } x > -y; \\ -1, & \text{if } x < -y. \end{cases}$$

Therefore, for all $x > 0$ and $y \leq 0$, $0 \geq l'(x-y) + \beta l'(|x+y|) \frac{\partial(|x+y|)}{\partial x}$, which implies that $\frac{\partial \underline{U}_x(W,C)}{\partial x} \leq 0$. Analogously, we can show that for $x < 0$, $\frac{\partial \underline{U}_x(W,C)}{\partial x} \geq 0$. ■

Lemma A3. *The win set is connected, $W = [-v, v]$.*

Proof. From Lemma A1, $0 \in W$. Suppose that $y > 0 \in W$. We will show that all voters who vote for y also vote for any $y' \in [0, y]$. For voters $x \in [0, y']$ who vote for y, $L_x(y) \geq \underline{U}_x(W,C)$ and since $L_x(y') \geq L_x(y)$, they also vote for y'. For voters $x \leq 0$ who vote for y, $L_x(y) \geq \overline{U}_x(W,C)$ and since $L_x(y') \geq L_x(y)$, they also vote for y'. Every voter $x \geq y'$ also votes for y' since $L_x(y') \geq L_x(0) \geq \underline{U}_x(W,C)$ where the last inequality comes from Lemma A1. Thus, y' receives at least as many votes as y, and we have $y' \in W$. The same argument applies to any $y < 0 \in W$. ■

Lemma A4. *The retrospective set of the median voter R_0 is contained in the win set $W = [-v, v]$.*

Proof. Let $y \in R_0$ and $y \geq 0$. Every voter $x \geq y$ votes for y since $L_x(y) \geq L_x(0) \geq \underline{U}_x(W,C)$, where the last inequality comes from Lemma A1. Every voter $x \in [0, y]$ also votes for y since $L_x(y) \geq L_0(y) \geq \underline{U}_0(W,C) \geq \underline{U}_x(W,C)$, where the last inequality comes from Lemma A2. Therefore x wins at least half of the votes and belongs to the win set. The same argument applies to $y \leq 0$. ■

From Lemma A3, every incumbent with ideology $x \in [0, v]$ adopts his own policy and is re-elected, and those incumbents with ideology $x > v$ who compromise will adopt policy v since $v = \arg\min_{y \in W}(|x-y|)$. Similarly, incumbents $x < -v$ who compromise will adopt policy $-v$. An incumbent $x > v$ will compromise to v if and only if $L_x(v) - \delta \underline{U}_x(W,C) \geq 0$. For incumbent $x = v$, $L_v(v) - \delta \underline{U}_v(W,C) > 0$. Hence, the necessary condition for the compromise set $\overline{C}$ to be connected is that $L_x(v) - \delta \underline{U}_x(W,C)$ crosses zero only once for $x \in [v, a]$. A sufficient condition is that $L_x(v) - \delta \underline{U}_x(W,C)$ is concave in x for any $x \in [v, a]$.

The analysis above implies that we can rewrite equation (11) as

$$\begin{aligned} \underline{U}_x(W,C) \quad = \quad & \frac{1}{1-\beta} \left\{ 2 \int_0^v \left[\frac{L_x(-y) + \beta L_x(y)}{1+\beta} \right] dF(y) + 2 \int_{\overline{C}} \left[\frac{L_x(-v) + \beta L_x(v)}{1+\beta} \right] dF(y) \right. \\ & \left. + (1-\delta) 2 \int_{\overline{E}} \left[\frac{L_x(-y) + \beta L_x(y)}{1+\beta} \right] dF(y) \right\}. \end{aligned}$$

Lemma A5. *There exists a uniform bound $0 < M'''$, such that if $|l'''| \leq M'''$ then the compromise set C consists of two symmetric intervals around the win set, i.e. $C = [-k, -v] \cup [v, k]$.*

Proof. We need to show that $L_x(w) - \delta \underline{U}_x(W,C)$ is concave in x for any $x \in [v,a]$.

$$\begin{aligned} L_x(v) - \delta \underline{U}_x(W,C) &= \frac{1}{1-\beta}\left\{2\int_0^v \left[L_x(v) - \delta\frac{L_x(-y)+\beta L_x(y)}{1+\beta}\right] dF(y)\right. \\ &+2\int_{\overline{C}} \left[L_x(v) - \delta\frac{L_x(-v)+\beta L_x(v)}{1+\beta}\right] dF(y) \\ &\left.+(1-\delta)2\int_{\overline{E}} \left[L_x(v) - \delta\frac{L_x(-y)+\beta L_x(y)}{1+\beta}\right] dF(y)\right\}, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial^2}{\partial x^2}\left[L_x(v) - \delta \underline{U}_x(W,C)\right] &= \frac{1}{1-\beta}\left\{2\int_0^v \left[\frac{\partial^2}{\partial x^2}L_x(v) - \delta\frac{\frac{\partial^2}{\partial x^2}L_x(-y)+\beta \frac{\partial^2}{\partial x^2}L_x(y)}{1+\beta}\right] dF(y)\right. \\ &+2\int_{\overline{C}} \left[\frac{\partial^2}{\partial x^2}L_x(v) - \delta\frac{\frac{\partial^2}{\partial x^2}L_x(-v)+\beta \frac{\partial^2}{\partial x^2}L_x(v)}{1+\beta}\right] dF(y) \\ &\left.+(1-\delta)2\int_{\overline{E}} \left[\frac{\partial^2}{\partial x^2}L_x(v) - \delta\frac{\frac{\partial^2}{\partial x^2}L_x(-y)+\beta \frac{\partial^2}{\partial x^2}L_x(y)}{1+\beta}]\right] dF(y)\right\}. \end{aligned}$$

If $l''' = 0$, then $\frac{\partial^2}{\partial x^2}L_x(y)$ is a constant, $l'' \leq 0$, and $\frac{\partial^2}{\partial x^2}\left[L_x(v) - \delta \underline{U}_x(W,C)\right] = l''(1-\delta) \leq 0$. Hence, there is a uniform bound $0 < M'''$ such that if $|l'''| \leq M'''$ then $L_x(v) - \delta \underline{U}_x(W,C)$ is concave.

