Extracting surplus by walking away from acquiring information*

Tingjun Liu
Faculty of Business and Economics,
The University of Hong Kong, tjliu@hku.hk

Dan Bernhardt
University of Illinois, danber@illinois.edu
University of Warwick

Odilon Câmara
University of Southern California, ocamara@marshall.usc.edu

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Abstract

A recent information design literature focuses on the value to buyers of acquiring strategically-designed information about their valuation to induce a monopolist to set lower prices. We highlight the value of commitments to *not* acquire information if excessively high prices are offered, thereby facilitating more advantageous signal designs. We endogenize commitment via costly information acquisition—after seeing the price, a buyer investigates if and only if benefits exceed the costs. Despite the inability to commit (to acquiring information or not), we identify when buyer payoffs from deferring information acquisition decisions exceed those from committing ex-ante to acquiring optimally-designed information.

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1 Introduction

A consumer without private information about her valuation of a product is at the mercy of a monopolist who can extract the consumer's surplus by charging a price equal to the consumer's expected valuation. Roesler and Szentes (2017, RS) show how a buyer can alleviate this problem by acquiring a strategically-designed signal about her valuation. When information is costless to acquire and trade is always efficient—when the seller's valuation is always below the buyer's—RS show a buyer-optimal signal induces a unit-elastic consumer demand, making the seller indifferent to setting any price on its support. In equilibrium, the seller selects the lowest price on the support and trade occurs, resulting in efficient outcomes.

Thus, RS show how acquiring information via a strategically-designed (limited) signal can increase buyer payoff. Our paper shows how an ability to commit to *not* observe the optimal signal if the seller offers a price that is too high further increases buyer payoffs. This commitment allows the buyer to design a more advantageous signal, incentivizing the seller to offer a lower price. We then show how costly information can provide the buyer this commitment power. We fully endogenize the buyer's choice of whether to acquire information. The buyer waits until after a price is posted to decide whether to incur the cost of investigating, acquiring information if and only if the expected information benefit—which depends on the posted price—outweighs the cost. Despite the complete inability to commit ex ante (either to acquiring or not acquiring information), we identify settings where buyer payoffs are strictly higher than when she commits ex ante to acquiring information according to an optimally-designed signal. In other words, we show how *waiting* until after a seller posts prices to decide whether or not to acquire information can raise buyer payoffs further.

Our starting point is the observation that a commitment to not acquire information if the posted price exceeds the buyer's expected valuation μ discourages a seller from setting such a high price, as an uninformed buyer would reject it. This commitment does not *directly* lower the seller's equilibrium price offer, because in the optimal signal design (without such a commitment) the price offer is already below μ. Rather, it lets the buyer concentrate signal mass on a high price without inducing the seller to post that price. In turn, this relaxes the Bayesian updating (mean-preserving spread) constraint that the buyer faces in the information design, allowing the buyer to induce unit-elastic demand at lower prices, incentivizing the seller to reduce his price offer.

We first illustrate the logic in a base model where the buyer's valuation always exceeds the seller's and information is costless. We then extend the analysis to a more general setting where (i) the seller's valuation may exceed the buyer's, and (ii) it costs a buyer C to see the signal realization. With full commitment, the buyer commits to pay C to observe signal L for certain prices, and commits to not observe L (and hence not pay C) for all other prices. As RS note, feature (i) introduces a tension in the optimal design: when the seller's valuation can exceed the buyer's, the information design that minimizes seller profit ceases to maximize social welfare, as it leads to inefficient trades; and when this matters enough, the design that minimizes seller profit does *not* maximize buyer payoff. Costly information acquisition exacerbates this tension because in the design that minimizes seller profit a buyer foregoes information acquisition, saving that cost; but a buyer must acquire information in order to reject surplus-reducing trades. We show how the buyer-optimal design weighs this tradeoff, and the resulting information design hinges on whether information costs are large or small.

Our costly information acquisition scheme has a natural interpretation:

- If a seller makes a very favorable or unfavorable offer, a buyer has an easy decision to make—accept or reject without devoting resources to acquiring information.
- If instead, the seller demands an intermediate price, then the buyer investigates to determine whether or not her valuation merits purchasing.

For example, in a real estate market, a buyer can propose to a seller: "If you offer q^*

(or less), I will waive my right to inspect the house. If you demand an intermediate price, I will inspect. If you demand a high price then I will not take the trouble to come and look at your house." Similarly, someone buying a used car online can email a seller: "If you give me a good price, then I will buy now; if you give me an intermediate price then I will test drive; if you demand a high price then I will not even bother to go see the car."

Finally, we drop the assumption that the buyer can fully commit to acquire or not acquire the signal depending on the price, endogenizing this choice. Because information has little value if the seller offers a very high or low price, the cost of information makes it incentive compatible for the buyer to not acquire information for such prices.

To develop intuition, we relax the ability to commit in two steps. We first consider a buyer who can commit to pay C to observe signal L if a seller offers certain prices; for all other prices the buyer decides, ex post, whether to pay to see L. With this partial commitment, buyer payoff weakly *increases* in C: costlier information makes it incentive compatible, ex post, not to acquire information for a *larger* range of high prices. The higher cost does not directly affect buyer payoff since, in equilibrium, the buyer does not observe the signal. The cost only affects buyer payoff via the stronger endogenous commitment to not acquire information after more high prices, improving the set of signals that the buyer can implement.

In the second and more important step, we fully endogenize a buyer's decision of whether or not to acquire information. Specifically, the buyer designs a signal L and then, after the seller posts a price, the buyer observes the signal realization *if and only if* the expected benefit exceeds the cost C. The cost affects both the buyer's ex-post choice of whether to acquire the signal and the optimal design of L. We focus on the case in which the buyer's expected valuation μ is below the seller's, so that information must be acquired for trade to be socially beneficial. We ask: is the buyer better off if she (i) commits ex ante to observe an optimally-designed signal at a cost C; or (ii) does not commit, instead waiting to see p before deciding whether to spend

C to acquire information? In the first "acquire information ex-ante" scenario, the buyer can optimally design the signal L and commit to always pay C to observe its realization; and if C is too high, she has the option of committing ex ante to never acquiring information (hence obtaining a payoff of zero). In the second "acquire information later" scenario, the buyer designs L but there is no commitment: after observing the price, the buyer endogenously chooses whether to pay C to observe L.

We identify conditions such that waiting to decide whether to acquire information is weakly better and, for an intermediate range of costs C, it is strictly better. This reinforces our message that a buyer can benefit from the option of not acquiring information, even when the information structure is optimally designed ex ante. When a buyer decides ex post whether to acquire information, she would not pay C to observe the signal if the seller offers a price that is too high (the expected benefit does not cover the cost)—and this relaxes the constraints on the signal design, leading to higher buyer payoffs. Moreover, the resulting buyer-optimal signal design increases social surplus: the optimal relaxed design extracts rents from trades more effectively, better aligning the buyer's objective of payoff maximization with surplus maximization. These two effects (relaxing the mean-preserving constraint and reducing surplus-decreasing trades) reinforce each other, further raising buyer payoffs.

Our finding that a buyer can be better off in the fully-endogenous investigation setting may seem contrary to the intuition that commitment must be valuable. The resolution of this "paradox" is that existing studies have focused on the value of acquiring information, but always acquiring information—even when optimally designed—comes with an implicit cost, as it forgoes the option of not acquiring information (ex-post incentive-compatible with costly information)—and we show the value of this option can be large.

Our paper relates to the broad literature that studies how information can affect trade.[1] Johnson and Myatt (2006) study how information can change the shape of the

[1] In Condorelli and Szentes (2020), a buyer can change the actual distribution of her valuation.

demand function, benefitting a monopolist. Bergemann, Brooks, and Morris (2015) study the welfare consequences of a monopolist having additional information about consumer tastes. Li and Shi (2017) and Terstiege and Wasser (2020) examine a seller who can disclose additional information to a privately-informed buyer. Recent papers study seller's behavior that is robust to different buyer information structures—e.g., Bergemann, Brooks, and Morris (2017) and Du (2018). Kartik and Zhong (2023) generalize RS to interdependent buyer-seller values and characterize *all* possible ex-ante expected payoffs that can obtain in equilibrium for the buyer and seller for *some* information structure; see Makris and Renou (2023) for related work.

Libgober and Mu (2021, LM) study the robust pricing strategy of a seller in a dynamic setting in which a buyer receives signals about her valuation over time and decides when to make a one-time purchase. They identify an information structure that minimizes seller payoffs when signals are costless and can depend on the seller's offer price (a different signal for each price). When a buyer's valuation always exceeds the seller's and information is costless, LM describe a price-dependent information structure that simultaneously minimizes seller payoff and maximizes buyer payoff (Proposition 9 in their online appendix).

In our base model a buyer either costlessly observes a single signal or obtains no information, depending on the price. Thus, the buyer in our base model has more commitment power than a buyer in RS (single price-independent signal) but less than in LM (a different signal for each price). We then depart from RS and LM to focus on our central setting: we endogenize a buyer's commitment to not investigate by considering information design when information acquisition is costly and the decision to investigate must be ex-post incentive compatible.[2]

[2]Our paper also relates to a recent literature on persuasion with costly information acquisition—see Bloedel and Segal (2021), Lipnowski et al. (2020, 2022), and Wei (2021). These papers focus on sender-receiver environments, while we focus on a buyer who both designs the signal and pays the cost to observe the signal realization. The signal design in our model persuades two receivers—the seller to choose a lower price, and the receiver to (i) acquire information or not, and (ii) purchase or not.

2 Base model: Exogenous commitment to not acquire information

Consider a seller who has an object to sell to a single buyer. We normalize the seller's valuation to zero. The buyer's valuation v is distributed according to a continuous cumulative distribution function F with support on $[\underline{v}, 1]$, where $\underline{v} \leq 0$. The buyer and seller do not know v but its distribution F is common knowledge. We denote the expected value of v by μ. Integration by parts yields

$$\mu = \int_{\underline{v}}^{1} v dF(v) = 1 - \int_{\underline{v}}^{1} F(v) dv. \tag{1}$$

There are 4 dates. At $t = 0$, the buyer announces and commits to a scheme of acquiring (or not acquiring) information about v at $t = 2$. The *information acquisition scheme* specifies two components: a *no-information set* S_N and a *signal distribution* L. The no-information set $S_N \subseteq \mathbb{R}_+$ defines prices for which the buyer commits to not acquire information about v if the seller offers a price $p \in S_N$. For all other prices (i.e., for $p \notin S_N$), the buyer can observe a signal s about v. Without loss of generality, we assume that the signal provides an unbiased estimate of the buyer's valuation, $E(v|s) = s$, and we describe it by a distribution L over s for which F is a mean-preserving spread of L, i.e.,

$$\text{(i)} \int_{\underline{v}}^{1} dL(s) = 1 - \mu \quad \text{and} \quad \text{(ii)} \int_{\underline{v}}^{x} L(s) ds \leq \int_{\underline{v}}^{x} F(v) dv \quad \text{for all } x \in [\underline{v}, 1].$$

At $t = 1$, after observing $\{S_N, L\}$, the seller makes a take-it-or-leave-it price offer of p to the buyer. At $t = 2$, the buyer does not acquire information if $p \in S_N$; otherwise, she costlessly observes the realization of s according to L.[3] At $t = 3$, trade occurs if and only if the buyer's expected valuation given her information is at least p. If trade occurs, the seller receives p and the buyer gets $v - p$. If trade does not occur, then

[3] We consider costly information acquisition in Section 3.

the buyer and seller receive payoffs of zero.

Both the seller and buyer maximize expected payoff. The equilibrium concept is perfect Bayesian equilibrium.

Analysis. To begin we identify the information acquisition scheme $\{S_N, L\}$ that maximizes buyer payoff. To ease exposition, this analysis focuses on $\underline{v} = 0$, so the buyer's value always exceeds the seller's. Section 3 extends the analysis to $\underline{v} < 0$.

Before proceeding, we develop some useful notation. Let $\mathbf{1}_{\{p \le \mu\}}$ be an indicator function that is 1 if $p \le \mu$ and it is zero otherwise. Given the information acquisition scheme $\{S_N, L\}$, let $\beta_L(s)$ be the probability of realization s given distribution L (i.e., $\beta_L(s)$ is the mass of any atom at s). If the buyer does not acquire information, $p \in S_N$, then she purchases the good if and only if $p \le \mu$. If she acquires information, $p \notin S_N$, then she purchases the good if and only if $p \le s$, which occurs with probability $(1 - L(p) + \beta_L(s))$. Therefore, the seller's expected payoff when he offers p is:

$$\pi(p) = \begin{cases} p\mathbf{1}_{\{p \le \mu\}} & \text{if } p \in S_N \\ p\left(1 - L(p) + \beta_L(s)\right) & \text{if } p \notin S_N \end{cases}.$$

The buyer's expected payoff is

$$\rho(p) = \begin{cases} (\mu - p)\,\mathbf{1}_{\{p \le \mu\}} & \text{if } p \in S_N \\ \int_p^1 (s - p)\, dL(s) & \text{if } p \notin S_N \end{cases}.$$

Let $\mathcal{L}_F$ be the set of CDFs for which F is a mean-preserving spread, i.e., $\mathcal{L}_F$ is the set of possible signal distributions. The problem of designing a buyer-optimal information acquisition scheme $\{S_N, L\}$ can be stated as follows:

$$\max_{S_N \subseteq \mathbb{R}_+; L \in \mathcal{L}_F} \rho(p)$$
$$\text{s.t. } p \in \arg\max_{\tilde{p}} \pi(\tilde{p}). \tag{2}$$

RS Benchmark. To place our analysis, we relate it to RS. In our base model, the buyer selects the no-information set S_N and the signal distribution L that maximizes her expected payoff. One can re-interpret the model in RS as a buyer who selects the signal distribution L that maximizes her expected payoff given that she sets $S_N = \emptyset$. In contrast, our buyer can commit to not acquire information if the seller offers certain prices.[4] Thus, the difference in buyer payoff from the two models shows how much a buyer can gain from being able to commit to not acquire information—i.e., from the ability to choose $S_N \neq \emptyset$.

RS define the following family of CDFs indexed by $q \in (0, 1]$ and $B \in [q, 1]$:

$$G_q^B(s) = \begin{cases} 0 & \text{if } s \in [0, q) \\ 1 - \frac{q}{s} & \text{if } s \in [q, B) \\ 1 & \text{if } s \in [B, 1] \end{cases} . \tag{3}$$

G_q^B induces a unit demand elasticity for the seller, leaving him indifferent between charging any price on its support $[q, B]$. The seller is strictly worse off if he charges a price below q or above B. RS show that when $\underline{v} = 0$, a buyer-optimal signal distribution comes from this family. It is given by the lowest q such that there exists a B for which G_q^B is a mean-preserving spread of F. Let q^{RS} and B^{RS} represent the optimal signal distribution $G_{q^{RS}}^{B^{RS}}$ from RS. Given $G_{q^{RS}}^{B^{RS}}$, the seller optimally offers $p = q^{RS}$, and the buyer purchases with probability one, receiving expected payoff $\mu - q^{RS}$.

Optimal Information Acquisition Scheme in our Setting. To solve the buyer's problem in (2), we jointly optimize over S_N and L. We identify a family of distributions from which a buyer-optimal signal comes. For each $q \in [0, \mu]$ and $B \in [\mu, 1]$,

[4] To be precise, RS study the exogenous signal that maximizes buyer's ex ante payoff. This is equivalent to the buyer strategically committing to a signal distribution at date $t = 0$.

define the CDF $L_q^B(\cdot)$ by:

$$L_q^B(s) = \begin{cases} 0 & \text{if } s \in [0, q) \\ 1 - \frac{q}{s} & \text{if } s \in [q, \mu) \\ 1 - \frac{q}{\mu} & \text{if } s \in [\mu, B) \\ 1 & \text{if } s \in [B, 1]\,. \end{cases} \tag{4}$$

We prove there exists an optimal signal in the family of distributions (4), for parameters q and B with $q \in [q_{\min}, \mu]$, where $q_{\min} \in (0, \mu)$ is the unique solution to

$$\mu = q_{min} \left(\ln(\mu) + \frac{1}{\mu} - \ln(q_{\min}) \right), \tag{5}$$

and B is the unique value $B(q) \in [\mu, 1]$ such that $L_q^{B(q)}$ has mean μ (see Lemmas A.1 to A.4 in Appendix A). Further, $B(q) = \mu$ for $q = \mu$ and $B(q) > \mu$ for $q < \mu$.

Proposition 1 *Suppose $\underline{v} = 0$. The information acquisition scheme $S_N = (\mu, 1]$ and $L_{q^*}^{B(q^*)}$ maximizes buyer payoff, where q^* is the lowest value of $q \in [q_{\min}, \mu]$ such that*

$$\int_0^x L_q^{B(q)}(s)ds \leq \int_0^x F(s)ds \tag{6}$$

for all $x \in [0, 1]$. With this scheme, the seller offers price $p = q^*$, the buyer purchases the good with probability one and receives an expected payoff $\mu - q^*$.

Proof: See Appendix A. □

The buyer commits to not acquire information if the price offer exceeds the mean μ, and she observes $L_{q^*}^{B(q^*)}$ otherwise. This discourages the seller from offering $p > \mu$ (an uninformed buyer would not purchase), which, in turn, lets the buyer improve the signal $L_{q^*}^{B(q^*)}$. The optimal signal re-allocates upwards probability mass on signal realizations above μ as far as possible to $B(q^*)$. This allows for the largest possible downward reallocation of probability mass below μ—while still satisfying the mean-preserving spread constraint—extending (weakly) downward the interval $[q^*, \mu]$ with unit elastic demand.

We next provide conditions for the value of commitment to not acquire information if the seller sets high prices to be strictly positive. Our buyer obtains a strictly higher payoff if and only if $q^{RS} > q^*$. The q^{RS} is given by the lowest q such that $\int_0^x G_{q^{RS}}^{B^{RS}}(s)ds \leq \int_0^x F(s)ds$, $\forall x \in [0,1]$, and there exists some $x \in [q^{RS}, 1]$ such that $\int_0^x G_{q^{RS}}^{B^{RS}}(s)ds = \int_0^x F(s)ds$. Let x^* be the smallest such x (if there are multiple x).

Proposition 2 *If a buyer can commit to not acquiring information, then (i) buyer payoff is always weakly higher ($q^* \leq q^{RS}$); and (ii) when F has no atom at $s = q^{RS}$ or the total mass under F is strictly positive for $s < q^{RS}$ (i.e., $F(q^{RS} - \epsilon) > 0$ for some $\epsilon > 0$), then buyer payoff is strictly higher ($q^* < q^{RS}$) if and only if $x^* > \mu$.*

Proof: See Appendix A. □

The signal design removes all probability mass over $s \in [\mu, B)$, which supports a reduction in the feasible q when $x^* > \mu$. This is because $\int_0^{x^*} G(s)ds$ decreases when mass from s above μ is removed, so a distribution with a lower q remains in the set of CDFs for which F is a mean-preserving spread. This reduction in q reduces seller payoff and increases buyer payoff. We next provide sufficient conditions for $x^* > \mu$.

Proposition 3 *The value of commitment to not acquire information is strictly positive if distribution F admits a density f such that:*

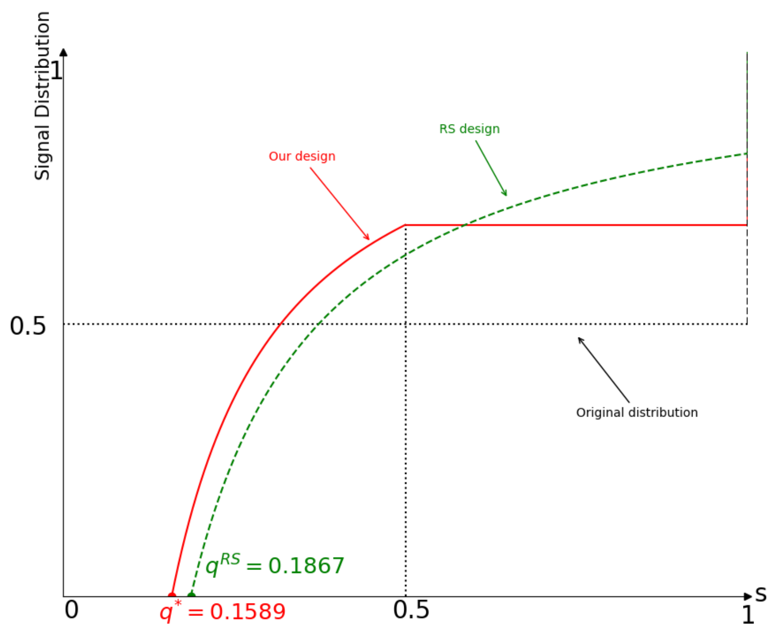
(i) Probability density f weakly decreases over $[0,1]$ with $f(1) \geq 0.57$.

(ii) Probability density f satisfies $f(v) \in [1-\Delta, 1+\Delta]$, $\forall v \in [0,1]$ where $\Delta = 0.12$.

Proof: See Online Appendix B. □

Proposition 3 says that buyer payoff in our setting always exceeds that in RS if the distribution of buyer valuations is "close enough" to uniform. The sufficient conditions in Proposition 3 are far from being tight or exhaustive. For example, the uniform distribution is a special case of the family of CDFs given by $F(s) = s^\alpha$, where $\alpha = 1$. Here, $\mu = \frac{\alpha}{\alpha+1}$, and numerical calculations yield that $x^* > \mu$ if and only if $\alpha < \alpha^* \approx 3$.

Figure 1: Optimal signal designs



Notes: Figure 1 plots the CDFs when $w = 0.5$ for the original distribution $F(s)$ (dotted black line), optimal RS design (dashed green line), and our design $L_{q^*}^{B(q^*)}(s)$ (solid red line).

Alternatively, consider the binary distribution analyzed by RS in their internet appendix that places probability mass w on $v = 0$ and mass $1 - w$ on $v = 1$. Then $F(v) = w$ for all $v \in (0, 1)$. Because F is flat over $(0, 1)$, the lowest $x \in [q^{RS}, 1]$ that satisfies $\int_0^x G_{q^{RS}}^{B^{RS}}(s)ds = \int_0^x F(s)ds$ is at $x = 1$, so $x^* = 1$. Since $\mu < 1$, Proposition 2 implies that the ability to commit to not acquiring information strictly increases buyer payoffs. For instance, when $w = 0.5$, $q^* = 0.159 < q^{RS} = 0.187$, implying a 15% higher buyer payoff. Figure 1 plots the CDFs when $w = 0.5$ for (a) the original distribution $F(s)$ (dotted black line), (b) the optimal RS design (dashed green line), and (c) our design $L_{q^*}^{B(q^*)}(s)$ (solid red line). Our design mimics RS on the lower portion of the support, but mass between 0.5 and 1 is removed—some mass is pushed up to $B = 1$, and this supports a reduction in the lower support from q^{RS} to q^*.

3 Extended Model

We now extend our base model in two ways: (i) we allow for the buyer's value to be less than the seller's, $\underline{v} \leq 0$, and (ii) we introduce a cost of acquiring information. We model costly information as follows: the buyer commits to pay C at date $t = 2$ to observe signal s if $p \notin S_N$, and commits to not observe the signal (and hence not pay C) if $p \in S_N$. The exogenous cost C is common knowledge. To ease exposition, we focus on $\mu > 0$ in this section. All other aspects of the model remain the same.

RS Benchmark. To start, we consider how $\underline{v} < 0$ changes optimal solutions. When $\underline{v} < 0$, RS show in their Internet Appendix the following solution: with a probability γ^{RS}, the signal reveals that $v < 0$, so the buyer does not purchase; and with probability $1 - \gamma^{RS}$, the signal structure mirrors that when v is always non-negative, and the buyer purchases at price q^{RS}. Setting $\gamma^{RS} = 0$ is optimal when the extent of negative values is small enough relative to that of positive values, while $\gamma^{RS} > 0$ is optimal when the extent of negative values is larger.

Optimal Information Acquisition Scheme. When the buyer's value always exceeds the seller's ($\underline{v} = 0$), Proposition 1 extends immediately to any cost $C \geq 0$. The optimal signal distribution $L_{q^*}^{B(q^*)}$ is exactly the same as that in Proposition 1, and we just adjust the no-information set to $S_N = [0, q^*] \cup (\mu, 1]$. The intuition is simple: for any price offer $p \in [0, q^*]$, the buyer will purchase the good independently of s, so the buyer gains nothing from observing s but would pay the cost C. Therefore, it is optimal to add $[0, q^*]$ to the no-information set, which, in equilibrium, saves the buyer the cost of investigation. In both the base and extended models, the seller offers $p = q^*$ and the buyer purchases the good, receiving expected payoff $\mu - q^*$ (since C is not incurred in equilibrium). Thus, Propositions 2 and 3 still hold when we compare our buyer's payoff $\mu - q^*$ to the payoff $\mu - q^{RS}$ in RS (with $C > 0$ and $\underline{v} = 0$, we assume the buyer in the RS design also saves the cost of investigation in equilibrium).

When the seller's value can exceed the buyer's ($\underline{v} < 0$), trade can reduce social welfare. With costless information acquisition, as RS note, if the surplus-reducing consequences of negative NPV (net present value) trades matter enough, the design that minimizes seller payoff does not maximize buyer payoff. We show costly information acquisition introduces another consideration. This is because in the design that minimizes seller payoff, a buyer does not acquire information in equilibrium; but to maximize gross-of-C buyer payoff, the buyer must acquire information to reject surplus-reducing trades. As a result, the optimal design hinges on C.

Next we solve the buyer's problem for our setting. Proposition 4 will show that an optimal information acquisition design is given by one of the following two options:

Information Scheme 1: Buyer always purchases the good. The no-information set is $S_N = [0, q^{**}] \cup (\mu, 1]$, and the signal distribution L_q^B is from the family described by (4). Let q^{**} be the lowest value of $q \in [q_{\min}, \mu]$ (where $q_{\min}$ solves (5)) such that $\int_{\underline{v}}^{x} L_q^{B(q)}(s)ds \le \int_{\underline{v}}^{x} F(s)ds$ for all $x \in [0,1]$. If this scheme is used in equilibrium, the seller offers $p = q^{**}$ and the buyer purchases with probability one without acquiring information, receiving payoff $\mu - q^{**}$.

Information Scheme 2: Buyer purchases with probability less than one. The no-information set is $S_N = (\mu, 1]$. The signal distribution $\bar{L}^B_{\gamma,\lambda,q}(s)$ is defined by four parameters $\{q, B, \gamma, \lambda\}$ as follows. With probability $\gamma \in (0,1)$ the signal is $s = \lambda$ with $\lambda < 0$; and with probability $1-\gamma$, the signal is given by L_q^B as in (4), where $q \in (0, \mu]$ and $B > \mu$ is such that $\bar{L}^B_{\gamma,\lambda,q}$ has mean μ. $\bar{L}^B_{\gamma,\lambda,q}$ has an atom of mass γ at $s = \lambda < 0$, leaving total mass $1-\gamma$ for $s > 0$. Conditional on a positive signal $s > 0$, the signal is distributed according to L_q^B. That is, $\bar{L}^B_{\gamma,\lambda,q}$ is given by

$$\bar{L}^B_{\gamma,\lambda,q}(s) = \begin{cases} 0 & \text{if } s < \lambda \\ \gamma & \text{if } s \in [\lambda, q] \\ \gamma + (1-\gamma)L_q^B(s) & \text{if } s > q. \end{cases} \tag{7}$$

The prior distribution F must be a mean-preserving spread of $\bar{L}^B_{\gamma,\lambda,q}$. Let $B(\gamma,\lambda,q)$ be the value of B such that $\bar{L}^{B(\gamma,\lambda,q)}_{\gamma,\lambda,q} \in \mathcal{L}_F$. Lemma A.5 in Appendix A shows that we can focus on the parameters $\bar{\gamma}, \bar{\lambda}, \bar{q}$ that solve:

$$\min_{\gamma,\lambda,q} \gamma\lambda + (1-\gamma)q$$
$$\text{s.t. } \bar{L}^{B(\bar{\gamma},\bar{\lambda},\bar{q})}_{\bar{\gamma},\bar{\lambda},\bar{q}} \in \mathcal{L}_F.$$

If this scheme is used in equilibrium, then the seller offers $p = \bar{q}$; the buyer pays C to acquire information, and then purchases with probability $(1-\bar{\gamma})$, obtaining an ex-ante expected payoff of $\mu - C - \bar{\gamma}\bar{\lambda} - (1-\bar{\gamma})\,\bar{q}$ (see the proof of Lemma A.5 for details).