The condition requires that the risk aversion of voters cannot increase too quickly in x (the second derivative cannot fall too quickly), else the compromise sets could be disconnected — some incumbents prefer to lose the election rather than compromise, but more extreme incumbents may be so risk averse that they prefer to compromise. In particular, these conditions are satisfied by Euclidean and quadratic loss functions. ■

Lemma A6. *If $L_x(0) - \overline{U}_x(W,C)$ does not increase in x for any $x > 0$, then the win set W is contained in the retrospective set of the median voter, R_0.*

Proof. First notice that if $L_x(0) - \overline{U}_x(W,C)$ does not increase in x for any $x > 0$, then $L_x(y) - \overline{U}_x(W,C)$ also does not increase in x for any $x > 0$ and $y < 0$, since $L_x(y)$ decreases at least as fast as $L_x(0)$ from concavity. Assume that is the case, we will show that if $y \notin R_0$, then $y \notin W$. Consider $y \notin R_0$ and $y < 0$. This implies that $0 > L_0(y) - \overline{U}_0(W,C)$ and for every voter $x > 0$, $L_0(y) - \overline{U}_0(W,C) \geq L_x(y) - \overline{U}_x(W,C)$ implies $\overline{U}_x(W,C) > L_x(y)$. All voters with ideology $x \in [0,a]$ vote for the challenger and the incumbent will not be reelected, therefore $y \notin W$. Analogously we can show that any $y \notin R_0$ and $y > 0$ will not belong to the win set. ■

Lemma A7. *There exists a uniform lower bound $\bar{M}'' < 0$ such that if $\bar{M}'' \leq l'' \leq 0$, then $L_x(0) - \overline{U}_x(W,C)$ does not increase in x for any $x > 0$.*

Proof.

$$\begin{aligned} L_x(0) - \overline{U}_x(W,C) &= \frac{1}{1-\beta}\left\{2\int_0^v \left[L_x(0) - \frac{L_x(y)+\beta L_x(-y)}{1+\beta}\right] dF(y)\right. \\ &\quad +2\int_v^k \left[L_x(0) - \frac{L_x(v)+\beta L_x(-v)}{1+\beta}\right] dF(y) \\ &\quad \left. +(1-\delta)2\int_k^a \left[L_x(0) - \frac{L_x(y)+\beta L_x(-y)}{1+\beta}\right] dF(y)\right\}, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial}{\partial x}\left[L_x(0) - \overline{U}_x(W,C)\right] &= \frac{1}{1-\beta}\left\{2\int_0^v \left[\frac{\partial}{\partial x}L_x(0) - \frac{\frac{\partial}{\partial x}L_x(y)+\beta\frac{\partial}{\partial x}L_x(-y)}{1+\beta}\right] dF(y)\right. \\ &\quad +2\int_v^k \left[\frac{\partial}{\partial x}L_x(0) - \frac{\frac{\partial}{\partial x}L_x(v)+\beta\frac{\partial}{\partial x}L_x(-v)}{1+\beta}\right] dF(y) \\ &\quad \left. +(1-\delta)2\int_k^a \left[\frac{\partial}{\partial x}L_x(0) - \frac{\frac{\partial}{\partial x}L_x(y)+\beta\frac{\partial}{\partial x}L_x(-y)}{1+\beta}\right] dF(y)\right\}. \end{aligned}$$

If $l'' = 0$, this quantity is indeed negative, because $\left|\frac{\partial}{\partial x}L_x(y)\right|$ is constant in x, y (negative for $y < x$ and positive for $y > x$). This implies that there is a uniform lower bound $\bar{M}'' < 0$ such that if $\bar{M}'' \leq l'' \leq 0$ then $L_x(0) - \overline{U}_x(W,C)$ decreases in x.

Note that $\frac{\partial}{\partial x}\left[L_x(0) - \overline{U}_x(W,C)\right] \leq 0$ holds for the Euclidean and quadratic loss functions. ■

Therefore, the median voter is decisive and for any equilibrium (v,k),

$$L_0(v) = \overline{U}_0(v,k) = \underline{U}_0(v,k) = U_0^*(v,k). \tag{13}$$

Lemma A8. *Every equilibrium (v,k) is interior,* $0 < v < k < a$.

Proof. By contradiction, let $v = 0$. Using equation (13), $L_0(v) = 0 = U_0^*(v,k)$, which implies that $k = a$, i.e., every incumbent compromises to 0, otherwise $U_0^*(v,k) < 0$. However, if every incumbent compromises to 0, then for every voter $|x| > 0$ we have $\underline{U}_0(v,k) = L_x(0) < 0$. Hence, $L_x(0) > \delta\underline{U}_x(v,k)$ and every incumbent $|x| > 0$ strictly prefers to adopt her own ideology rather than compromise, a contradiction.

Hence, it must be the case that $v > 0$. By contradiction, let $v > 0$ and $k = a$. Every incumbent $x \geq v$ adopts policy v and every incumbent $x \leq -v$ adopts policy $-v$. Both cases yield utility $L_0(v)$ to the median voter. Every incumbent $x \in (-v,v)$ adopts as policy her own moderate ideology x, which yields utility $L_0(x) > L_0(v)$ to the median voter. Hence, $U_0^*(v,k) > L_0(v)$, which contradicts (13).

Finally, let $v > 0$ and $k = v$. In this case, for every incumbent x we have $\underline{U}_x(v,k) < 0$. We can find $\epsilon > 0$ small enough so that, for incumbent $\tilde{x} \equiv v + \epsilon$, $L_{\tilde{x}}(v) > \delta\underline{U}_{\tilde{x}}(v,k)$. Hence, incumbent $\tilde{x} > v$ strictly prefers to compromise to v, a contradiction that completes the proof. ■

Therefore, the median voter is decisive and any equilibrium is fully characterized by a pair (v, k), $0 < v < k < a$, that satisfies the following equations

$$\begin{aligned} L_0(v) &= \overline{U}_0(v,k) = \underline{U}_0(v,k) = U_0^*(v,k) \\ L_k(v) &= \delta \underline{U}_k(v,k). \end{aligned}$$

Lemma A9. *There exists a lower bound $\tilde{M}'' < 0$, such that if $\tilde{M}'' \leq l'' \leq 0$ then for any equilibrium (v, k) and any voter $x \in [-a, a]$,*

$$\frac{L_x(v) + L_x(-v)}{2} \geq \left[\frac{\overline{U}_x(v,k) + \underline{U}_x(v,k)}{2}\right] \equiv U_x^*(v,k). \tag{14}$$