Proposition 4 *Information Scheme 1 is an optimal scheme if*

$$q^{**} \leq \bar{\gamma}\bar{\lambda} + (1-\bar{\gamma})\,\bar{q} + C. \tag{8}$$

Otherwise, Information Scheme 2 is an optimal scheme.

Proof: See Appendix A. □

Intuitively, the buyer's payoff from scheme 1 does not depend on cost C because, in equilibrium, the buyer does not observe the signal and purchases the product for sure. The buyer's payoff in scheme 2 is a decreasing function of C since, in equilibrium, she pays to observe the signal. Thus, a high information cost induces the buyer to change from the more informative scheme 2 (where the buyer sometimes rejects negative NPV trades) to the less informative scheme 1 (where the buyer always purchases the good).

4 Endogenizing the Extent of Commitment

We have analyzed a buyer who can commit to not acquiring information for prices $p \in S_N$. We now show how costly information acquisition endogenously provides that commitment power. We make our points in two steps. We first consider a buyer who can (only) commit to acquire costly information according to an information structure if the offer price p is in a given set; for p outside that set, she decides ex post, after observing p, whether to incur the investigation cost. We then consider a buyer who cannot commit either to investigating or not: the buyer always decides ex post whether or not to pay C to observe the signal.

Partial Commitment. With partial commitment, a buyer can commit to incur a cost C to acquire information according to an information structure L^{PC} (PC stands for "partial commitment") if $p \in S^{PC}$ for an optimally-designed set of prices S^{PC}. However, if $p \notin S^{PC}$, she only decides ex-post, after the seller offers p, whether or not to acquire information according to L^{PC}, i.e., information acquisition must be ex-post incentive compatible. The buyer jointly designs L^{PC} and S^{PC}. For simplicity, assume $\underline{v} = 0$. This framework features the same degree of commitment as RS: with costly information, one may interpret the buyer in RS as committing to acquire information only for prices that exceed q^{RS}, saving the cost C in equilibrium.

To proceed, we use $L_{q^*}^{B(q^*)}$ from Proposition 1 to define

$$\bar{C} \equiv \int_{\mu}^{1} (s - \mu)\, dL_{q^*}^{B(q^*)}(s),$$

Remark 1 *If $C \geq \bar{C}$, then Proposition 1 holds with partial commitment:*

(i) The commitment to not acquire information extends ex post: if a buyer commits to acquire information via $L_{q^*}^{B(q^*)}$ for all $p \in (q^*, \mu]$, then she will not acquire information, ex post, for any $p \in [0, q^*] \cup (\mu, \infty)$.

(ii) In equilibrium, the seller offers q^* and the buyer accepts without investigation, obtaining maximal profit $\mu - q^*$.

Proof: See Online Appendix B. $\square$

The intuition is simple: if a seller offers a high price that reduces the expected surplus from purchase below the cost of information, it is ex-post incentive compatible for the buyer to not acquire information. The buyer exploits this to design a better signal.

To illustrate equilibrium outcomes when $C < \bar{C}$, we use our running example: a binary buyer valuation distribution, with probability mass 0.5 on $v = 0$ and mass 0.5

on $v = 1$. Then $\bar{C} = 0.159$. Consider the following design of our partial commitment model: the buyer commits to acquire information for $p \in S^{PC} = (q^{PC}, k]$, where $q^{PC} \in (0, k)$ and $k \in (\mu, 1)$ are functions of C ($C < \bar{C}$) that are uniquely determined by

$$0.5 - C - \frac{Ck}{1-k} - \frac{Ck}{1-k} \ln \frac{1-k}{C} = 0 \quad \text{and} \quad q^{PC} = \frac{Ck}{1-k}. \tag{9}$$

The optimal information structure features unit-elastic demand for $s \in [q^{PC}, k)$, with $k > \mu$ and a point mass at $s = 1$:

$$L^{PC}(s) = \begin{cases} 0 & \text{if } s \in [0, q^{PC}) \\ 1 - \frac{q^{PC}}{s} & \text{if } s \in [q^{PC}, k) \\ 1 - \frac{q^{PC}}{k} & \text{if } s \in [k, 1) \\ 1 & \text{if } s = 1. \end{cases}$$

Remark 2 *In this design,*

(i) Ex post, the buyer will not acquire information for $p \in \left[0, q^{PC}(C)\right] \cup (k(C), \infty)$. *The seller offers* q^{PC} *and the buyer purchases without investigation.*

(ii) Both k *and* q^{PC} *strictly decline with* $C \in (0, \bar{C})$: *increasing* C *widens the range of high prices with no information acquisition, strictly increasing buyer payoffs.*

(iii) The buyer's payoff of $\mu - q^{PC}$ *strictly exceeds that in RS of* $\mu - q^{RS}$ *for all* $C > 0$.

Proof: See Online Appendix B. □

Costlier information acquisition makes it incentive compatible, ex post, for a buyer to not acquire information after more high prices, discouraging a seller from setting such prices and relaxing the mean-preserving constraint in the information design. As a result, the higher is C, the more a buyer benefits (as the cost is never incurred).

No Commitment. We now consider a buyer who cannot commit ex ante either to acquire or not acquire information. The buyer can only commit to a signal structure L. Then, after the seller posts a price, the buyer decides whether or not to pay C to acquire information using L.

When the buyer's valuation always exceeds the seller's ($\underline{v} = 0$) and the buyer cannot commit to which prices she will or will not investigate, her payoff is typically less than what she would get if she could commit to investigating. This is because a buyer has no incentive to acquire information if a seller offers a price slightly above q^{RS}, leading the seller to offer $p > q^{RS}$. When $\underline{v} = 0$, this typically reduces buyer payoff as it more than offsets the value of endogenous commitment to not acquire information for prices above μ that allows the optimal information structure to be pushed downwards.

However, when $\underline{v} < 0$, a striking change occurs: the possibility that the seller's valuation exceeds a buyer's can make it ex post incentive compatible for the buyer to acquire information when the seller offers q^{RS}, raising buyer payoff above what she gets if she commits to always investigating when the price offered is sufficiently high. The possibility that a seller's valuation exceeds the buyer's is especially germane in a search setting, where the value to a seller of not selling is the future possibility of finding a buyer with a higher valuation.

Let L^{NC} denote the information structure in our no-commitment framework, where the buyer's choice to acquire or not information at $t = 2$ is fully endogenous. To convey our insights sharply, we focus on $\mu \leq 0$, so that seller and buyer payoffs would be zero if the buyer does not acquire information.[5]

As a benchmark, consider the optimal information design from RS described in Section 3 for $\mu \leq 0$. When information is costless ($C = 0$), with the optimal signal $G^{RS}_{C=0}$, the buyer purchases the good with probability $(1 - \gamma^{RS}_{C=0})$ at price $q^{RS}_{C=0}$. When $C > 0$, we reinterpret the RS benchmark as follows: at $t = 0$, the RS buyer can either commit to not observe any signal (if C is too large) or commit to pay C to observe signal $G^{RS}_{C=0}$. Consequently, the buyer receives a payoff of zero if she commits to not observe any signal. If the buyer, instead, chooses to commit to pay C to observe some signal, then $G^{RS}_{C=0}$ is an optimal signal and the buyer's payoff becomes $\rho^{RS}_{C=0} - C$, where

[5]Without information acquisition, expected social surplus is non-positive, so neither the buyer nor seller can have a strictly positive payoff, when satisfying individual rationality for the other party.

$\rho^{RS}_{C=0}$ denotes the buyer's expected payoff in the original RS model with costless information. Consequently, it is optimal for the buyer to commit to pay C to see $G^{RS}_{C=0}$ if $\rho^{RS}_{C=0} - C > 0$, and she commits to not observe any signal otherwise. Thus, the relevant benchmark is the (adjusted) RS buyer payoff of $\rho^{RS}_{C} = \max\{\rho^{RS}_{C=0} - C, 0\}$.

Proposition 5 *Suppose $\mu \leq 0$ but there is a non-zero probability that the valuation is positive so that $\int_0^1 F(x)\,dx < 1$. Then in the optimal no-commitment design*

(i) buyer payoffs are at least as high as in the (adjusted) RS design.

(ii) for an interval of costs $C > 0$ with strictly positive measure, buyer payoffs strictly exceed those in the (adjusted) RS design.

Proof: See Appendix A. $\square$

The information cost grants to the buyer the ability to credibly commit to walking away from acquiring information if the seller offers a price that is too high, allowing the buyer to extract more surplus by designing a more advantageous signal. To understand the result, first consider $C < \rho^{RS}_{C=0}$. If our no-commitment buyer uses the same information design as RS ($L^{NC} = G^{RS}$), then the seller will offer the price q^{RS} and the buyer will ex post choose to pay C to investigate, so outcomes are the same as in RS. Thus, there is an information design that does at least as well as in RS, but the buyer may be able to do even better. The proof shows that if C exceeds $\rho^{RS}_{C=0}$, but not by too much, then one can design L^{NC} so the buyer's payoff net of the information acquisition cost is strictly positive, which exceeds the payoff of zero from the (adjusted) RS design.

Proposition 5 sheds light on a fundamental question: When a buyer can optimally design a signal distribution L at date 0, is the buyer better off if she (i) commits to acquire information (commits to always pay C to see L), or (ii) does not commit to acquire information (waits to see p and then decides whether to pay C to see L)? Under the conditions of Proposition 5, waiting is weakly better than commitment to acquire information, and, for a range of costs C, waiting is strictly better. This reinforces our message that a buyer can benefit from not acquiring information. When the buyer endogenously chooses ex post whether to acquire information, she can credible threaten to not pay C to observe signal L if the seller offers a price that is too high. This relaxes the constraints on information design, leading to a more advantageous signal L.

To derive the *buyer optimal* no-commitment information structure, we now focus on a binary distribution F over buyer valuations that places probability mass $w \in (0,1)$ on $\underline{v} < 0$, mass $1-w$ on $\bar{v} > 0$, with mean $\mu = w\underline{v} + (1-w)\,\bar{v} \leq 0$. We consider $C \in (0, (1-w)\,\bar{v})$, as trade is not possible if C exceeds the maximum possible gross social surplus of $(1-w)\,\bar{v}$. We show an optimal information structure takes the form:

$$L^{NC}(s) = \begin{cases} \gamma & \text{if } s \in [\underline{v}, q^{NC}) \\ 1-(1-\gamma)\frac{q^{NC}}{s} & \text{if } s \in [q^{NC}, k) \\ 1-(1-\gamma)\frac{q^{NC}}{k} & \text{if } s \in [k, \bar{v}) \\ 1 & \text{if } s = \bar{v}, \end{cases} , \tag{10}$$

where $\gamma \in [0, w]$, $k \in (0, \bar{v})$, and $q^{NC} \in (0, k)$ solve the following system of equations:

$$\gamma\underline{v} + \frac{C\bar{v}}{\bar{v}-k} + \frac{Ck}{(\bar{v}-k)} \ln\left(\frac{(1-\gamma)(\bar{v}-k)}{C}\right) = \mu, \tag{11}$$

$$q^{NC} = \frac{Ck}{(1-\gamma)(\bar{v}-k)}, \tag{12}$$

with

$$\gamma = \arg\max_{\hat{\gamma}\in[0,w]} \hat{\Pi}(\hat{\gamma}), \text{ where } \hat{\Pi}(\hat{\gamma}) \equiv \mu - \hat{\gamma}\underline{v} - (1-\hat{\gamma})\, q^{NC}(\hat{\gamma}, C). \tag{13}$$

The design has a mass of γ at $s = \underline{v}$, a mass of $(1-\gamma)\frac{q^{NC}}{k}$ at $s = \bar{v}$, and unit-elastic demand between q^{NC} and k. Equations (11) – (13) determine γ, k and q^{NC} as functions of C. Here, (11) determines k as a function of C and γ, ensuring that the design has the same mean as the original distribution. Plugging this k in (12) determines q^{NC} as a function of C and γ. (12) ensures that k is the cutoff at which the value of acquiring information just equals the cost: the value of information is less than C for all $p > k$, and it is greater than C for all $p \in [0, k)$. Accordingly, the information structure in (10) places no probability mass on $s \in (k, \bar{v})$, facilitating a reduction in the price q offered in equilibrium by the seller. The indifference point at k coincides with the point where the probability mass on the signal is shifted upward: this coincidence condition ensures buyer optimality for a given γ.

Equation (13) says that the choice of the probability mass γ on the negative signal maximizes the buyer's gross-of-information-costs equilibrium payoff of $\hat{\Pi}(\gamma)$ for the given C. To see this, note that $(1 - \hat{\gamma})\, q^{NC}$ is the seller's expected payoff; and $\mu - \hat{\gamma}\underline{v}$ is the expected social surplus (gross of information costs). That is, social surplus would be μ if trades always take place and $-\underline{v}\hat{\gamma}$ is the increase in social welfare from the equilibrium rejection with probability $\hat{\gamma}$ of welfare-decreasing trades with $s = \underline{v}$. A tradeoff arises: a lower $\hat{\gamma}$ moves more mass of negative value to the positive side and leads to trade, reducing social welfare; but this shift allows for a lower q, reducing the seller's payoff. A similar tradeoff exists in RS.