Proof. Let (v, k) be an equilibrium. From median voter indifference, we have

$$\frac{L_0(v) + L_0(-v)}{2} = U_0^*(v,k). \tag{15}$$

To show that (14) holds we use equation (15) and show that

$$\frac{L_x(v) + L_x(-v) - L_0(v) - L_0(-v)}{2} \geq U_x^*(v,k) - U_0^*(v,k). \tag{16}$$

Similar to Lemma A2, the expected utility $U_x^*(v,k)$ decreases as voter x becomes more extreme. In particular,

$$\begin{aligned} U_x^*(v,k) - U_0^*(v,k) &= \frac{1}{1-\beta}\left\{2\int_0^v \left[\frac{L_x(y) + L_x(-y) - L_0(y) - L_0(-y)}{2}\right] dF(y)\right. \\ &\quad +2\int_v^k \left[\frac{L_x(v) + L_x(-v) - L_0(v) - L_0(-v)}{2}\right] dF(y) \\ &\quad \left.+ (1-\delta)2\int_k^a \left[\frac{L_x(y) + L_x(-y) - L_0(y) - L_0(-y)}{2}\right] dF(y)\right\} \\ &\leq 0, \end{aligned} \tag{17}$$

where the last inequality comes from the concavity of L. Similar to Lemma A1, any voter x prefers an incumbent who adopts policy 0 to an untried challenger drawn from at large. Using concavity of L and equation (12),

$$\begin{aligned} L_x(0) - U_x^*(v,k) &= \frac{1}{1-\beta}\left\{2\int_0^v \left[L_x(0) - \frac{L_x(y) + L_x(-y)}{2}\right] dF(y)\right. \\ &\quad +2\int_v^k \left[L_x(0) - \frac{L_x(v) + L_x(-v)}{2}\right] dF(y) \\ &\quad \left.+ (1-\delta)2\int_k^a \left[L_x(0) - \frac{L_x(y) + L_x(-y)}{2}\right] dF(y)\right\} \\ &\geq 0. \end{aligned} \tag{18}$$

If $l'' = 0$ (Euclidean loss function) the first derivative is a constant. For every x such that $|x| \geq v$, $L_x(v) + L_x(-v) = 2L_x(0)$ and the LHS of equation (16) becomes $L_x(0) - L_0(v)$. From equation (18), $L_x(0) \geq U_x^*(v,k)$, and from median voter's indifference condition, $L_0(v) = U_0^*(v,k)$. Hence, equation (16) holds. For every x such that $|x| < v$, $L_x(v) + L_x(-v) = L_0(v) + L_0(-v)$ and the LHS of equation (16) becomes 0, while the RHS is non-positive from equation (17), concluding the proof.

This implies that there is a lower bound $\tilde{M}'' < 0$ such that if $\tilde{M}'' \leq l'' \leq 0$ then equation (16) holds. In particular, equation (16) holds for a quadratic loss function: the LHS becomes $-x^2 - v^2 + v^2$ and the RHS becomes $-x^2 - v^2 + v^2$. ■

Let $M'' = \max\left\{\bar{M}'', \tilde{M}''\right\} < 0$ and assume that the conditions $M'' \leq l'' \leq 0$ and $|l'''| \leq M'''$ are satisfied. We conclude the proof showing that the solution is unique. In particular, there exists a $Z > 2$ such that our results hold for any power function $l(x) = -|x|^z$ with $z \in [1, Z]$, which includes both Euclidean and quadratic loss functions.

Lemma A10. *The system*

$$\begin{aligned} L_0(v) &= \overline{U}_0(v,k) = \underline{U}_0(v,k) = U_0^*(v,k) \\ L_k(v) &= \delta \underline{U}_k(v,k) \end{aligned}$$

has a unique solution (v,k).

Proof. Existence follows from a fixed point argument on the expected utility of the median voter. $U_0^*(v,k)$ is at least $L_0(a)$ (no incumbent compromises) and at most zero (all incumbents compromise to the median voter's position). For any x in the compact set $[L_0(a), 0]$, let $v \in [0,a]$ be the unique value that solves $L_0(v) = x$, and k be the maximum $k \in [v,a]$ such that $L_k(v) \geq \delta \underline{U}_k(v,k)$. This pair (v,k) yields a unique $x' = U_0^*(v,k) \in [L_0(a), 0]$. Notice that v is continuous on x, k is continuous on v, and x' is continuous on both v and k. Therefore, the two equilibrium conditions define a continuous function that maps $[L_0(a), 0]$ into itself, and a fixed point x^* exists.

To prove uniqueness, by contradiction, suppose (v,k) and (v',k') are both equilibria. Without loss of generality, let $v' \geq v$.

Case 1: Suppose $k' \geq k$ such that $(v,k) \neq (v',k')$. Since both are equilibria, from median voter condition

$$\begin{aligned}
L_0(v') - L_0(v) &= U_0^*(v', k') - U_0^*(v, k) \\
&= 2\int_v^{v'} [L_0(y) - L_0(v)]dF(y) + 2\int_{v'}^{k} [L_0(v') - L_0(v)]dF(y) \\
&\quad +2\int_k^{k'} [L_0(v') - (1-\delta)L_0(y) - \delta U_0^*(v,k)]dF(y) + 2\int_{k'}^{a} \delta[^*U_0(v', k') - U_0^*(v, k)]dF(y) \\
&> 2\int_v^{v'} [L_0(v') - L_0(v)]dF(y) + 2\int_{v'}^{k} [L_0(v') - L_0(v)]dF(y) \\
&\quad +2\int_k^{k'} [L_0(v') - (1-\delta)L_0(v) - \delta L_0(v)]dF(y) \\
&\quad +2\int_{k'}^{a} \delta[L_0(v') - L_0(v)]dF(y)[L_0(v') - L_0(v)]\left[2\int_v^{k'} dF(y) + 2\delta\int_{k'}^{a} dF(y)\right],
\end{aligned}$$

a contradiction since $L_0(v') - L_0(v) \leq 0$ and $1 > 2\int_v^{k'} dF(y) + 2\delta \int_{k'}^{a} dF(y)$.

Case 2: Suppose $k' < k$. We will show that, if (v', k') is an equilibrium, then for any pair $(v, k) \neq (v', k')$ such that $v \leq v'$ and $k' \leq k$ we have $\underline{U}_{k'}(v', k') < \underline{U}_{k'}(v, k)$, that is, the change to more moderate cutoffs (v, k) increases the expected utility of incumbent k', and since compromising becomes more costly with a lower v, this implies less compromising, a contradiction to $k' < k$.

We first prove that $U_{k'}^*(v', k') < U_{k'}^*(v, k)$.

$$\begin{aligned}
U_{k'}^*(v', k') - U_{k'}^*(v, k) &= \int_v^{v'} [L_{k'}(y) + L_{k'}(-y) - L_{k'}(v) - L_{k'}(-v)]dF(y) \\
&\quad + \int_{v'}^{k'} [L_{k'}(v') + L_{k'}(-v') - L_{k'}(v) - L_{k'}(-v)]dF(y) \\
&\quad + \int_{k'}^{k} [(1-\delta)(L_{k'}(y) + L_{k'}(-y)) + 2\delta U_{k'}^*(v', k') - L_{k'}(v) - L_{k'}(-v)]dF(y) \\
&\quad +2\delta \int_k^a [U_{k'}^*(v', k') - U_{k'}^*(v, k)]dF(y).
\end{aligned}$$

Rewriting,

$$\begin{aligned}
&[U_{k'}^*(v', k') - U_{k'}^*(v, k)][1 - 2\delta \int_k^a dF(y)] \\
= &\int_v^{v'} [L_{k'}(y) + L_{k'}(-y) - L_{k'}(v) - L_{k'}(-v)]dF(y) \\
&+ \int_{v'}^{k'} [L_{k'}(v') + L_{k'}(-v') - L_{k'}(v) - L_{k'}(-v)]dF(y) \\
&+ \int_{k'}^{k} [(1-\delta)(L_{k'}(y) + L_{k'}(-y)) + 2\delta U_{k'}^*(v', k') - L_{k'}(v) - L_{k'}(-v)]dF(y).
\end{aligned}$$

The first and second integrals are negative from the concavity of the loss function. So it is sufficient to show that

$$\int_{k'}^{k} [(1-\delta)(L_{k'}(y) + L_{k'}(-y)) + 2\delta U^*_{k'}(v', k') - L_{k'}(v) - L_{k'}(-v)]dF(y) \leq 0,$$

or simply

$$L_{k'}(v) + L_{k'}(-v) \quad \geq \quad (1-\delta)(L_{k'}(k') + L_{k'}(-k')) + 2\delta U^*_{k'}(v', k'). \tag{19}$$

Rewriting equation (19), using $L_{k'}(k') = 0$ yields

$$L_{k'}(v) + L_{k'}(-v) - L_{k'}(k') - L_{k'}(-k') \geq \delta[2U^*_{k'}(v', k') - L_{k'}(-k')].$$

The LHS is positive from concavity. If the RHS is negative, this concludes the first step of the proof. If the RHS is positive, it is sufficient to show that

$$\frac{L_{k'}(v) + L_{k'}(-v)}{2} \geq U^*_{k'}(v', k').$$

From the concavity of the loss function, $\frac{L_{k'}(v)+L_{k'}(-v)}{2} \geq \frac{L_{k'}(v')+L_{k'}(-v')}{2}$, and since (v', k') is an equilibrium, Lemma A9 implies $\frac{L_{k'}(v')+L_{k'}(-v')}{2} \geq U^*_{k'}(v', k')$, concluding this step of the proof.

As the final step, by contradiction, suppose $\underline{U}_{k'}(v', k') > \underline{U}_{k'}(v, k)$, which implies that $\overline{U}_{k'}(v', k') < \overline{U}_{k'}(v, k)$ since $U^*_{k'}(v', k') < U^*_{k'}(v, k)$. Then

$$\begin{aligned}
\underline{U}_{k'}(v', k') - \underline{U}_{k'}(v, k) \quad &= \quad 2\int_{v}^{v'} [L_{k'}(-y) - L_{k'}(-v)]dF(y) + 2\int_{v'}^{k'} [L_{k'}(-v') - L_{k'}(-v)]dF(y) \\
&\quad +2\int_{k'}^{k} [(1-\delta)L_{k'}(-y) + \delta\overline{U}_{k'}(v', k') - L_{k'}(-v)]dF(y) \\
&\quad +\delta 2\int_{k}^{a} [\overline{U}_{k'}(v', k') - \overline{U}_{k'}(v, k)]dF(y).
\end{aligned}$$

The first, second and fourth integrals are negative. Hence, it suffices to show that

$$2\int_{k'}^{k} [(1-\delta)L_{k'}(-y) + \delta\overline{U}_{k'}(v', k') - L_{k'}(-v)]dF(y) \quad \leq \quad 0.$$

Since

$$\int_{k'}^{k} [(1-\delta)L_{k'}(-y) + \delta\overline{U}_{k'}(v', k') - L_{k'}(-v)]dF(y) \quad \leq \quad \int_{k'}^{k} [(1-\delta)L_{k'}(-k') + \delta\overline{U}_{k'}(v', k') - L_{k'}(-v)]dF(y),$$

it is sufficient to show

$$(1-\delta)L_{k'}(-k') + \delta\overline{U}_{k'}(v', k') - L_{k'}(-v) \quad \leq \quad 0.$$

From indifference condition of incumbent k', we have $\delta\underline{U}_{k'}(v', k') - L_{k'}(v') = 0$. Hence, $\delta\underline{U}_{k'}(v', k') - L_{k'}(v) > 0$. Thus,

$$\begin{aligned}
&\quad (1-\delta)L_{k'}(-k') + \delta\overline{U}_{k'}(v', k') - L_{k'}(-v) \\
&\leq \quad (1-\delta)L_{k'}(-k') + \delta\overline{U}_{k'}(v', k') + \delta\underline{U}_{k'}(v', k') - L_{k'}(-v) - L_{k'}(v) \\
&= \quad (1-\delta)L_{k'}(-k') + 2\delta U^*_{k'}(v', k') - L_{k'}(-v) - L_{k'}(v).
\end{aligned}$$

Hence, it suffices to show that $(1-\delta)L_{k'}(-k')+2\delta U^*_{k'}(v',k')-L_{k'}(-v)-L_{k'}(v)\leq 0$, or equivalently that

$$L_{k'}(v)+L_{k'}(-v)-L_{k'}(-k')\geq \delta[2U^*_{k'}(v',k')-L_{k'}(-k')].$$

But we have already established that this final inequality holds. ■ ■

Proof of Proposition 1. Case 1) Suppose $w=v$ and $c=k$. From the equilibrium condition, $L_c(w)=\delta U_c(w,c)$, and from Lemma 1, we have $\delta U_c(w,c)>\delta \underline{U}_c(v,k)$. Since from equilibrium condition $\delta \underline{U}_c(v,k)=L_c(v)$, we have $L_c(w)>L_c(v)$, a contradiction since $w=v$.