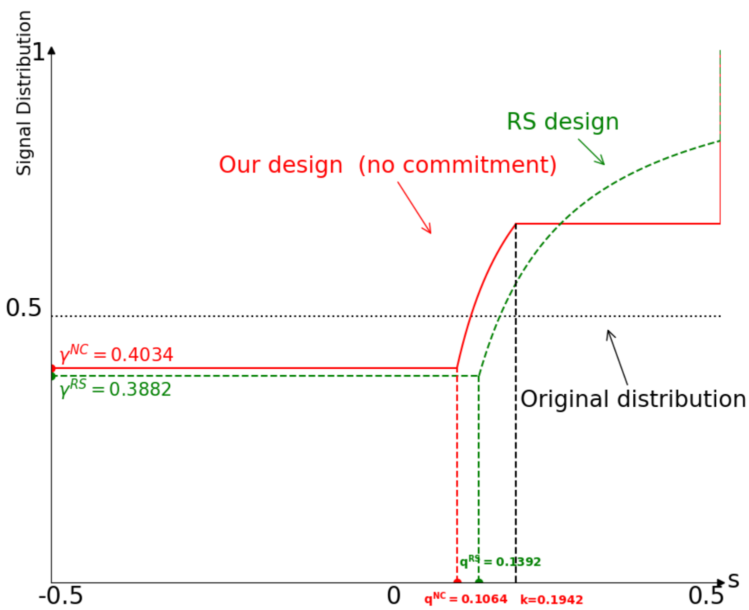
Proposition 6 *The information structure in (10) is the optimal no-commitment design. Ex post, the buyer acquires information if $p \in [0, k(C)]$, but not if $p > k(C)$. The seller offers q^{NC}. The buyer incurs cost C to acquire information, accepting the offer if and only if $s \geq q^{NC}$.*

Proof: We prove in Appendix A that L^{NC} is optimal, including satisfying the requirement that the original distribution is a mean-preserving spread of L^{NC}. With this result, observe that for any price $p > 0$, a buyer's payoff from not acquiring information is zero as no purchase is made, and her payoff from acquiring information is

$$\Pi(p) \equiv \int_p^{\bar{v}} (s - p)\, dL^{NC}(s) - C. \tag{14}$$

Substituting (12) yields $\Pi(k) = (\bar{v} - k)(1 - \gamma)\, q^{NC}/k - C = 0$. Thus, at price k, the buyer is indifferent to information acquisition. Since the buyer's payoff from acquiring information (equation (14)) strictly decreases in p, the buyer acquires information if and only if $p \leq k$. Standard arguments then yield that the seller offers q^{NC} and the buyer acquires information, accepting the offer if and only if $s \geq q^{NC}$. $\square$

Figure 2: Optimal signal designs with costly information



To illustrate equilibrium outcomes, we numerically solve equations (11) – (13) when $w = 0.5$, $\underline{v} = -0.5$, $v = 0.5$ for $C \in (0, 0.25)$. For $C = 0.1$, Figure 2 illustrates our design (solid red line), the CDF for the original distribution $F(s)$ (dotted black line) and the optimal RS design (dashed green line). In our design, $q^{NC} = 0.1064$, $k = 0.1942$, with a probability mass of $\gamma = 0.4034$ at $s = -0.5$, a mass of $(1-\gamma)\frac{q^{NC}}{k} = 0.3269$ at $s = 0.5$, and unit-elastic demand between q^{NC} and k.[6] The RS design features $q^{RS} = 0.1392$ and $\gamma^{RS} = 0.3882$.

The top panel of Figure 3 shows that buyer payoff in our design is higher for all $C \in (0, 0.25)$. It plots how net-of-C buyer payoff in our design (solid red line) and the (adjusted) RS design (dotted green line) vary with C. The bottom panel plots how the probability mass γ on negative valuations for our design (solid red line) and the

[6] One can verify that the buyer is indifferent between acquiring information or not when the seller offers k: the value of acquiring information is $0.3269 * (0.5 - k)$, which equals the cost of 0.1. The buyer's payoff is 0.1382 gross of C, and 0.0382 net of $C = 0.1$—by the same logic as (13) or (A.14).

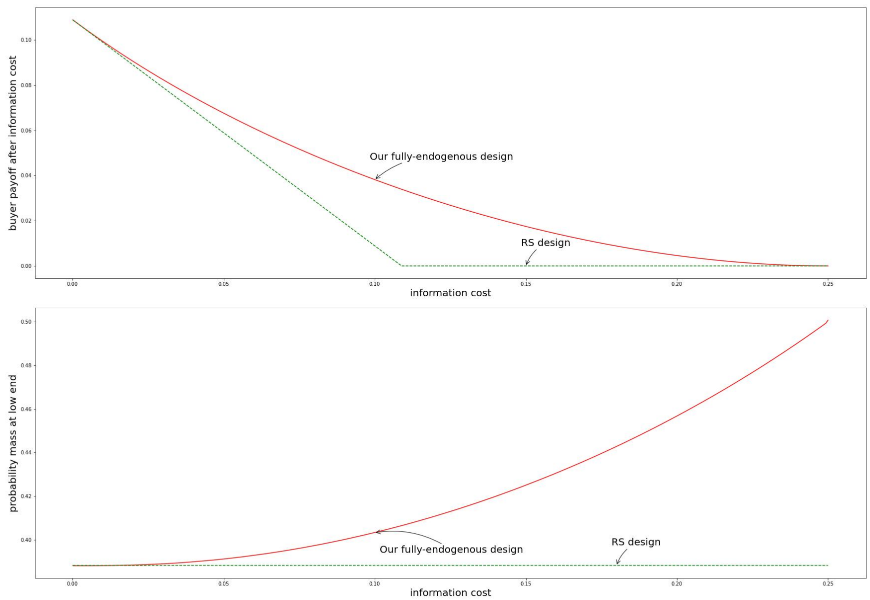


Figure 3: Top: Net (of C) buyer payoff in our design (solid red line) and (adjusted) RS design (dotted green line) vs. C. Bottom: probability mass γ on negative valuations for our design (solid red line) and (adjusted) RS design (dashed green line) vs. C.

(adjusted) RS design (dotted green line) varies with C. In the (adjusted) RS design, $\gamma = 0.389$ for all C. In our design, γ increases from 0.389 at $C = 0$ to 0.5 at $C = 0.25$. Thus, with costly information, social surplus is higher in our design than in RS due to the lower mass γ on negative valuations, and the gap widens with C.

Figure 4 illustrates how k and q vary with C. As C rises from zero to 0.25, the top panel shows k falls from 0.5 to 0 (which is μ), i.e., increases in C widen the range of high prices for which the buyer will not acquire information ex post. The middle panel shows that as C increases, the reductions in k support reductions in q from 0.1392 (which is q^{RS}) to 0. That is, with increases in C, the resulting optimal signal design induces the seller to make lower offers. In turn, the bottom panel shows gross buyer payoffs rise with C, starting from 0.1092 (the payoff in RS) at $C = 0$ to 0.25 at $C = 0.25$.

The sources for the higher buyer payoffs with our design are: $q^{NC} < q^{RS}$ and $\gamma^{NC} > \gamma^{RS}$. In our design, increases in C (i) induce the seller to make lower offers,

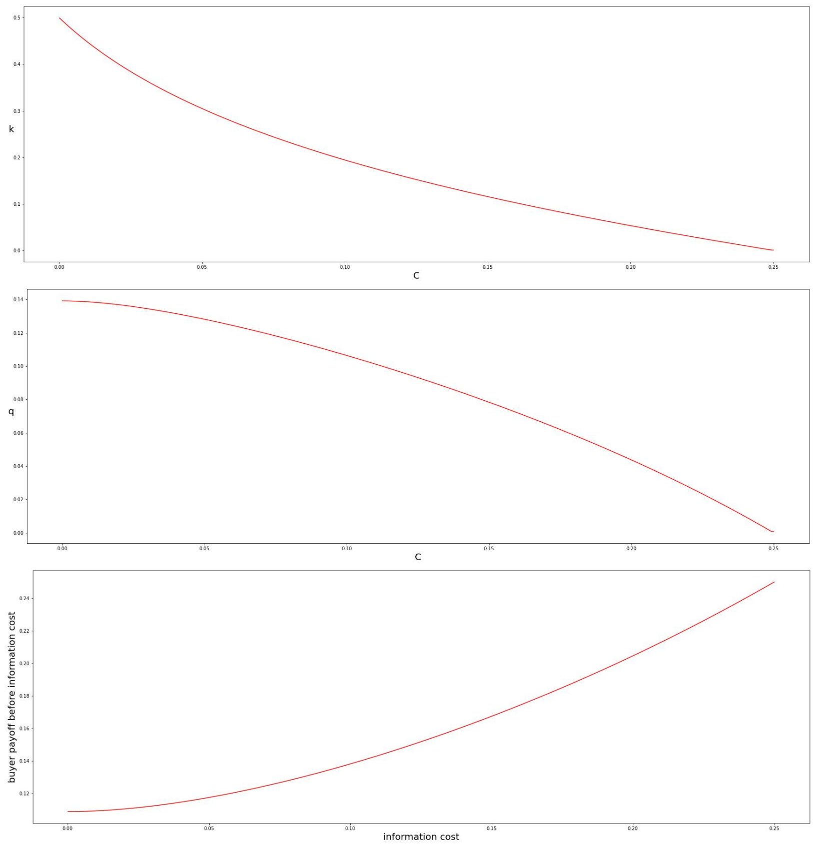


Figure 4: Top: q vs. C. Center: k vs. C. Bottom: gross buyer payoff vs. C.

and (ii) increase γ and hence social surplus. Relaxing the mean-preserving constraint lets a buyer extract rents from trades more effectively, better aligning her objective of payoff maximization with surplus maximization, making it optimal to further reduce surplus-decreasing trades. These two effects (relaxing the mean-preserving constraint and reducing surplus-decreasing trades) reinforce each other in raising buyer payoff.

5 Conclusion

Beginning with Roesler and Szentes (2017, RS), an information design literature focuses on the potential value to a buyer of committing to acquire limited information about her valuation in order to induce a monopolist to set a lower price. Our paper highlights the value to a buyer of being able to commit to *not* acquiring this strategically designed information if the seller posts either particularly unattractive or attractive prices, capturing a buyer's ability to commit to simply walking away from acquiring information if the seller offers an excessively high price. The ability to commit to walking away from acquiring information allows the buyer to extract more surplus by shifting probability mass upwards to signals of higher valuations. This upward shift, in turn, facilitates a downward shift of probability mass on lower signals, inducing the seller to offer an even lower price.

To underscore the value to a buyer of being able to commit to not acquiring information, we fully endogenize this choice via costly information acquisition, so that a buyer will acquire information if and only if the potential value exceeds the cost. We identify conditions such that waiting to decide whether to acquire information is strictly better than committing ex-ante to acquiring information. We show that, when the possibility of surplus-reducing trades matters enough that information must be acquired for trade to arise, there is always a range of information acquisition costs for which the buyer is strictly better off than if she commits ex ante to always acquiring optimally-designed, limited information.

A Appendix

Lemma A.1 *Define* $q_{\min}$ *to be the value of* $q \in (0, \mu)$ *such that* L_q^1 *(i.e.,* $L_q^{B=1}$*) satisfies* $\int_0^1 L_q^1(s)ds = \int_0^1 F(s)ds$*. By (1), this is equivalent to*

$$\int_0^1 L_{q_{\min}}^1(s)ds = 1 - \mu. \tag{A.1}$$

A unique $q_{\min} \in (0, \mu)$ *satisfies (A.1). This* $q_{\min}$ *is the unique solution to:*

$$\mu = q_{min}\left(\ln(\mu) + \frac{1}{\mu} - \ln(q_{\min})\right). \tag{A.2}$$

Proof: From (4), for any $q_{min} \in (0, \mu)$, we have

$$\begin{aligned}\int_0^1 L^1_{q_{min}}(s)ds &= \int_0^{q_{min}} 0ds + \int_{q_{min}}^{\mu} \left(1 - \frac{q_{min}}{s}\right) ds + \int_{\mu}^1 \left(1 - \frac{q_{min}}{\mu}\right) ds \\ &= \mu - q_{min} - q_{min} \ln\left(\frac{\mu}{q_{min}}\right) + \left(1 - \frac{q_{\min}}{\mu}\right)(1-\mu). \qquad \text{(A.3)}\end{aligned}$$

Substitute (A.3) into (A.1) and rewrite to obtain

$$\mu - q_{min} - q_{min} \ln\left(\frac{\mu}{q_{min}}\right) - \frac{q_{\min}}{\mu}(1-\mu) = 0. \qquad \text{(A.4)}$$

Our first claim is that (A.4) has a unique solution $q_{\min} \in (0, \mu)$. This follows since the LHS of (A.4) is a strictly decreasing continuous function of $q_{\min}$.[7] The limit of the LHS of (A.4) as $q_{\min} \to 0$ is $\mu > 0$, and as $q_{\min} \to \mu$ it is $\mu - 1 < 0$. The claim then follows from the intermediate value theorem. Solution (A.2) follows from (A.4) and differentiation reveals that q_{min} increases with μ. □

Lemma A.2 *For each $q \in [q_{\min}, \mu]$, there is a unique $B \in [\mu, 1]$ such that the distribution L^B_q has a mean of μ.*

Proof: For each $q \in [q_{\min}, \mu]$, the definition of L in (4) implies:

$$\int_0^1 L^{\mu}_q(s)ds \geq \int_{\mu}^1 L^{\mu}_q(s)ds = \int_{\mu}^1 1ds = 1 - \mu.$$

Because $L^B_q(s)$ decreases in q, we have $\int_0^1 L^1_q(s)ds \leq \int_0^1 L^1_{q_{\min}}(s)ds = 1 - \mu$, where the inequality follows from $q \geq q_{\min}$ and the equality follows from (A.1). By the intermediate value theorem and fact that $\int_0^1 L^B_q(s)ds$ strictly and continuously decreases in B, there is a unique $B \in [\mu, 1]$ such that $\int_0^1 L^B_q(s)ds = 1 - \mu$. □

Lemma A.3 *Let $B(q)$ be the solution for B in Lemma A.2 as a function of $q \in [q_{min}, \mu]$, so $\int_0^1 L^{B(q)}_q(s)ds = 1-\mu$. For any distribution function $H(s)$ with $\int_0^1 H(s)ds = 1 - \mu$,*

[7] The derivative is $1 - 1/\mu - \ln(\mu/q_{min}) < 0$, for $\mu \in (0, 1)$ and $q_{min} \in (0, \mu)$.

(i) $\int_0^x L_q^{B(q)}(s)ds \leq \int_0^x H(s)ds,\ \forall x \in [B(q),1]$. *The inequality is strict if* $H(x) < 1$.

(ii) *If* $\int_0^x L_q^1(s)ds \leq \int_0^x H(s)ds,\ \forall x \in [0,1]$, *then* $\int_0^x L_q^{B(q)}(s)ds \leq \int_0^x H(s)ds,\ \forall x \in [0,1]$.

Proof: By definition of $L_q^{B(q)}$, for all $s \geq B(q)$, we have $H(s) \leq L_{\hat{q}}^{B(\hat{q})}(s) = 1$. Combine this fact with $\int_0^1 L_q^{B(q)}(s)ds = \int_0^1 H(s)ds = 1-\mu$ to find that for all $x \in [B(q), 1]$:

$$\int_0^x L_q^{B(q)}(s)ds = 1 - \mu - \int_x^1 L_q^{B(q)}(s)ds \leq 1 - \mu - \int_x^1 H(s)ds = \int_0^x H(s)ds, \quad \text{(A.5)}$$

where the inequality is strict if $H(x) < 1$, which completes the proof of part (i).

To prove part (ii), first note that for all $x \in [0, B(q))$, the definition of L in (4) implies that $\int_0^x L_q^{B(q)}(s)ds = \int_0^x L_q^1(s)ds \leq \int_0^x H(s)ds$. For all $x \in [B(q),1]$, the result follows from part (i), completing the proof. □

Lemma A.4 *Let* $B(q)$ *be the solution for* B *in Lemma A.2, as a function of* q. *Then* $B(q) = \mu$ *for* $q = \mu$ *and* $B(q) > \mu$ *for* $q < \mu$. *At* $q = \mu$ *(so* $B(q) = \mu$*), for all* $x \in [0,1]$,

$$\int_0^x L_\mu^\mu(s)ds \leq \int_0^x F(s)ds, \quad \text{(A.6)}$$

where the inequality is strict for all $x < 1$ *in the support of* F, *that is,* $F(x) \in (0,1)$.

Proof: From the definition of L in (4), the LHS of (A.6) equals zero if $x < \mu$ and it equals $x - \mu$ if $x \geq \mu$. Therefore, inequality (A.6) trivially holds for all $x \in [0, \mu)$, and it is strict if $F(x) > 0$. For all $x \in [\mu, 1]$, because $B(q) = \mu$, Lemma A.3(i) implies that (A.6) holds in this interval, where the inequality is strict if $F(x) < 1$. □

Proof of Proposition 1: Let q^* denote the lowest value of $q \in [q_{\min}, \mu]$ such that

$$\int_0^x L_q^{B(q)}(s)ds \leq \int_0^x F(s)ds \quad \text{(A.7)}$$

for all $x \in [0,1]$. By Lemma A.4, $q = \mu$ satisfies (A.7), and hence q^* is well-defined. Moreover, $q^* < \mu$ because F is not degenerate by assumption.

First, note that given $L_{q^*}^{B(q^*)}$ and S_N in the proposition, the seller optimally offers $p = q^*$. In particular, seller payoff is $p^* > 0$ when he offers $p = q^*$. His payoff would be less if he offers $p < p^*$ (his payoff would be p); and his payoff would be zero if he offers $p > \mu$ since the buyer would not acquire information (i.e., $p \in S_N$) and hence would reject the offer. Finally, the seller's payoff from offering any price $p \in (p^*, \mu]$ is $p\left(1 - L_{q^*}^{B(q^*)}\right) = p^*$. Thus, the seller cannot do better than offering $p = p^*$. Because trade is efficient and always occurs in equilibrium, this outcome generates the maximal social welfare of μ, yielding a buyer payoff of $\mu - p^*$.

Next, we show there is no design in which seller payoff is less than q^*. Suppose by way of contradiction seller payoff is only $\hat{q} < q^*$ in a design. Let $\hat{L}$ and $\hat{S}_N$ denote the associated signal structure and no information acquisition set. Then for any offer $p \in (\hat{q}, \mu]$, it must be that $p \notin \hat{S}_N$ so that the buyer acquires information—else the buyer would purchase the good (because $p \leq \mu$) and the seller's payoff would be $p > \hat{q}$. Thus, $\hat{q}$ must be weakly higher than the expected payoff $p(1 - \hat{L}(p) + \beta_{\hat{L}}(p))$ from offering a price $p \in (\hat{q}, \mu]$. Thus, for all $s \in (\hat{q}, \mu]$,

$$\hat{q} \ \geq \ p(1 - \hat{L}(p) + \beta_{\hat{L}}(p)) \ \Rightarrow \ \hat{L}(s) - \beta_{\hat{L}}(s) \geq 1 - \frac{\hat{q}}{s} \ \Rightarrow \ \hat{L}(s) \geq 1 - \frac{\hat{q}}{s},$$

where the last inequality follows from $\beta_{\hat{L}} \geq 0$. $\hat{L}(s)$ is non-decreasing, so $\hat{L}(s) \geq \hat{L}(\mu) \geq 1 - \frac{\hat{q}}{\mu} = L_{\hat{q}}^1(s)$ for all $s \in (\mu, 1)$. Moreover, $\hat{L}(s) \geq L_{\hat{q}}^1(s) = 0$ for all $s \in [0, \hat{q}]$, and $\hat{L}(1) = L_{\hat{q}}^1(1) = 1$. Hence,

$$\hat{L}(s) \geq L_{\hat{q}}^1(s), \quad \text{for all } s \in [0,1]\,. \tag{A.8}$$

Because $\int_0^1 \hat{L}(s)ds = 1 - \mu$ by (1), (A.8) yields $\int_0^1 L_{\hat{q}}^1(s)ds \leq 1 - \mu$. This implies $\hat{q} \geq q_{\min}$ by equation (A.1) and the fact that $L_q^B(s)$ decreases in q.

Next, we claim that

$$\int_0^x L_{\hat{q}}^{B(\hat{q})}(s)ds \leq \int_0^x \hat{L}(s)ds \leq \int_0^x F(s)ds, \ \forall x \in [0,1]. \tag{A.9}$$

The claim implies that $\hat{q} \in [q_{\min}, \mu]$ satisfies (A.7) for all $x \in [0,1]$, contradicting the premise that q^* is the smallest value satisfying (A.7). To prove the claim, note that the second inequality in (A.9) follows because F is a mean-preserving spread of $\hat{L}$. From (A.8), we have $\int_0^x \hat{L}(s)ds \geq \int_0^x L_{\hat{q}}^1(s)ds$ for all $x \in [0,1]$—we can then use Lemma A.3(ii) to establish the first inequality in (A.9), which completes the proof of our claim.

Finally, because valuations are always nonnegative ($\underline{v} = 0$), the maximum social welfare gain is μ, so no design can generate a buyer payoff that exceeds $\mu - p^*$. $\square$

Proof of Proposition 2: The proof of part (i) is immediate, since the buyer in our model can guarantee the same payoff as the buyer in RS by choosing $S_N = \emptyset$ and signal $G_{q^{RS}}^{B^{RS}}(s)$. To prove part (ii), we prove the following claims.

Claim 1: $q_{\min} < q^{RS}$. Proof: note that $L_{q^{RS}}^1(s) \leq G_{q^{RS}}^{B^{RS}}(s)$ for all $s \in [0,1]$, and the inequality is strict for a positive measure of s. Therefore, $\int_0^1 L_{q^{RS}}^1(s)ds < \int_0^1 G_{q^{RS}}^{B^{RS}}(s)ds = 1 - \mu$. By equation (A.1) and the fact that $L_q^B(s)$ decreases in q, we have $q_{\min} < q^{RS}$.

Next, for $q \in (0, \mu)$, define $\hat{B}(q)$ to be such that the distribution $G_q^{\hat{B}(q)}$ has mean μ:

$$\int_0^1 G_q^{\hat{B}(q)}(s)ds = \int_0^1 F(s)ds = 1 - \mu. \tag{A.10}$$

Then $B^{RS} = \hat{B}\left(q^{RS}\right)$.

Claim 2: $x^* \leq B^{RS}$. Proof: by contradiction, suppose that $x^* > B^{RS}$. Since x^* is the smallest $x \in [q^{RS}, 1]$ such that $\int_0^x G_{q^{RS}}^{B^{RS}}(s)ds = \int_0^x F(s)ds$, we have $\int_0^x G_{q^{RS}}^{B^{RS}}(s)ds < \int_0^x F(s)ds$, $\forall x \in \left[q^{RS}, B^{RS}\right]$, or $\int_0^x \left(F(s) - G_{q^{RS}}^{B^{RS}}(s)\right) ds > 0$ (as $q^{RS} < B^{RS}$ since F is not degenerate). Because the interval $\left[q^{RS}, B^{RS}\right]$ is closed, there exists some $\delta > 0$ such that $\int_0^x \left(F(s) - G_{q^{RS}}^{B^{RS}}(s)\right) ds > \delta$ for all $x \in \left[q^{RS}, B^{RS}\right]$. Then, by continuity, for all $\epsilon \in \left(0, q^{RS}\right)$ sufficiently close to zero, we have $\int_0^x \left(F(s) - G_{q^{RS}-\epsilon}^{\hat{B}\left(q^{RS}-\epsilon\right)}(s)\right) ds > 0$, $\forall x \in \left[q^{RS} - \epsilon, \hat{B}\left(q^{RS} - \epsilon\right)\right]$ under the premise of the proposition.[8] From (3),

[8]Either there is no atom in F at $s = q^{RS}$, or the total mass under F is strictly positive for $s < q^{RS}$.

$\int_0^x \left(F(s) - G_{q^{RS}-\epsilon}^{\hat{B}\left(q^{RS}-\epsilon\right)}(s)\right) ds \geq 0$, $\forall x \in \left[0, q^{RS}-\epsilon\right]$. The proof of Lemma A.3(i) also applies to $G_q^{B(q)}$, so $\int_0^x \left(F(s) - G_{q^{RS}-\epsilon}^{\hat{B}\left(q^{RS}-\epsilon\right)}(s)\right) ds \geq 0$, $\forall x \in \left[\hat{B}\left(q^{RS}-\epsilon\right), 1\right]$. Thus, $\int_0^x G_{q^{RS}-\epsilon}^{\hat{B}\left(q^{RS}-\epsilon\right)}(s)ds \leq \int_0^x F(s)ds$ for all $x \in [0,1]$. This contradicts the premise that q^{RS} is the smallest q such that $\int_0^x G_q^{\hat{B}(q)}(s)ds \leq \int_0^x F(s)ds$ for all $x \in [0,1]$, establishing Claim 2.

Claim 3: if $x^* > \mu$, then $q^* < q^{RS}$. Proof: suppose $x^* > \mu$. By Claim 2 and the definition of $L_{q^{RS}}^1(s)$, we have $L_{q^{RS}}^1(s) < G_{q^{RS}}^{B^{RS}}(s)$ for all $s \in (\mu, x^*)$, and $L_{q^{RS}}^1(s) \leq G_{q^{RS}}^{B^{RS}}(s)$ for all $s \in [0,1]$. Thus, for all $x \in (\mu, 1]$,

$$\int_0^x L_{q^{RS}}^1(s)ds < \int_0^x G_{q^{RS}}^{B^{RS}}(s)ds \leq \int_0^x F(s)ds, \tag{A.11}$$

where the second inequality obtains since F is a mean-preserving spread of $G_{q^{RS}}^{B^{RS}}$. Since $x^* > \mu$ is the smallest $x \in [q^{RS}, 1]$ such that $\int_0^x G_{q^{RS}}^{B^{RS}}(s)ds = \int_0^x F(s)ds$, we have:

$$\int_0^x L_{q^{RS}}^1(s)ds = \int_0^x G_{q^{RS}}^{B^{RS}}(s)ds < \int_0^x F(s)ds, \quad \text{for all } x \in \left[q^{RS}, \mu\right]. \tag{A.12}$$

From (A.11) and (A.12), there exists some $\delta > 0$ such that $\int_0^x F(s)ds - \int_0^x L_{q^{RS}}^1(s)ds > \delta$ for all $x \in [q^{RS}, 1]$. Thus, there exists a $\hat{q} \in [q_{\min}, q^{RS})$ sufficiently close to q^{RS} such that $\int_0^x L_{\hat{q}}^1(s)ds \leq \int_0^x F(s)ds$ for all $x \in [\hat{q}, 1]$ — see footnote 8. The inequality extends to the entire interval $x \in [0,1]$ because $L_{\hat{q}}^1(s) = 0$ when $s \in [0, \hat{q}]$. We can then apply the result from Lemma A.3(ii) to conclude that $\int_0^x L_{\hat{q}}^{B(\hat{q})}(s)ds \leq \int_0^x F(s)ds$ for all $x \in [0,1]$. Because $q^* \leq \hat{q}$ and $\hat{q} < q^{RS}$, we have $q^* < q^{RS}$, establishing Claim 3.

Claim 4: If $x^* \leq \mu$, then $q^* = q^{RS}$. Proof: suppose $x^* \leq \mu$. Because Proposition 2(i) already shows that $q^* \leq q^{RS}$, it suffices to show that $q^* \geq q^{RS}$.