Case 2) Suppose $w<v$ and $k<c$. Following the proof of case 2 on Lemma A10, since (v,k) is an equilibrium, incumbent k has a higher expected utility under any more moderate cutoffs (w,c) where $w<v$ and $k<c$, which implies $\delta \underline{U}_k(w,c)>\delta \underline{U}_k(v,k)$. From Lemma 1, $\delta U_k(w,c)>\delta \underline{U}_k(w,c)$. Since $w<k<c$, incumbent k is compromising to w in the at-large economy, thus equilibrium indifference condition implies $L_k(w)\geq \delta U_k(w,c)$. Finally, $L_k(v)>L_k(w)$, so that $L_k(v)>\delta \underline{U}_k(v,k)$, a contradiction to the equilibrium indifference condition, $L_k(v)=\delta \underline{U}_k(v,k)$.

From lemma 2, this implies that the equilibrium must such that $v<w$ and $c<k$. ■

Proof of Theorem 1. From Proposition 1, $0<v<w<c<k$. We must compare at-large ex-ante welfare $U_x(c,w)$, with party-competition ex-ante welfare,

$$U^*_x(k,v)=\frac{1}{2}[\underline{U}_x+\overline{U}_x]=\int_{-a}^{-k}\left(L_x(y)(1-\delta)+\delta\overline{U}_x\right)dF(y)+\int_{-k}^{-v}L_x(-v)dF(y)$$
$$+\int_{-v}^{0}L_x(y)dF(y)+\int_{0}^{v}L_x(y)dF(y)+\int_{v}^{k}L_x(v)dF(y)+\int_{k}^{a}\left(L_x(y)(1-\delta)+\delta\underline{U}_x\right)dF(y).$$

Simple algebraic manipulations give:

$$\begin{aligned}U^*_x(k,v)-U_x(c,w) &= \frac{1}{1-2\delta\int_k^a dF(y)}\left\{\int_v^w\{L_x(v)+L_x(-v)-[L_x(y)+L_x(-y)]\}dF(y)\right.\\ &\quad+\int_w^c\{L_x(v)+L_x(-v)-[L_x(w)+L_x(-w)]\}dF(y)\\ &\quad\left.+\int_c^k\{L_x(v)+L_x(-v)-[(1-\delta)(L_x(y)+L_x(-y))+2\delta U_x(c,w)]\}dF(y)\right\}.\end{aligned}$$

We need to show that $U^*_x(k,v)-U_x(c,w)>0$. From the concavity of the loss function, the first and second integrals are positive. Hence, it suffices to show that the third integral is not negative, or equivalently that

$$\int_c^k\{L_x(v)+L_x(-v)-[L_x(y)+L_x(-y)]\}dF(y)\geq\delta\int_c^k\{2U_x(c,w)-[L_x(y)+L_x(-y)]\}dF(y).$$

From concavity, the LHS is positive. If the RHS is negative, we are done. If not, it is sufficient to show that $L_x(v) + L_x(-v) \geq 2U_x(c,w)$. From concavity, $L_x(v) + L_x(-v) \geq L_x(w) + L_x(-w)$, and from Theorem A1, $L_x(w) + L_x(-w) \geq 2U_x(c,w)$. This concludes the proof. ■

Proof of Proposition 2. Let (v,k) be the unique equilibrium when each of the two most moderate parties has size a. The equilibrium is characterized by equations

$$L_0(v) = \overline{U}_0(v,k) = \underline{U}_0(v,k), \tag{20}$$
$$L_k(v) = \delta \underline{U}_k(v,k). \tag{21}$$

Suppose F is uniform and the loss function is homogeneous. For any $m > 0$, let $\tilde{a} = ma$ be the size of the most moderate party. Let $\tilde{v} = mv$ and $\tilde{k} = mk$. To show that $(\tilde{v}, \tilde{k})$ is the corresponding equilibrium, we need to prove that the following equations hold:

$$L_0(\tilde{v}) = \overline{U}_0\left(\tilde{v}, \tilde{k}\right) = \underline{U}_0\left(\tilde{v}, \tilde{k}\right), \tag{22}$$
$$L_{\tilde{k}}(\tilde{v}) = \delta \underline{U}_{\tilde{k}}\left(\tilde{v}, \tilde{k}\right). \tag{23}$$

Since L is homogeneous, for any ideology x, policy y and $m > 0$, $L_{mx}(my) = l(|mx - my|) = l(m|x-y|) = g(m)l(|x-y|) = g(m)L_x(y)$. Therefore $L_0(\tilde{v}) = g(m)L_0(v)$ and $L_{\tilde{k}}(\tilde{v}) = g(m)L_k(v)$. For any $x \geq 0$, since F is uniform

$$\underline{U}_{mx}(\tilde{v},\tilde{k}) = \frac{1}{1-\beta}\left\{2\int_0^{mv} \frac{L_{mx}(-y) + \beta L_{mx}(y)}{1+\beta} d\frac{y}{2ma} + 2\int_{mv}^{mk} \frac{L_{mx}(-mv) + \beta L_{mx}(mv)}{1+\beta} d\frac{y}{2ma} \right.$$
$$\left. +2(1-\delta)\int_{mk}^{ma} \frac{L_{mx}(-y) + \beta L_{mx}(y)}{1+\beta} d\frac{y}{2ma}\right\}.$$

Recall that $\beta = 2\delta \int_{\tilde{k}}^{\tilde{a}} dF(y)$. Since F is uniform, $2\delta \int_{mk}^{ma} d\frac{y}{2ma} = 2\delta \int_k^a d\frac{y}{2a}$. The term inside the second integral is a constant in y, therefore $2\int_{mv}^{mk} \frac{L_{mx}(-mv)+\beta L_{mx}(mv)}{1+\beta} d\frac{y}{2ma} = 2\int_v^k \frac{L_{mx}(-mv)+\beta L_{mx}(mv)}{1+\beta} d\frac{y}{2a}$. By changing variables in the first and third integrals, we obtain,

$$\underline{U}_{mx}(\tilde{v},\tilde{k}) = \frac{1}{1-\beta}\left\{2\int_0^{v} \frac{L_{mx}(-my) + \beta L_{mx}(my)}{1+\beta} d\frac{y}{2a} + 2\int_{v}^{k} \frac{L_{mx}(-mv) + \beta L_{mx}(mv)}{1+\beta} d\frac{y}{2a} \right.$$
$$\left. +2(1-\delta)\int_{k}^{a} \frac{L_{mx}(-my) + \beta L_{mx}(my)}{1+\beta} d\frac{y}{2a}\right\}. \tag{24}$$