By definition, $L_{q^{RS}}^{B\left(q^{RS}\right)}(s) = G_{q^{RS}}^{B\left(q^{RS}\right)}(s)$ for all $s \leq \mu$. Thus, at x^* we have $\int_0^{x^*} L_{q^{RS}}^{B\left(q^{RS}\right)}(s)ds = \int_0^{x^*} G_{q^{RS}}^{B\left(q^{RS}\right)}(s)ds = \int_0^{x^*} F(s)ds$. Since $\int_0^{x^*} L_{\hat{q}}^{B(\hat{q})}(s)ds > \int_0^{x^*} L_{q^{RS}}^{B\left(q^{RS}\right)}(s)ds$ for all $\hat{q} < q^{RS}$, it follows that the mean-spreading condition $\int_0^{x^*} L_{\hat{q}}^{B(\hat{q})}(s)ds \leq \int_0^{x^*} F(s)ds$ is violated for all $\hat{q} < q^{RS}$, which implies that $q^* \geq q^{RS}$. $\square$

Lemma A.5 *(i) The information structure* $\bar{L}_{\bar{\gamma},\bar{\lambda},\bar{q}}^{B\left(\bar{\gamma},\bar{\lambda},\bar{q}\right)}$ *with* $S_N = (\mu, 1]$ *maximizes buyer payoff in Scenario 2, where the parameters* $\bar{\gamma}, \bar{\lambda}, \bar{q}$ *solve:*

$$\min_{\gamma,\lambda,q} \gamma\lambda + (1-\gamma)q$$
$$s.t.\ \ \bar{L}_{\bar{\gamma},\bar{\lambda},\bar{q}}^{B\left(\bar{\gamma},\bar{\lambda},\bar{q}\right)} \in \mathcal{L}_F.$$

(ii) In equilibrium the seller offers $\bar{q}$*; and the buyer acquires information with net payoff*

$$\mu - C - \bar{\gamma}\bar{\lambda} - (1-\bar{\gamma})\,\bar{q}. \tag{A.13}$$

Proof: We first show that for any distribution H of which F is a mean-preserving spread, we can find γ, λ, q such that the distribution $\bar{L}_{\gamma,\lambda,q}^{B(\gamma,\lambda,q)}$ generates weakly higher buyer payoff than H under scenario 2 (where the buyer acquires information and buys the good with probability less than one). Let γ^H be the probability of receiving a negative signal $s < 0$ under H. Consider 2 cases.

Case 1: If $\gamma^H \in (0,1)$ then set $\gamma = \gamma^H$. Let H^- and H^+ be the distributions of the signal, conditional on the signal being negative and nonnegative, respectively, where $H = \gamma H^- + (1-\gamma)\,H^+$. Trade does not occur when the signal is negative. Thus, under H, buyer payoff is $1-\gamma$ times the payoff in a hypothetical setting where her valuation is always drawn from H^+.

Set λ to be the mean of the distribution H^-. We can replace H^- by a degenerate distribution that places all probability mass at λ: neither the seller's equilibrium offer nor the buyer's payoff changes.

Next, consider the following distribution $L_q^B(s)$ constructed via (4), where the value of μ in the second and third line of (4) refers to the mean for the original distribution of F (not the mean for the distribution H^+), and B is such that the mean for the distribution $L_q^B(s)$ equals the mean for distribution H^+. Using the same logic as in the proof of Proposition 1, there exist $q \in (0, \mu]$ and $B \in [\mu^+, 1]$ such that H^+

is a mean-preserving spread of L_q^B,[9] and buyer's payoff under L_q^B is weakly higher.

Note that B depends on γ, λ, q: as the mean of L_q^B equals the mean μ^+ of H^+, $\bar{L}^B_{\gamma,\lambda,q}$ has mean μ, where $\mu = \gamma\lambda + (1-\gamma)\mu^+$ implies

$$\mu^+ = \frac{1}{1-\gamma^H}\left(\mu - \gamma\lambda\right).$$

Case 2: If $\gamma^H = 0$, then the same logic as in Case 1 applies (and the steps are simpler).

This proves Lemma A.5 (i). To prove (ii), note that buyer payoff under $\bar{L}^B_{\gamma,\lambda,q}$ is

$$\begin{aligned}(1-\gamma)\left(\mu^+ - q\right) - C &= (1-\gamma)\left(\frac{1}{1-\gamma}\left(\mu - \gamma\lambda\right) - q\right) - C \\ &= \mu - C - \gamma\lambda - (1-\gamma)q, \end{aligned} \tag{A.14}$$

which is just (A.13). □

Proof of Proposition 4: In equilibrium, either the buyer purchases the good with probability one or with probability strictly less than one. If the buyer purchases with probability one, then the logic of Proposition 1 yields that the signal $L_{q^{**}}^{B(q^{**})}$ described by scheme 1 together with the no information set $S_N = [0, q^{**}] \cup (\mu, 1]$ is optimal. This maximizes buyer payoff from trade and the buyer does not incur information cost C. If Scheme 1 is used, expected buyer payoff is $\mu - q^{**}$. If the buyer purchases with probability less than one, Lemma A.5 yields that the signal detailed in Scheme 2 is optimal and expected buyer payoff is $\mu - C - \bar{\gamma}\bar{\lambda} - (1-\bar{\gamma})\bar{q}$.

Therefore, Scheme 1 is optimal if $\mu - q^{**} \geq \mu - C - \bar{\gamma}\bar{\lambda} - (1-\bar{\gamma})\bar{q}$, which implies (4). Otherwise, Scheme 2 is optimal. □

Proof of Proposition 5: To prove part (i), note that buyer payoff in the (adjusted) RS setting is zero for all $C \geq \rho^{RS}_{C=0}$, so the buyer can always do weakly better under no commitment (the buyer can ensure a zero payoff by not acquiring information). It remains to prove that the buyer is weakly better off for lower costs, and strictly

[9] If the distribution H^+ has no probability mass below μ, then set $q = \mu$ and $B = \mu^+$: the resulting distribution $L_{q=\mu}^{B=\mu^+}$ assigns probability one to $s = \mu^+$.

better off for a range of costs.

Claim 1: For all $C \in [0, \rho^{RS}_{C=0})$, setting $L^{NC} = G^{RS}$ in our design (i.e., the buyer commits to the information structure G^{RS} but decides ex post whether to acquire information) yields a buyer payoff of $\rho^{RS}_{C=0} - C$. Hence, the optimal signal L must be weakly better for the buyer.

Proof: To prove the claim, we show that in our design with $L^{NC} = G^{RS}$, the seller offers q^{RS} and the buyer investigates. Thus, equilibrium outcomes are the same as in RS.

First we show the buyer acquires information for any $p \in (0, q^{RS}]$. The payoff from not acquiring information is zero, as no purchase will be made (since $p > 0 \geq \mu$). The payoff from acquiring information is $\int_p^1 (s-p)\, dG^{RS}(s) - C$, which decreases in p. This payoff is at least

$$\int_{q^{RS}}^1 (s-p)\, dG^{RS}(s) - C = \rho^{RS}_{C=0} - C > 0,$$

implying that the buyer will acquire information. Since the probability of purchase is the same for all $p \in (0, q^{RS}]$, offering q^{RS} dominates offering $p < q^{RS}$ for the seller.

Next consider $p > q^{RS}$. If the buyer investigates, seller payoff cannot exceed that from offering q^{RS} (this follows from the finding in RS that the seller optimally offers q^{RS} if the buyer always investigates). If the buyer does not investigate, seller payoff is zero because the buyer will not purchase ($\mu \leq 0$). This proves Claim 1. □

To prove part (ii) we show there is a positive measure of $C > \rho^{RS}_{C=0}$ for which we can design L^{NC} so that buyer payoff net of information acquisition costs is strictly positive, which exceeds the payoff of zero in the RS setting. Define

$$\bar{\epsilon} \equiv \frac{1 - \int_0^1 F(x)dx}{1 + \int_{\underline{v}}^0 F(x)dx}.$$

By the proposition's premise that $\int_0^1 F(x)dx < 1$, we have $\bar{\epsilon} \in (0,1)$.

Claim 2: For all $\epsilon \in (0, \bar{\epsilon}]$, there is a unique $B \in (\epsilon, 1]$ such that $\Delta(\epsilon, B) = 0$, where

$$\Delta(\epsilon, B) \equiv (1-\epsilon) F(0)\epsilon + F(0)(B-\epsilon) + (1-B) - \epsilon \int_{\underline{v}}^{0} F(x)dx - \int_{0}^{1} F(x)dx. \quad \text{(A.15)}$$

Proof: $\Delta(\epsilon, B)$ decreases in B. Moreover, at $B = \epsilon$,

$$\begin{aligned}\Delta(\epsilon, \epsilon) &= (1-\epsilon) F(0)\epsilon + 1 - \epsilon - \epsilon \int_{\underline{v}}^{0} F(x)dx - \int_{0}^{1} F(x)dx \\ &> 1 - \epsilon - \epsilon \int_{\underline{v}}^{0} F(x)dx - \int_{0}^{1} F(x)dx > 1 - \bar{\epsilon} - \bar{\epsilon} \int_{\underline{v}}^{0} F(x)dx - \int_{0}^{1} F(x)dx = 0,\end{aligned}$$

and at $B = 1$,

$$\begin{aligned}\Delta(\epsilon, 1) &= (1-\epsilon) F(0)\epsilon + F(0)(1-\epsilon) - \epsilon \int_{\underline{v}}^{0} F(x)dx - \int_{0}^{1} F(x)dx \\ &< F(0)\epsilon + F(0)(1-\epsilon) - \epsilon \int_{\underline{v}}^{0} F(x)dx - \int_{0}^{1} F(x)dx \\ &= F(0) - \epsilon \int_{\underline{v}}^{0} F(x)dx - \int_{0}^{1} F(x)dx < F(0) - \int_{0}^{1} F(x)dx < 0.\end{aligned}$$

The intermediate value theorem then yields Claim 2. □

For $\epsilon \in (0, \bar{\epsilon}]$, let $B(\epsilon)$ solve $\Delta(\epsilon, B) = 0$. Consider the information design:

$$L_{\epsilon}^{PC}(s) = \begin{cases} (1-\epsilon) F(s) & \text{if } s \leq 0 \\ (1-\epsilon) F(0) & \text{if } s \in (0, \epsilon) \\ F(0) & \text{if } s \in [\epsilon, B(\epsilon)) \\ 1 & \text{if } s \geq B(\epsilon). \end{cases} \quad \text{(A.16)}$$

The design has a point mass of $\epsilon F(0)$ at $s = \epsilon$, a point mass of $1 - F(0)$ at $s = B(\epsilon)$, and the distribution over negative valuations is scaled down from F by a factor $(1-\epsilon)$.

Claim 3: F is a mean-preserving spread of $L_{\epsilon}^{PC}(s)$.

Proof: We have

$$\int_{\underline{v}}^{1} L_{\epsilon}^{NC}(x)dx = (1-\epsilon) \int_{\underline{v}}^{0} F(x)dx + (1-\epsilon) F(0)\epsilon + F(0)(B-\epsilon) + (1-B)$$

and

$$\int_{\underline{v}}^{1} F(x)dx = \int_{\underline{v}}^{0} F\left(x\right)dx + \int_{0}^{1} F(x)dx.$$

Taking the difference yields

$$\int_{\underline{v}}^{1} L_{\epsilon}^{NC}(x)dx - \int_{\underline{v}}^{1} F(x)dx = \Delta\left(\epsilon, B\left(\epsilon\right)\right).$$

By Claim 2 and the fact that $L_{\epsilon}^{NC}(x) \leq F(x)$ for all $x < B\left(\epsilon\right)$ and $L_{\epsilon}^{NC}(x) \geq F\left(x\right)$ for all $x > B\left(\epsilon\right)$, Claim 3 follows. □

Define $\pi^{surp} \equiv \int_0^1 vdF\left(v\right)$, to be the maximum possible social surplus (gross of C). Note that $\rho_{C=0}^{RS} < \pi^{surp}$ because the design cannot simultaneously achieve all three of the following: trade always occurs when $v > 0$; trade never occurs when $v < 0$; seller payoff is zero. Defining $\kappa\left(\epsilon\right) \equiv \left(B\left(\epsilon\right) - \epsilon\right)\left(1 - F(0)\right)$, we show:

Claim 4: There exists $\epsilon^* \in (0, \bar{\epsilon})$ such that $\kappa\left(\epsilon^*\right) \in \left(\rho_{C=0}^{RS}, \pi^{surp}\right)$.

Proof: From (A.15), the term $\Delta\left(\epsilon, B\right)$ decreases in B and ϵ. Therefore, the implicit function theorem yields that $B\left(\epsilon\right)$ decreases in ϵ. As ϵ goes to zero, $\Delta\left(\epsilon, B\right)$ approaches $1 - \int_0^1 F(x)dx - B\left(1 - F(0)\right)$, hence $\Delta\left(\epsilon, B\left(\epsilon\right)\right) = 0$ yields

$$\lim_{\epsilon \to 0} B\left(\epsilon\right) = \frac{1 - \int_0^1 F(x)dx}{1 - F(0)} = \frac{\pi^{surp}}{1 - F\left(0\right)}.$$

The final equality uses $\int_0^1 xdF(x) = 1 - \int_0^1 F(x)dx$. Thus, $\lim_{\epsilon \to 0} \kappa\left(\epsilon\right) = \pi^{surp}$. Since $\kappa\left(\epsilon\right)$ continuously decreases in ϵ, Claim 4 follows. □

Claim 5: There exists $v^* \in (\epsilon^*, \min\{\frac{1-(1-\epsilon^*)F(0)}{1-F(0)}\epsilon^*, B(\epsilon^*)\})$ with $\eta(v^*) > \rho_{C=0}^{RS}$, where

$$\eta\left(v^*\right) \equiv \left(B\left(\epsilon^*\right) - v^*\right)\left(1 - F(0)\right).$$

Proof: $\eta\left(\epsilon^*\right) = \left(B\left(\epsilon^*\right) - \epsilon^*\right)\left(1 - F(0)\right) = \kappa\left(\epsilon^*\right) > \rho_{C=0}^{RS}$, where the inequality follows from Claim 4. Because $\eta\left(v^*\right)$ is continuous in v^*, Claim 5 follows. □

Because $\eta\left(\cdot\right)$ is a decreasing function and $v^* > \epsilon^*$, we have $\eta\left(v^*\right) < \eta\left(\epsilon^*\right)$.

Claim 6: For any $C \in (\eta(v^*), \eta(\epsilon^*))$, the design $L^{NC}_{\epsilon^*}(s)$ (which is given by (A.16) with ϵ^* replacing ϵ) leads to strictly positive buyer payoff.