Since L is homogeneous,

$$\underline{U}_{mx}(\tilde{v},\tilde{k}) = \frac{1}{1-\beta}\left\{g(m)2\int_0^{v} \frac{L_{x}(-y) + \beta L_{x}(y)}{1+\beta} d\frac{y}{2a} + g(m)2\int_{v}^{k} \frac{L_{x}(-v) + \beta L_{x}(v)}{1+\beta} d\frac{y}{2a} \right.$$
$$\left. +g(m)2(1-\delta)\int_{k}^{a} \frac{L_{x}(-y) + \beta L_{x}(y)}{1+\beta} d\frac{y}{2a}\right\},$$

which simplifies to $\underline{U}_{mx}(mv,mk) = g(m)\underline{U}_x(v,k)$. Similarly, $\overline{U}_{mx}(mv,mk) = g(m)\overline{U}_x(v,k)$. Therefore, $\underline{U}_0(\tilde{v},\tilde{k}) = g(m)\underline{U}_0(v,k)$ and $\underline{U}_{\tilde{k}}(\tilde{v},\tilde{k}) = g(m)\underline{U}_k(v,k)$. Hence, $(\tilde{v},\tilde{k})$ is the new equilibrium.

Finally, rewrite equation (24) for any ideology $x > 0$,

$$\begin{aligned}\underline{U}_x(mv, mk) \quad = \quad & \frac{1}{1-\beta}\left\{2\int_0^v \frac{L_x(-my)+\beta L_x(my)}{1+\beta} d\frac{y}{2a} + 2\int_v^k \frac{L_x(-mv)+\beta L_x(mv)}{1+\beta} d\frac{y}{2a}\right. \\ & \left. +2(1-\delta)\int_k^a \frac{L_x(-my)+\beta L_x(my)}{1+\beta} d\frac{y}{2a}\right\}.\end{aligned}$$

Similarly, write $\overline{U}_x(mv, mk)$ and combine the two equations to find the expected utility of voter x,

$$\begin{aligned}\frac{\overline{U}_x(mv, mk)+\underline{U}_x(mv, mk)}{2} \quad = \quad & \frac{1}{1-\beta}\left\{2\int_0^v [l(|x-my|)+l(|x+my|)]d\frac{y}{2a}\right. \\ & +2\int_v^k [l(|x-mv|)+l(|x+mv|)]d\frac{y}{2a} \\ & \left. +2(1-\delta)\int_k^a [l(|x-my|)+l(|x+my|)]d\frac{y}{2a}\right\}.\end{aligned}$$

As we explained above, β does not vary with m. From the concavity of the loss function it follows that $\partial\left[l(|x-my|)+l(|x+my|)\right]/\partial m < 0$ and hence $\partial[\frac{\overline{U}_x+\underline{U}_x}{2}]/\partial m < 0$, concluding the proof. ■

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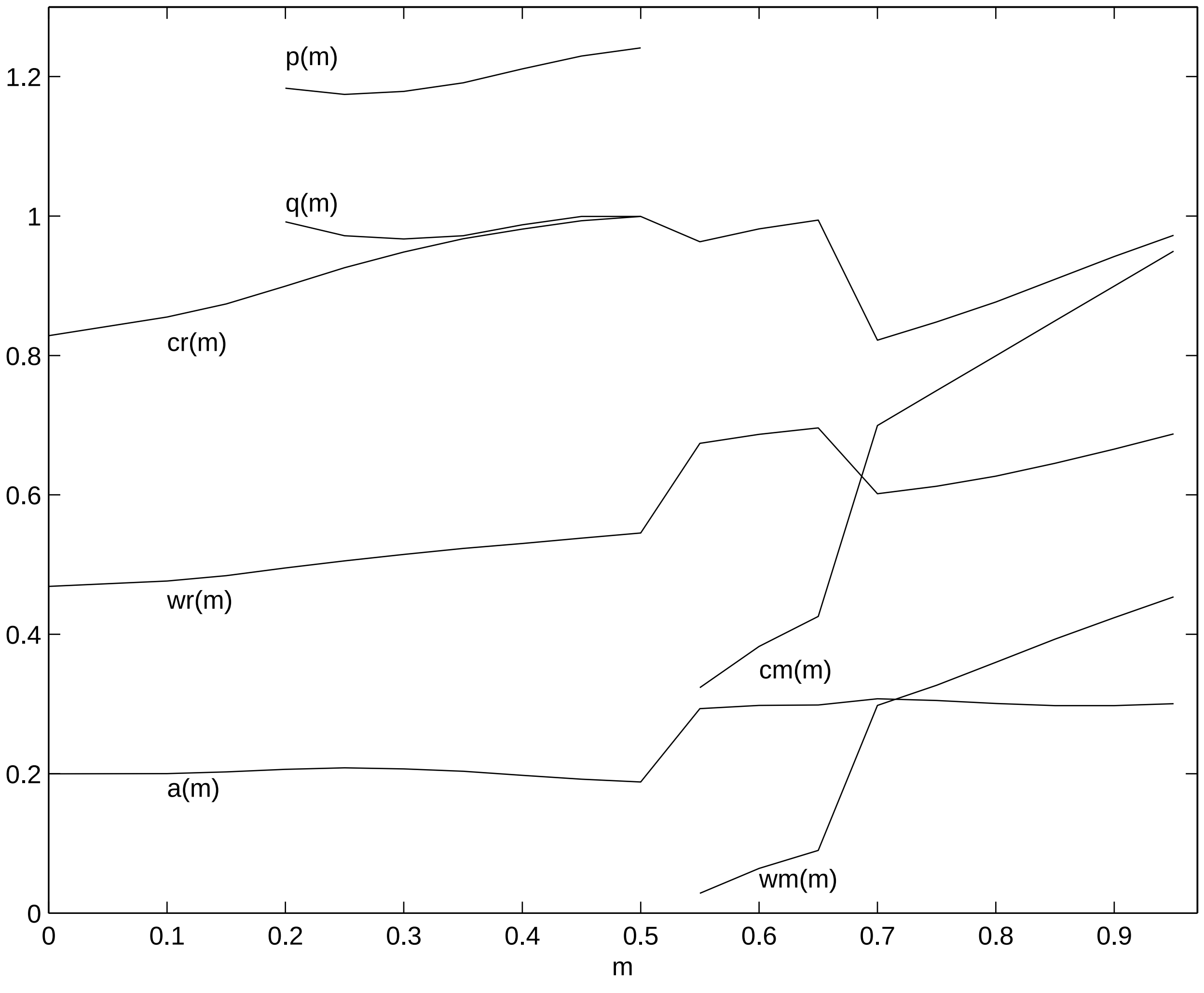

Fig. 1. wr, cr, ar, p, q, wm, and cm as a function of m for delta=0.3

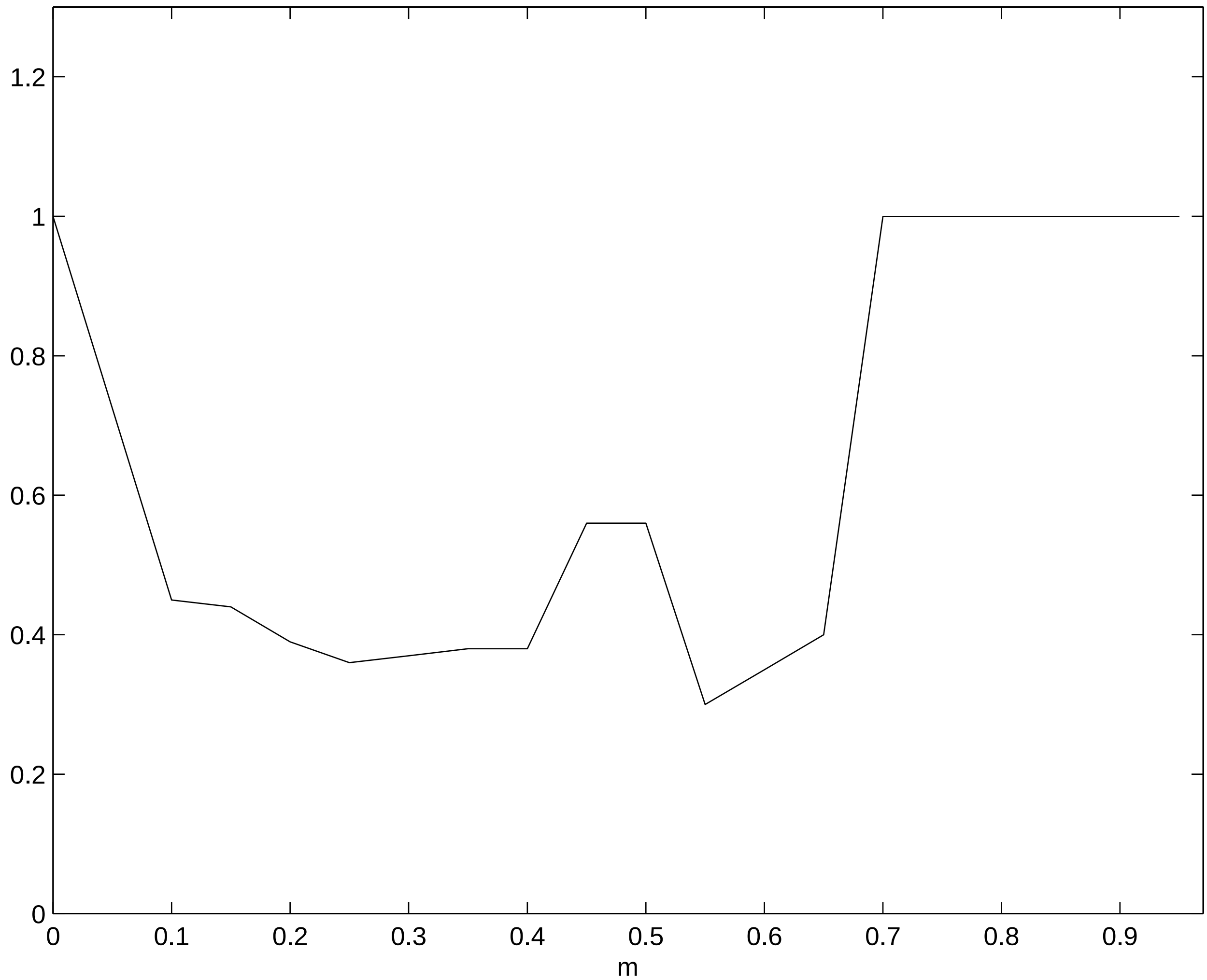

Fig. 2. Fraction of voters that prefer 3 parties over 2 parties as a function of m for delta=0.3