Proof: For any $p \in [\epsilon^*, B(\epsilon^*)]$, the payoff from not acquiring information is zero as no purchase is made (since $p \geq \epsilon^* > 0 \geq \mu$). Buyer payoff from acquiring information is

$$\Pi(p) \equiv \int_p^1 (s-p)\, dL^{NC}_{\epsilon^*}(s) - C$$
$$= (B(\epsilon^*) - p)(1 - F(0)) - C = \eta(p) - C, \tag{A.17}$$

which strictly decreases in p. By the premise that $C \in (\eta(v^*), \eta(\epsilon^*))$, we have $\Pi(\epsilon^*) > 0$ and $\Pi(v^*) < 0$. Thus, there exists $\hat{v} \in (\epsilon^*, v^*)$ such that $\Pi(p) = 0$. Therefore, the buyer will investigate if and only if $p \in [\epsilon^*, \hat{v}]$.

Given the buyer's investigation strategy, it is not optimal for the seller to offer $p > \hat{v}$ or $p < \epsilon^*$. Denote the expected seller payoff from offering $p \in [\epsilon^*, \hat{v}]$ by $\pi_{seller}(p)$. Recalling that there is a point mass $\beta_{L^{NC}}(\epsilon^*) = \epsilon^* F(0)$ at ϵ^*, and the buyer accepts an offer when her payoff is nonnegative, the seller's payoff from offering ϵ^* is

$$\pi_{seller}(\epsilon^*) = \epsilon^*(1 - F(0) + \epsilon^* F(0)) = \epsilon^*(1 - (1-\epsilon^*)F(0)),$$

and the seller's payoff from offering $p \in (\epsilon^*, \hat{v}]$ is

$$\pi_{seller}(p) = p(1 - F(0)) \leq \hat{v}(1 - F(0)).$$

From Claim 5, we have $\hat{v} < v^* < \frac{1-(1-\epsilon^*)F(0)}{1-F(0)}\epsilon^*$. Thus, for $p \in (\epsilon^*, \hat{v}]$,

$$\pi_{seller}(p) < \left(\frac{1-(1-\epsilon^*)F(0)}{1-F(0)}\epsilon^*\right)(1 - F(0)) = \pi_{seller}(\epsilon^*).$$

It is then optimal for the seller to offer $p = \epsilon^*$, yielding buyer payoff (equation (A.17)):

$$\Pi(\epsilon^*) = \eta(\epsilon^*) - C > 0. \quad \square$$

The optimally-designed L^{NC} can only do better than a specific choice of L^{NC}, and the (adjusted) RS buyer payoff is zero[10] for $C > \eta(v^*)$, so part (ii) follows. $\square$

Proof of Claims in Proposition 6: We first show that the original distribution F is a mean-preserving spread of L^{NC}. By (10),

$$\begin{aligned}\int_{\underline{v}}^{\bar{v}} L^{NC}(s)ds &= \int_{\underline{v}}^{q^{NC}} \gamma ds + \int_{q^{NC}}^{k} \left[1-(1-\gamma)\frac{q^{NC}}{s}\right] ds + \int_{k}^{\bar{v}} \left[1-(1-\gamma)\frac{q^{NC}}{k}\right] ds \\ &= \gamma(q^{NC}-\underline{v}) + \left(k-q^{NC}\right) - (1-\gamma)\int_{q^{NC}}^{k} \frac{q^{NC}}{s} ds + \left(1-(1-\gamma)\frac{q^{NC}}{k}\right)(\bar{v}-k) \\ &= \gamma(q^{NC}-\underline{v}) + \left(k-q^{NC}\right) - (1-\gamma)q^{NC}\ln\frac{k}{q^{NC}} + \left(1-(1-\gamma)\frac{q^{NC}}{k}\right)(\bar{v}-k) \\ &= \bar{v} - \gamma\underline{v} - (1-\gamma)q^{NC}\left(\frac{\bar{v}}{k} + \ln\frac{k}{q^{NC}}\right), \qquad \text{(A.18)}\end{aligned}$$

which is the left-hand side of (11) upon plugging in (12) for q^{NC}, plus $\bar{v}-\mu$. Thus, $\int_{\underline{v}}^{\bar{v}} L^{NC}(s)ds = \bar{v}-\mu$, which yields $\int_{\underline{v}}^{\bar{v}} sdL^{NC}(s)ds = \mu$ through integration by parts. Hence $L^{NC}(s)$ has the same mean as the original distribution F. This guarantees that F is a mean-preserving spread of $L^{NC}(s)$ (since F is flat over $(\underline{v}, \bar{v})$).

Next we show this design is optimal. We first establish some general results. Consider any distribution K with support $[\underline{v}, \bar{v}]$ and mean $\mu \in (-\infty, \infty)$. For $p \geq \max\{\mu, 0\}$, the gross-of-C payoff to the buyer from acquiring information at price p is

$$\Pi(p) \equiv \int_{p}^{\bar{v}} (s-p)\, dK(s).$$

If $C \geq \Pi(0)$, the buyer's expected payoff is zero, so any signal distribution (including our proposed L^{NC}) is weakly better than K. Thus, for the rest of the proof we only consider $C < \Pi(0)$, so there exist nonnegative prices for which the buyer investigates.

Lemma A.6 *Let $\tau \in [\max\{\mu, 0\}, \bar{v}]$ be the offer price at which the buyer is indiffer-*

[10] Claim 5 established that $\eta(v^*) > \rho^{RS}_{C=0}$, so the (adjusted) RS buyer will not acquire information.

ent between acquiring information or not, i.e., $\Pi(\tau) \equiv C$*. Then*

$$(\bar{v} - \tau)(1 - K(\tau)) \geq C \tag{A.19}$$

and

$$\int_{\underline{v}}^{\tau} K(s)ds - \tau = C - \mu. \tag{A.20}$$

Proof: Using integration by parts, we have

$$\begin{aligned} \Pi(\tau) = -\int_{\tau}^{\bar{v}} (s-\tau)\, d(1 - K(s)) &= \int_{\tau}^{\bar{v}} (1 - K(s))\, ds \qquad \text{(A.21)} \\ &\leq \int_{\tau}^{\bar{v}} (1 - K(\tau))\, ds = (\bar{v} - \tau)(1 - K(\tau)), \end{aligned}$$

which yields (A.19). Since $\Pi(\tau) = C$, (A.21) implies $\int_{\tau}^{\bar{v}} (1 - K(s))\, ds = C$, hence

$$\int_{\tau}^{\bar{v}} K(s)ds = \bar{v} - \tau - C. \tag{A.22}$$

Next, use

$$\mu = \int_{\underline{v}}^{\bar{v}} s dK(s) = \bar{v} - \int_{\underline{v}}^{\bar{v}} K(s)ds$$

to obtain

$$\int_{\underline{v}}^{\tau} K(s)ds = \bar{v} - \mu - \int_{\tau}^{\bar{v}} K(s)ds. \tag{A.23}$$

Result (A.20) follows from substituting (A.22) into the last term of (A.23):

$$\int_{\underline{v}}^{\tau} K(s)ds = \bar{v} - \mu - (\bar{v} - \tau - C). \quad \square$$

Note that in the special case with costless information ($C = 0$) and $\bar{v}$ being in the support of K, we have $\tau = \bar{v}$, and (A.20) reduces to $\int_{\underline{v}}^{\bar{v}} K(s)ds = \bar{v} - \mu$, which is the standard relation between a distribution and its mean. (A.20) extends this to settings with an endogenous decision to acquire costly information. For future reference, note also that the right-hand side of (A.20) is independent of the details of the distribution K.

Lemma A.7 *Consider two distributions* K_1 *and* K_2 *with support* $[\underline{v}, \bar{v}]$ *and mean* μ. *Let* τ_1 *and* τ_2 *be the associated offer prices that leave the buyer indifferent between acquiring information or not (as defined in Lemma A.6). Suppose*

$$\int_{\underline{v}}^{\tau_1} K_1(s)ds \geq \int_{\underline{v}}^{\tau_1} K_2(s)ds \tag{A.24}$$

and $K_2(\tau_2) < 1$. *Then* $\tau_1 \geq \tau_2$.

Proof: By the premises of the lemma,

$$\int_{\underline{v}}^{\tau_2} K_2(s)ds - \tau_2 = \int_{\underline{v}}^{\tau_1} K_1(s)ds - \tau_1 \geq \int_{\underline{v}}^{\tau_1} K_2(s)ds - \tau_1, \tag{A.25}$$

where the equality follows from (A.20) and the inequality follows from (A.24). The inequality $\int_{\underline{v}}^{\tau_2} K_2(s)ds - \tau_2 \geq \int_{\underline{v}}^{\tau_1} K_2(s)ds - \tau_1$ implies $\int_{\tau_1}^{\tau_2} K_2(s)ds \geq \tau_2 - \tau_1$, which, by the premise $K_2(\tau_2) < 1$, yields $\tau_1 \geq \tau_2$. $\square$

Lemma A.8 *Suppose that in equilibrium the buyer acquires information according to* K *with lower support* $\underline{v}$ *and mean* μ. *Then the gains from trade (gross of* C*) satisfy*

$$\textit{Gains from trade (gross of } C\textit{)} \leq \mu - \int_{\underline{v}}^{0} sdK(s) = \mu + \int_{\underline{v}}^{0} K(s)ds, \tag{A.26}$$

where equality holds if trades always occur conditional on $s > 0$.

Proof: If all trades take place regardless of realization of s, then the gains from trade would be μ. Next note that trades will not take place for any $s < 0$, increasing gains from trade by $-\int_{\underline{v}}^{0} sdK(s)$. Furthermore, if trades do not take place for some $s > 0$, it would lead to a reduction in the gains from trade. This establishes the lemma. $\square$

Next, consider any distribution H for which the binary distribution F is a mean-preserving spread. Set

$$\bar{\gamma} \equiv \frac{-1}{\underline{v}} \int_{\underline{v}}^{0} H(s)\, ds, \tag{A.27}$$

and let $\bar{q}^{NC}$ and $\bar{k}$ be the solutions for q^{NC} and k to (11) and (12), for the given $\bar{\gamma}$.[11] Denote the resulting L^{NC} by $\bar{L}^{NC}$. We show buyer payoff under H does not exceed that under $\bar{L}^{NC}$. We can restrict attention to H for which the buyer acquires information in equilibrium (her payoff from not acquiring information is zero, which is less than that in our optimal design).

Lemma A.9 *Seller payoff under H is no less than that under $\bar{L}^{NC}$.*

Proof: Seller payoff under $\bar{L}^{NC}$ is $(1-\bar{\gamma})\,\bar{q}$, while it is $p(1-H(p)+\beta_H(p))$ under H for any price p such that the buyer acquires information. By contradiction, suppose H generates strictly lower seller payoff than $\bar{L}^{NC}$. Let $\tau \in (0,\bar{v})$ be the offer price that leaves the buyer indifferent between acquiring information or not under H. Then for all $p \in (0,\tau]$

$$\begin{aligned}(1-\bar{\gamma})\,\bar{q} > p(1-H(p)+\beta_H(p)) &\Rightarrow H\left(p\right)-\beta_H(p) > 1-\frac{(1-\bar{\gamma})\,\bar{q}}{p}. \\ &\Rightarrow H\left(p\right) > 1-\frac{(1-\bar{\gamma})\,\bar{q}}{p}. \end{aligned} \tag{A.28}$$

Claim 1: For all $s \in (0,\tau]$, $H(s) \geq \bar{L}^{NC}\left(s\right)$. Proof: We have

$$\bar{L}^{NC}(s) = \max\left\{\bar{\gamma}, 1-\frac{(1-\bar{\gamma})\,\bar{q}}{s}\right\} \text{ for } s \in (0,\bar{k}],$$

$$\bar{L}^{NC}(s) = \bar{L}^{NC}\left(\bar{k}\right) < 1-\frac{(1-\bar{\gamma})\,\bar{q}}{s} \text{ for } s \in (\bar{k},\bar{v}),$$

and

$$H(s) \geq \max\left\{H\left(0\right), 1-\frac{(1-\bar{\gamma})\,\bar{q}}{s}\right\} \text{ for } s \in (0,\tau].$$

By (A.27) and $H(s)$ nondecreasing, $\bar{\gamma} \leq H(0)$. These relations establish Claim 1.

In addition, (A.27) and the definition of $\bar{L}^{NC}$ yield $\int_{\underline{v}}^{0} H(s)ds = \int_{\underline{v}}^{0} \bar{L}^{NC}(s)ds$. Thus Claim 1 yields $\int_{\underline{v}}^{\tau} H(s)ds \geq \int_{\underline{v}}^{\tau} \bar{L}^{NC}\left(s\right)ds$. By Lemma A.7, $\tau \geq \bar{k}$, which

[11] Note that $\underline{v} < 0$ and $\int_{\underline{v}}^{0} H\left(s\right)ds \geq 0$, hence $\bar{\gamma} \geq 0$. Because F is a mean preserving spread of H and $F(s) = \omega$ for $s \in [\underline{v}, 0]$, we have $(-1/\underline{v})\int_{\underline{v}}^{0} H(s)ds \leq (-1/\underline{v})\int_{\underline{v}}^{0} F(s)ds = \omega$. Hence, $\bar{\gamma} \leq \omega$.

implies $(\bar{v}-\tau) \leq (\bar{v}-\bar{k})$. Then (A.28) yields $H(\tau) > 1 - \frac{(1-\bar{\gamma})\bar{q}}{\bar{k}}$. Rewriting yields

$$1 - H(\tau) < \frac{(1-\bar{\gamma})\bar{q}}{\bar{k}}$$
$$\Rightarrow (\bar{v}-\tau)(1-H(\tau)) < (\bar{v}-\bar{k})\frac{(1-\bar{\gamma})\bar{q}}{\bar{k}} = C, \tag{A.29}$$

where the last equality follows from (12), contradicting (A.19), establishing the lemma.