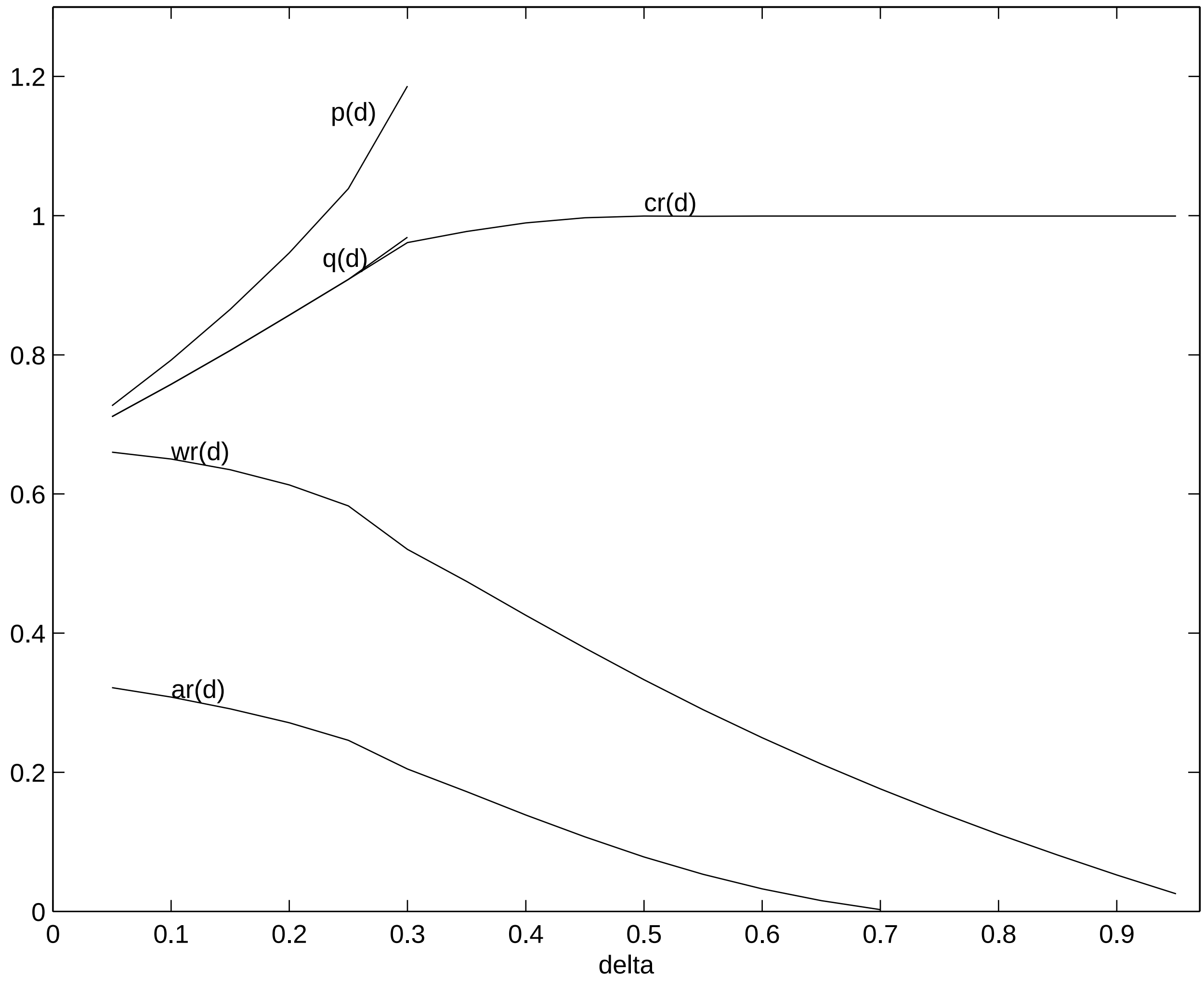

Fig. 3. wr, cr, ar, p, and q as a function of delta for m=1/3

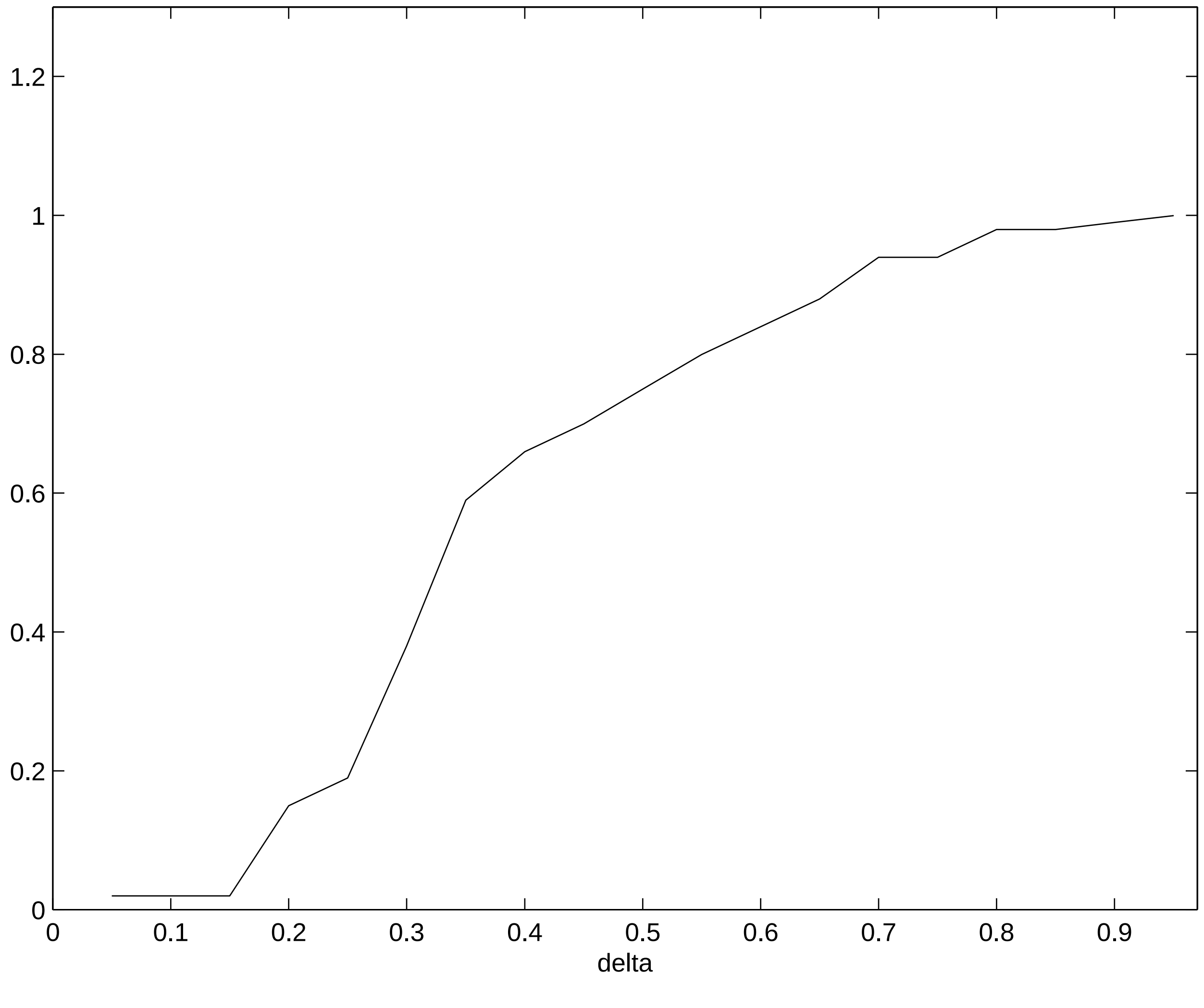

Fig. 4. Fraction of voters that prefer 3 parties over 2 parties as a function of delta for m=1/3