Next, note that buyer payoff equals expected social surplus (expected gains from trade) minus the seller's payoff and the information cost. Refer to (A.26). Note that H and $\bar{L}^{NC}$ have the same mean, $\int_{\underline{v}}^{0} H(s)ds = \int_{\underline{v}}^{0} \bar{L}^{NC}(s)ds$, and that (A.26) holds as an equality for $\bar{L}^{NC}$. By this and Lemma A.9, buyer payoff under $\bar{L}^{NC}$ is at least that under H. Because γ in our design is optimally chosen, this proves its optimality. □

References

Bergemann, D., B. Brooks, and S. Morris, "The limits of price discrimination." *American Economic Review* 105.3, 2015: 921-957.

Bergemann, D., B. Brooks, and S. Morris "First-Price Auctions With General Information Structures: Implications for Bidding and Revenue," *Econometrica*, 85, 2017: 107-143.

Bloedel, A. and I. Segal. "Persuading a Rationally Inattentive Agent". WP, 2021.

Condorelli, D., and B. Szentes, "Information design in the holdup problem," *Journal of Political Economy* 128.2, 2020: 681-709.

Du, S., "Robust mechanisms under common valuation," *Econometrica* 86, 2018: 1569-1588.

Johnson, J. and D. Myatt, "On the simple economics of advertising, marketing, and product design," *American Economic Review*, 96.3, 2006: 756-784.

Kartik, N. and W. Zhong, "Lemonade from Lemons: Information Design and Adverse

Selection", WP, 2023.

Li, H. and X. Shi, "Discriminatory information disclosure," *American Economic Review* 107.11, 2017: 3363-3385.

Libgober, J. and X. Mu, "Informational robustness in intertemporal pricing," *Review of Economic Studies*, 88(3), 2021: 1224-1252.

Lipnowski, E., L. Mathevet, and D. Wei. "Attention management." *American Economic Review, Insights* 2 (1), 2020: 17-32.

Lipnowski, E., L. Mathevet, and D. Wei "Optimal attention management: A tractable framework," *Games and Economic Behavior*, 133, 2022: 170-180.

Makris, M. and L. Renou, "Information Design in Multi-Stage Games," *Theoretical Economics*, forthcoming.

Roesler, A. and B. Szentes, "Buyer-optimal learning and monopoly pricing." *American Economic Review* 107(7), 2017: 2072-2080.

Terstiege, S. and C. Wasser, "Buyer-optimal extension-proof information." *Journal of Economic Theory* 188, 2020: 105070.

Wei, D., "Persuasion under costly learning," *Journal of Mathematical Economics*, 94, 2021: 102451.

B Online Appendix

Proof of Proposition 3. We first prove part (i). We show that $x^* > \mu$, and hence the result follows from Proposition 2(ii).

The buyer-optimal information structure in RS is $G_{q^{RS}}^{B^{RS}}$. By Claim 2 in the proof of Proposition 2, $x^* < B^{RS}$. Moreover, $x^* > q^{RS}$. Therefore, using the definition of $G_{q^{RS}}^{B^{RS}}(s)$,

$$\int_0^{x^*} G_{q^{RS}}^{B^{RS}}(s)ds = \int_0^{q^{RS}} 0ds + \int_{q^{RS}}^{x^*} \left(1 - \frac{q^{RS}}{s}\right) ds = x^* - q^{RS} - q^{RS} \ln\left(\frac{x^*}{q}\right).$$

Because $\int_0^x G_{q^{RS}}^{B^{RS}}(s)ds \le \int_0^x F(s)ds$ for all $x \in [0,1]$ and $\int_0^{x^*} G_{q^{RS}}^{B^{RS}}(s)ds = \int_0^{x^*} F(s)ds$, the function $\int_0^x \left(G_{q^{RS}}^{B^{RS}}(s) - F(s)\right) ds$ obtains a maximum at $x = x^*$. The first-order condition yields $G_{q^{RS}}^{B^{RS}}(x^*) - F(x^*) = 0$, hence

$$F(x^*) = G_{q^{RS}}^{B^{RS}}(x^*) = 1 - \frac{q^{RS}}{x^*}. \tag{B.1}$$

Because $\frac{dF}{dx}$ weakly decreases, F is concave. Thus, $F(\alpha x^* + (1-\alpha)0) \ge \alpha F(x^*) + (1-\alpha)F(0)$ for all $\alpha \in [0,1]$. Substitute $F(0) = 0$, $\alpha = x/x^*$, and (B.1) to obtain $F(x) \ge \frac{x}{x^*}\left(1 - \frac{q^{RS}}{x^*}\right)$ for all $x \in [0, x^*]$. Thus,

$$\int_0^{x^*} F(s)ds \ge \int_0^{x^*} \left[\frac{s}{x^*}\left(1 - \frac{q^{RS}}{x^*}\right)\right] ds = 0.5x^*\left(1 - \frac{q^{RS}}{x^*}\right).$$

Since $\int_0^{x^*} G_{q^{RS}}^{B^{RS}}(s)ds = \int_0^{x^*} F(s)ds$ (by definition), we have

$$\int_0^{x*} G_{q^{RS}}^{B^{RS}}(s)ds = x^* - q^{RS} - q^{RS} \ln\left(\frac{x^*}{q^{RS}}\right) \ge 0.5x^*\left(1 - \frac{q^{RS}}{x^*}\right) = 0.5x^* - 0.5q^{RS}.$$

Simplifying this inequality yields:

$$\frac{x^*}{q^{RS}} - 2\ln\left(\frac{x^*}{q^{RS}}\right) \ge 1.$$

This gives $\frac{x^*}{q^{RS}} \geq 3.513$. To see why, define $J(t) \equiv t - 2\ln(t)$ for $t \in [1, \infty)$. Note that $\frac{x^*}{q^{RS}} > 1$ and that $J(t)$ is strictly convex. Further, $J(1) = J(3.513) = 1$. Hence, $t > 1$ and $J(t) = 1$ implies that $t \geq 3.513$.

Defining $f_A = 0.57$, we also have

$$1 - F(x^*) = \int_{x^*}^{1} f(s)ds \geq f_A (1 - x^*).$$

Then (B.1) and $\frac{x^*}{q^{RS}} \geq 3.513$ yield $F(x^*) \geq 1 - \frac{1}{3.513}$, or $1 - F(x^*) \leq \frac{1}{3.513}$. Thus,

$$f_A (1 - x^*) \leq \frac{1}{3.513} = 0.2847 \tag{B.2}$$

yielding

$$x^* \geq 1 - \frac{0.2847}{f_A}.$$

Substituting $f_A = 0.57$ yields $x^* \geq 1 - \frac{0.2847}{0.57} > 0.5$, and, since f is weakly decreasing, we have $\mu \leq 0.5$. Thus, $x^* > 0.5 \geq \mu$, establishing part (i) of the proposition.

Next we prove part (ii) for $\Delta = 0.12$. We again show that $x^* > \mu$. By the proposition's premise, we have $F(x) \geq (1 - \Delta) x$ for all $x \in [0, 1]$. Hence

$$\int_0^{x^*} F(s)ds \geq \int_0^{x^*} [(1 - \Delta) s] \, ds = 0.5 (1 - \Delta) (x^*)^2. \tag{B.3}$$

By the proposition's premise, we also have $F(x^*) \leq (1 + \Delta) x^*$. Thus, (B.1) yields $(1 + \Delta) x^* \geq 1 - \frac{q^{RS}}{x^*}$, which gives $x^* \geq \frac{1 - \frac{q^{RS}}{x^*}}{1+\Delta}$. Plugging this into (B.3) (by replacing only one x^* on the right-hand side) yields

$$\int_0^{x^*} F(s)ds \geq 0.5 (1 - \Delta) x^* \frac{1 - \frac{q^{RS}}{x^*}}{1 + \Delta} = \frac{1 - \Delta}{2(1 + \Delta)} x^* \left(1 - \frac{q^{RS}}{x^*}\right). \tag{B.4}$$

Since $\int_0^{x^*} G_{q^{RS}}^{B^{RS}}(s)ds = \int_0^{x^*} F(s)ds$ (by definition), we have

$$\int_0^{x*} G_{q^{RS}}^{B^{RS}}(s)ds = x^* - q^{RS} - q^{RS} \ln\left(\frac{x^*}{q^{RS}}\right) \geq \frac{1 - \Delta}{2(1 + \Delta)} x^* \left(1 - \frac{q^{RS}}{x^*}\right).$$

Simplifying this inequality yields

$$\frac{x^*}{q^{RS}} - \frac{2+2\Delta}{1+3\Delta}\ln\left(\frac{x^*}{q^{RS}}\right) \geq 1.$$

Substituting $\Delta = 0.12$ into this equation yields $\frac{x^*}{q^{RS}} > 2.5$. To see why, define $\hat{J}(t) \equiv t - \frac{2+2\Delta}{1+3\Delta}\ln t = t - \frac{2.24}{1.36}\ln t$ for $t \in [1, \infty)$. Note that $\frac{x^*}{q} > 1$ and $\hat{J}(t)$ is strictly convex. Further, $\hat{J}(1) = 1$ and $\hat{J}(2.5) < 1$. Thus, $t > 1$ and $\hat{J}(t) = 1$ imply that $t > 2.5$. Then, analogous to (B.2), we have

$$(1-\Delta)(1-x^*) < \frac{1}{2.5} \Rightarrow x^* > 1 - \frac{\frac{1}{2.5}}{1-\Delta} = 0.545.$$

Next we bound the mean μ from above. μ is maximized when the mass is pushed to the right as far as possible. Given $1-\Delta \leq f(v) \leq 1+\Delta$, this occurs at a $\hat{v} \in (0,1)$ where $f(v) = 1+\Delta$ for all $v > \hat{v}$ and $f(v) = 1-\Delta$ for all $v < \hat{x}$. Probability conservation pins down $\hat{v}$:

$$\hat{v}(1-\Delta) + (1-\hat{v})(1+\Delta) = 1,$$

yielding $\hat{v} = 0.5$. Thus, the maximum possible mean is:

$$\mu_{\max} = 0.25\frac{1-\Delta}{2} + 0.75\frac{1+\Delta}{2} = 0.5 + \frac{\Delta}{4} = 0.53.$$

Thus, $x^* > 0.545 > \mu_{\max} \geq \mu$, establishing part (ii) of the proposition. $\square$

Proof of Remark 1: (i) For $p > \mu$, the payoff from no information is zero, as no purchase will be made. The payoff from acquiring information is $\int_p^1 (s-p)\, dL_{q^*}^{B(q^*)}(s) - C$, which decreases in p. Thus, if $C \geq \bar{C}$, then the payoff from acquiring information is less than zero, implying no incentives to acquire information:

$$\int_p^1 (s-p)\, dL_{q^*}^{B(q^*)}(s) - C \leq \int_\mu^1 (s-\mu)\, dL_{q^*}^{B(q^*)}(s) - C \leq \bar{C} - C \leq 0.$$

For $p < q^*$, the buyer has no incentive to acquire information because she will accept the trade regardless of whether or not information is acquired.

(ii) Follows from (i), Proposition 1, and the fact that endogenous information acquisition only adds a constraint on the buyer (requiring ex post incentive compatibility to not acquire information for some prices) that is slack if $C \geq \bar{C}$. □

Proof of Remark 2: Part (i): For $p < q^{PC}$, the buyer always accepts the offer and hence has no incentive to acquire information.

For $p > \mu = 1/2$, the payoff from no information is zero, as no purchase is made. The payoff from acquiring information is

$$\rho^*(p) \equiv \int_p^1 (s-p)\, dL^{PC}(s) - C.$$

From (9), this yields $\rho^*(k) = \frac{q^{PC}}{k}(1-k) - C = 0$. Thus, at price k, the buyer is indifferent to information acquisition. Furthermore, ρ^* strictly decreases in p for $p \in (\mu, 1)$. Hence, the buyer will not acquire information for $p > k$ (where $k > \mu$).

From (1), $L^{PC}(s)$ and the binary distribution have the same mean of $\mu = 0.5$ if

$$\begin{aligned} \int_0^1 L^{PC}(s)ds = 0.5 \Rightarrow \int_0^{q^{PC}} 0ds + \int_{q^{PC}}^k \left(1 - \frac{q^{PC}}{s}\right) ds + \int_k^1 \left(1 - \frac{q^{PC}}{k}\right) d(s) = 0.5 \\ \Rightarrow k - q^{PC} - q^{PC} \ln \frac{k}{q^{PC}} + \left(1 - \frac{q^{PC}}{k}\right)(1-k) = 0.5. \\ \Rightarrow 0.5 - \frac{q^{PC}}{k} - q^{PC} \ln \frac{k}{q^{PC}} = 0. \qquad \text{(B.5)} \end{aligned}$$

Note that (9) implies (B.5): plugging $C = \frac{q^{PC}(1-k)}{k}$ from the second part of (9) into the first part of (9) yields (B.5). Moreover, because the binary distribution is flat over $(0,1)$, (9) ensures that the binary distribution is a mean-preserving spread of $L^{PC}(s)$.

Part (ii): Differentiation of (9) yields that k strictly decreases in C. Further, the LHS of (B.5) strictly increases with k and strictly decreases with q^{PC}.[12] Thus, (B.5) implies that a strict decrease in k is accompanied by a strict decrease in q^{PC}. Thus,

[12]The derivative of the LHS of (B.5) with respect to k is $q^{PC}(1-k)/k^2 > 0$ and with respect to q^{PC} is $1 - 1/k - \ln(k/q^{PC}) < 0$. The inequalities hold since $0 < q^{PC} < k < 1$.

increasing C strictly decreases k and q^{PC}, so buyer payoff $(\mu - q^{PC})$ strictly increases.

Part (iii): Buyer payoff is $\mu - q^{PC}$, and buyer payoff in RS is $\mu - q^{RS}$. Thus, it suffices to show $q^{PC} < q^{RS}$ for all $C > 0$. First note that $q^{PC} = q^{RS}$ when $C = 0$. Part (ii) proved that q^{PC} strictly decreases with $C \in (0, \bar{C})$. Note that q^{PC} reaches its lowest value and becomes constant when $C \geq \bar{C}$. Thus, $q^{PC} < q^{RS}$ for all $C > 0$. $\square$